Grade 8 – Book A

(Teacher's Guidelines)

(Revised CAPS edition)

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ISBN 978-0-958443-31-9

Chapter A1

Integers

A1.1 Number systems and properties of integers:

Exercise 1: Date: ____________

Complete: * Natural numbers: N = { 1; 2; 3; 4; - - - }

* Whole numbers N_0 = { 0; 1; 2; 3; - - - }

* Integers: Z = { - - - ; -2; -1; 0; 1; 2; - - - }

The integers are expanded to include the fractions:

Rational numbers (ℚ): Include all fractions which can be written as $\frac{a}{b}$, with a and b as integers and b ≠ 0. This includes all finite and recurring decimal fractions.
E.g. $\frac{1}{3}$; $0,\dot{7}$; $-3\frac{5}{8}$; 2,34; $\sqrt{25}$; 9; $\sqrt[3]{27}$ etc.

Irrational numbers (ℚ'): Includes all infinite and non-recurring decimal fractions.
E.g. 3,68463….. ; π ; $\sqrt{10}$; $\sqrt[3]{4}$ etc.

Real numbers (ℝ) consist of all rational and irrational numbers in union: $\mathbb{Q} \cup \mathbb{Q}'$

Non-real numbers for example are: $\sqrt{-4}$; $\sqrt{-12}$ etc.
$\sqrt[3]{-8}$ and $\sqrt[5]{-243}$ however, are real numbers, because $\sqrt[3]{-8} = -2$ and $\sqrt[5]{-243} = -3$.

Properties of 1 and 0:

* $m \times 0 = 0$
* $0 \div m = 0$
* $m \div 0$ = undefined
* $m \times 1 = m$
* $m \div 1 = m$

Identity elements:

* 0 is the identity element of addition, because $m + 0 = m$
* 1 is the identity element of multiplication, because $m \times 1 = m$

Inverse:

* The sum of a number and its additive inverse is 0.
  E.g. **3** is the additive inverse of **-3**, because $3 + (-3) = 3 - 3 = 0$
* The multiplicative inverse (reciprocal) is the number multiplied with a certain number with a result of 1. E.g. the multiplicative inverse of **3** is $\frac{1}{3}$, because $3 \times \frac{1}{3} = \frac{3}{1} \times \frac{1}{3} = 1$

Other properties:

* Commutative operation: $m \times n = n \times m$ or $m + n = n + m$
* Associative operation: $(m \times n) \times p = m \times (n \times p)$ or $(m + n) + p = n + (m + p)$
* Distributive operation: $p \times (m + n) = p \times m + p \times n$ or $p \times (m - n) = p \times m - p \times n$

A1.2 Rules for divisibility:

Divisor:	Rules for divisibility:
2	Last digit must be an even number or a 0.
3	Sum of all the digits must be divisible by 3.
4	Two last digits must be divisible by 4.
5	Last digit must be 5 or 0.
6	Rules for divisibility for 2 and 3 must apply.
8	Last three digits must be divisible by 8.
9	Sum of all the digits must be divisible by 9.
10	Last digit must be 0.
11	Calculate the sums of the alternate digits. The difference between these sums must be 0, or it must be divisible by 11.

E.g. 1 Determine whether 10 527 is divisible by the numbers in the above table:

2: NO, because the number (10 527) does not end on an even number.
3: YES, because the sum of the digits, 1+0+5+2+7=15 is divisible by 3.
4: NO, because 27(10 5<u>27</u>) is not divisible by 4.
5: NO, because the number does not end on a 5 or a 0.
6: NO, because the rule of divisibility for 2 does not apply.
8: NO, because the last three digits, 527, are not divisible by 8.
9: NO, because the sum of the digits viz. 1+0+5+2+7=15 is not divisible by 9.
10: NO, because the last digit is not 0.
11: YES, because the difference between 1+5+7=<u>13</u> and 0+2=<u>2</u> with 13 - 2 = 11.

Exercise 2: Date: ____________________

Determine whether the following numbers are divisible by the numbers in the above table:

(1) 1 275:
2: No, end on uneven.
3: Yes, 1+2+7+5=15 and 15 divisible by 3.
4: No, 75 not divisible by 4.
5: Yes, end on 5.
6: No, not divisible by 2.
8: No, not divisible by 4.
9: No, 1+2+7+5=15 and 15 is not divisible by 9.
10: No, last digit not 0.
11: No, (1+7)-(2+5)=8-7=1 and that is not 0 or 11.

(2) 2 772:
2: Yes, ends on even.
3: Yes, 2+7+7+2=18 and 18 divisible by 3.
4: Yes, 72 divisible by 4.
5: No, do not end on 0 or 5.
6: Yes, divisible by 2 and 3.
8: No, 772 not divisible by 8.
9: Yes, 2+7+7+2=18 and 18 is divisible by 9.
10: No, last digit not 0.
11: Yes, (2+7)-(7+2)=0.

(3) 7 920: 2: yes, ends on 0.	8: yes, 920 is divisible by 8.
3: yes, 7+9+2+0 = 18 and 18 divisible by 3.	9: yes, 7+9+2+0 = 18 and 18 divisible by 9.
4: yes, 20 divisible by 4.	10: yes, ends on 0.
5: yes, ends on 0.	11: yes, (7+2)−(9+0) = 0.
6: yes, divisible by 2 and 3.	

☺ **A certain number is divisible by 2, 3, 5 and 11. This number is not divisible by 8 and 9, but it is divisible by 4. Determine the smallest number that meets these conditions.**

4 × 3 × 5 × 11 = 660

(4 is also divisible by 2!)

A1.3 Factors:

E.g. 2 The factors of 10 are: $F_{10} = \{1 ; 2 ; 5 ; 10\}$

Exercise 3: Date: __________

Complete:

(1) $F_{20} = \{1; 2; 4; 5; 10; 20\}$

(2) $F_{16} = \{1; 2; 4; 8; 16\}$

(3) $F_{5} = \{1; 5\}$

(4) $F_{32} = \{1; 2; 4; 8; 16; 32\}$

(5) $F_{15} = \{1; 3; 5; 15\}$

(6) $F_{28} = \{1; 2; 4; 7; 14; 28\}$

(7) $F_{12} = \{1; 2; 3; 4; 6; 12\}$

(8) $F_{7} = \{1; 7\}$

(9) $F_{36} = \{1; 2; 3; 4; 6; 9; 12; 18; 36\}$

(10) $F_{11} = \{1; 11\}$

A1.4 Multiples:

E.g. 3 The multiples of 10 are: $M_{10} = \{10 ; 20 ; 30 ; \ldots\ldots\ldots\ldots\}$

Exercise 4: Date: ____________

Complete:

(1) M_6 = {6; 12; 18; ---}

(2) M_{20} = {20; 40; 60; ---}

(3) M_7 = {7; 14; 21; ---}

(4) M_{12} = {12; 24; 36; ---}

(5) M_{36} = {36; 72; 108; ---}

(6) M_9 = {9; 18; 27; ---}

(7) M_{35} = {35; 70; 105; ---}

(8) M_{16} = {16; 32; 48; ---}

(9) M_{11} = {11; 22; 33; ---}

(10) M_3 = {3; 6; 9; ---}

☺ **Determine the multiples of 6 which are also factors of 120.**

{6; 12; 24; 30; 60; 120}

A1.5 Prime numbers and compound numbers:

Exercise 5: Date: ____________

Complete:

(1) The definition of a prime number is: all natural numbers with only 2 factors → 1 and the number itself.

(2) The smallest prime number is: 2

(3) The only even prime number is: 2

(4) The definition of a compound number is: all natural number with more than 2 factors.

(5) Which natural number is **neither** a prime number nor a compound number? 1

(6) Which natural numbers smaller than 50, are prime numbers?
(Do the following: Encircle 2 ; 3 ; 5 and 7 and cross out all the multiples of 2 ; 3 ; 5 ; and 7. The numbers which are left will be the prime numbers. Remember to cross out 1 as well!)

~~1~~	(2)	(3)	~~4~~	(5)	~~6~~	(7)	~~8~~	~~9~~	~~10~~
(11)	~~12~~	(13)	~~14~~	~~15~~	~~16~~	(17)	~~18~~	(19)	~~20~~
~~21~~	~~22~~	(23)	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	(29)	~~30~~
(31)	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	(37)	~~38~~	~~39~~	~~40~~
(41)	~~42~~	(43)	~~44~~	~~45~~	~~46~~	(47)	~~48~~	~~49~~	~~50~~

∴ The prime numbers smaller than 50 are: {2 ; 3 ; 5 ; 7 ; 11 ; 13 ; 17 ; 19 ; 23 ; 29 ; 31 ; 37 ; 41 ; 43 ; 47}

A1.6 Prime factors:

E.g. 4 The factors of 6 are: $F_6 = \{1 ; 2 ; 3 ; 6\}$
∴ The prime factors of 6 are: 2 and 3. (In other words they are factors which are prime numbers)

E.g. 5 The factors of 20 are: $F_{20} = \{1 ; 2 ; 4 ; 5 ; 10 ; 20\}$
∴ The prime factors of 20 are: 2 and 5.

E.g. 6 Determine the prime factors of 60:

2	60
2	30
3	15
5	5
	1

$\therefore 60 = 2 \times 2 \times 3 \times 5$
$= 2^2 \times 3 \times 5$

Exercise 6: Date: ____________________

Determine the prime factors of:

(1)

2	12
2	6
3	3
	1

$12 = 2^2 \times 3$

(2)

5	35
7	7
	1

$35 = 3 \times 7$

(3)

2	32
2	16
2	8
2	4
2	2
	1

$32 = 2^5$

(4)

2	44
2	22
11	11
	1

$44 = 2^2 \times 11$

(5)

2	48
2	24
2	12
2	6
3	3
	1

$48 = 2^4 \times 3$

(6)

3	27
3	9
3	3
	1

$27 = 3^3$

(7)

2	56
2	28
2	14
7	7
	1

$56 = 2^3 \times 7$

(8)

2	100
2	50
5	25
5	5
	1

$100 = 2^2 \times 5^2$

(9)

2	18
3	9
3	3
	1

$18 = 2 \times 3^2$

(10)

2	168
2	84
2	42
3	21
7	7
	1

$168 = 2^3 \times 3 \times 7$

(11)

2	588
2	294
3	147
7	49
7	7
	1

$588 = 2^2 \times 3 \times 7^2$

(12)

2	450
3	225
3	75
5	25
5	5
	1

$450 = 2 \times 3^2 \times 5^2$

A1.7 LCM and HCF:

LCM = Lowest common multiple.
HCF = Highest common factor.

E.g.7 Determine the LCM of 8 ; 12 and 20
[First determine the prime factors!]

$8 = \boxed{2 \times 2} \times 2$
$12 = \boxed{2 \times 2} \times 3$
$20 = \boxed{2 \times 2} \times 5$ $\quad \therefore LCM = \boxed{2 \times 2} \times 2 \times 3 \times 5 = \underline{\underline{120}}$

E.g.8 Determine the HCF of 36 and 60.
[First determine the prime factors!]

$36 = \boxed{2 \times 2 \times 3} \times 3$
$60 = \boxed{2 \times 2 \times 3} \times 5$ $\quad \therefore HCF = \boxed{2 \times 2 \times 3} = \underline{\underline{12}}$

Exercise 7: Date: ____________

(1) Determine the HCF of the following by finding the prime factors first:

(a) 14 = 2 x 7
21 = 3 x 7 ∴HCF = 7
35 = 5 x 7 = ______

(b) 27 = 3 x 3 x 3
45 = 3 x 3 x 5 ∴HCF = 3 x 3
72 = 2 x 2 x 2 x 3 x 3 = 9

(c) 12 = 2 x 2 x 3 ∴HCF = 2 x 2 x 3
168 = 2 x 2 x 2 x 3 x 7 = 12

(d) 38 = 2 x 19
57 = 3 x 19 ∴HCF = 19
95 = 5 x 19 = ______

(e) 10 = 2 x 5
15 = 3 x 5 ∴HCF = 5
105 = 3 x 5 x 7 = ______

(2) Determine the LCM of the following by finding the prime factors first:

(a) 6 = 2 x 3

12 = 2 x 2 x 3 ∴LCM = 2 x 3 x 2 x 3

18 = 2 x 3 x 3 = 36

(b) 8 = 2 x 2 x 2 ∴LCM = 2 x 2 x 2 x 5

20 = 2 x 2 x 5 = 40

(c) 2 = 2

6 = 2 x 6 ∴LCM = 2 x 3 x 11

11 = 11 = 66

(d) 21 = 3 x 7 ∴LCM = 7 x 3 x 7

49 = 7 x 7 = 147

(e) 3 = 3

9 = 3 x 3

12 = 2 x 2 x 3 ∴LCM = 3 x 2 x 2 x 3 x 5

60 = 2 x 2 x 3 x 5 = 180

(f) 15 = 3 x 5

45 = 3 x 3 x 5 ∴LCM = 3 x 5 x 3 x 2 x 3

270 = 2 x 3 x 3 x 3 x 5 = 270

(3) Determine the LCM and the HCF:

(a) 16 = 2 x 2 x 2 x 2 ∴LCM = 2 x 2 x 2 x 2 x 3 x 7

48 = 2 x 2 x 2 x 2 x 3 = 336

56 = 2 x 2 x 2 x 7 ∴HCF = 2 x 2 x 2

= 8

(b) 5 = 5 ∴LCM = 5×2×2×2×3

24 = 2×2×2×3 = 120

∴HCF = 1

☺ **Mounting boards of (a) 24cm², (b) 36cm² and (c) 18cm² have to be cut. Determine the size of the smallest mounting board panel (determine the area) that should be used so that any combination of (a), (b) and/or (c) can be cut from it, without wasting any board. [Make use of prime factors**

24 = 2 × 2 × 2 × 3

36 = 2 × 2 × 3 × 3

18 = 2 × 3 × 3

∴ LCM = 2 × 3 × 2 × 3 × 2 = 72 cm²

2	24
2	12
2	6
3	3
	1

2	36
2	18
3	9
3	3
	1

2	18
3	9
3	3
	1

A1.8 <u>Square roots and cube roots</u>:

E.g.9 Determine the following by using prime factors:

(a) $\sqrt{784}$ *(b)* $\sqrt[3]{3\,375}$

****

2	784
2	392
2	196
2	98
7	49
7	7
	1

3	3 375
3	1 125
3	375
5	125
5	25
5	5
	1

$\therefore\ 784 = 2\times2\times2\times2\times7\times7 = 2^2 \times 2^2 \times 7^2$

$\therefore\ \sqrt{784} = 2 \times 2 \times 7 = \underline{28}$

$\therefore\ 3\,375 = 3\times3\times3\times5\times5\times5 = 3^3 \times 5^3$

$\therefore\ \sqrt[3]{3\,375} = 3 \times 5 = \underline{15}$

<u>Exercise 8</u>: Date: ____________

Calculate: (by using prime factors)

(1) $\sqrt{576}$ = $\sqrt{2\times2\times2\times2\times2\times2\times3\times3}$

= $\sqrt{2^2 \times 2^2 \times 2^2 \times 3^2}$

= 2 × 2 × 2 × 3

= 24

2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

(2) $\sqrt[3]{343}$ = $\sqrt[3]{7\times7\times7}$

= $\sqrt[3]{7^3}$

= 7

7	343
7	49
7	7
	1

(3) $\sqrt{225}$ = $\sqrt{3\times3\times5\times5}$

= $\sqrt{3^2\times5^2}$

= 3×5

= 15

3	225
3	75
5	25
5	5
	1

(4) $\sqrt{1\,024}$ = $\sqrt{2\times2\times2\times2\times2\times2\times2\times2\times2\times2}$

= $\sqrt{2^2\times2^2\times2^2\times2^2\times2^2}$

= $2\times2\times2\times2\times2$

= 32

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(5) $\sqrt[3]{1\,000}$ = $\sqrt[3]{2\times2\times2\times5\times5\times5}$

= $\sqrt[3]{2^3\times5^3}$

= 2×5

= 10

2	1 000
2	500
2	250
5	125
5	25
5	5
	1

(6) $\sqrt[3]{4\,096}$ = $\sqrt[3]{2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2}$

= $\sqrt[3]{2^3\times2^3\times2^3\times2^3}$

= $2\times2\times2\times2$

= 16

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(7) $\sqrt[3]{729}$ = $\sqrt[3]{3\times3\times3\times3\times3\times3}$

= $\sqrt[3]{3^3\times3^3}$

= 3×3

= 9

3	729
3	243
3	81
3	27
3	9
3	3
	1