

Grade 10 – Book B **(Revised CAPS edition)**

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Chapter B1

Functions and relations

B1.1 Revision – grade 9:

Study the following table:

Number of balls bought (N):	1	2	3	4	8	12
Total cost (C):	R2	R4	R6			

In grade 9 we already saw that we could complete the table by determining the relationship between N and C and then also by compiling a formula or equation. In this case the formula is:
 $C = 2 \times N$

We also saw that C is called the dependent variable and N the independent variable, because C's value depends on N's value. Tables are also represented graphically.

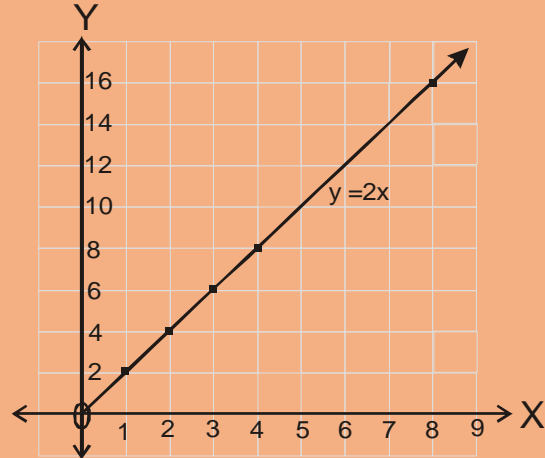
With functions and relations, x is used more times as the independent variable and y as the dependent variable. In other words, the table above will be as follows, with equation $y = 2x$.

Completed table:

Number of balls bought (x):	1	2	3	4	8
Total cost (y):	R2	R4	R6	R8	R16

From the table we can write down the following ordered pairs and then represent them graphically:

- (1 ; 2)
- (2 ; 4)
- (3 ; 6)
- (4 ; 8)
- (8 ; 16)



These ordered pairs can be written as a set:

$$\{(1 ; 2) ; (2 ; 4) ; (3 ; 6) ; (4 ; 8) ; (8 ; 16)\}$$

From the above set, we can also write down the domain (all the possible x -coordinates in the set) and the range (all the possible y -coordinates in the set):

Domain: $x \in \{1 ; 2 ; 3 ; 4 ; 8\}$

Range: $y \in \{2 ; 4 ; 6 ; 8 ; 16\}$

B1.2 Function concept:

A set of ordered pairs, as above, is then called a **relation**.

When each element in the domain is linked with only **one** element in the range, the relation is called a **function**.

The following are examples of relations: $\{(1; 1); (1; 2); (1; 3); (2; 1); (2; 2); (2; 3)\}$
 $\{(-1; 0); (0; 1); (1; 2); (2; 1); (3; -1)\}$
 $\{(1; 2); (2; 4); (3; 6); (4; 8); (8; 16)\}$

However, only the following are functions: $\{(-1; 0); (0; 1); (1; 2); (2; 1); (3; -1)\}$
 $\{(1; 2); (2; 4); (3; 6); (4; 8); (8; 16)\}$

Because the x values are not repeated.

B1.3 Notations:

Because one frequently works with more than one function, the different functions are named. We use different notations (ways of writing) and the letters of the alphabet can be used to give the different functions “names”.

B1.3.1 Set-builder notation:

$$f = \{(x; y) / y = x + 1\} \quad \text{or} \quad g = \{(x; y) : y = -2x; x > 0\}$$

It reads: “ f is the set of ordered pairs $(x; y)$ with y equal to $x + 1$ ”

or “ g is the set of ordered pairs $(x; y)$ with y equal to $-2x$ where x is greater than 0”

The functions under discussion here are then: $y = x + 1$ and $y = -2x$

B1.3.2 Function notation:

$$f(x) = x + 1 \quad \text{or} \quad g(x) = -2x$$

Thus, it is the same as: $y = x + 1$ or $y = -2x$

This means: in stead of writing “ y ”, $f(x)$, $g(x)$, $h(x)$ etc. is written. The f and the g therefore, indicates the “name” of each specific function, so one can differentiate between the different functions, where x indicates the elements of the domain that must be substituted into the equation to obtain the range.

B1.3.3 Map notation:

$f: x \rightarrow x + 1$	or	$g: x \rightarrow -2x$
Thus, it is the same as: $y = x + 1$	or	$y = -2x$
It reads: “ f maps (takes) x onto $x + 1$ ”	or	“ g maps x onto $-2x$ ”

E.g.1 $f: x \rightarrow -3x + 2$

(a) Write down the expression for $f(x)$.

(b) Calculate: (i) $f(-1)$ (ii) $f(m)$ (iii) x if $f(x) = 8$

(a) $f(x) = -3x + 2$

(b) (i) $f(-1) = -3(-1) + 2$

$$f(-1) = 3 + 2$$

$$f(-1) = 5$$

$$\therefore y = 5 \text{ if } x = -1$$

$$\therefore \text{Ordered number pair is: } (-1; 5)$$

(ii) $f(x) = -3x + 2$

$$f(m) = -3(m) + 2$$

$$f(-1) = -3m + 2$$

$$\therefore y = -3m + 2 \text{ if } x = m \quad \therefore \text{Ordered number pair is: } (m; -3m + 2)$$

(iii) $f(x) = -3x + 2$

$$8 = -3x + 2$$

$$3x = 2 - 8$$

$$3x = -6$$

$$x = -2$$

$$\therefore x = -2 \text{ if } f(x) = 8 = y \quad \therefore \text{Ordered number pair is: } (-2; 8)$$

E.g.2 $p = \{(x; y) / y = x^2 - 1\}$ and $q(x) = \frac{x+1}{2}$

Calculate: (a) $q(0)$

(b) $p\left(\frac{1}{2}\right)$

(c) $p(-2) + q(3)$

(d) x if $p(x) = q(x)$

(a) $q(x) = \frac{x+1}{2}$

(b) $p(x) = x^2 - 1$

$$q(0) = \frac{0+1}{2}$$

$$q(0) = \frac{1}{2}$$

$$\therefore \left(0; \frac{1}{2}\right)$$

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 1$$

$$p\left(\frac{1}{2}\right) = \frac{1}{4} - \frac{4}{4}$$

$$p\left(\frac{1}{2}\right) = \frac{-3}{4}$$

$$\therefore \left(\frac{1}{2}; \frac{-3}{4}\right)$$

Chapter B2

Linear function

B2.1 Standard form:

The standard form for a straight line is: $y = mx + c$

Where m represents the line's gradient and c the line's y intercept!

The gradient (m) is the incline of a line, $\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$

B2.2 Graphic representations:

As already seen in grade 9, the linear function or straight line can then be sketched in different ways, namely table method, dual intercept method or gradient intercept method.

E.g.1 Sketch f by using the table method and write down the range of f :

$$f = \{(x; y) : -x + 1 = y; x \in (-1; 4]\}$$

\therefore Sketch $y = -x + 1$ for the interval $(-1; 4]$.

\therefore The domain is $x \in (-1; 4]$ with -1 not included and 4 included!

$$\begin{aligned} y &= -(-1) + 1 \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

$$\begin{aligned} y &= -(0) + 1 \\ &= 0 + 1 \\ &= 1 \end{aligned}$$

$$\begin{aligned} y &= -(4) + 1 \\ &= -4 + 1 \\ &= -3 \end{aligned}$$

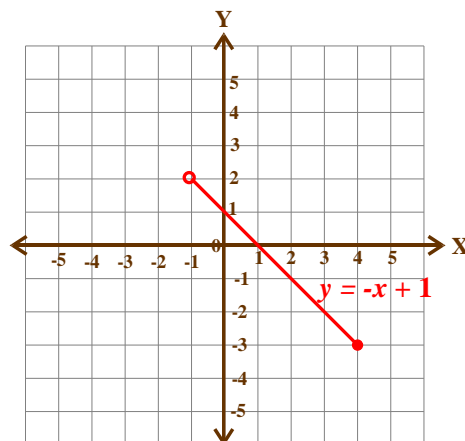
x	-1	0	4
y	2	1	-3

Ordered pairs:

$$(-1; 2)$$

$$(0; 1)$$

$$(4; -3)$$



The range of f : $R_f \in [-3; 2)$

E.g. 2 Sketch the following by using the dual intercept method: $2x - 4y = 8$

x -intercept: ($y = 0$)

$$\therefore 2x - 4(0) = 8$$

$$2x - 0 = 8$$

$$2x = 8$$

$$x = 4 \quad \therefore (4; 0)$$

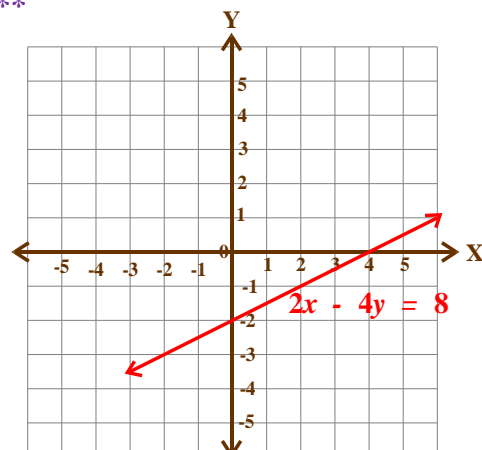
y -intercept: ($x = 0$)

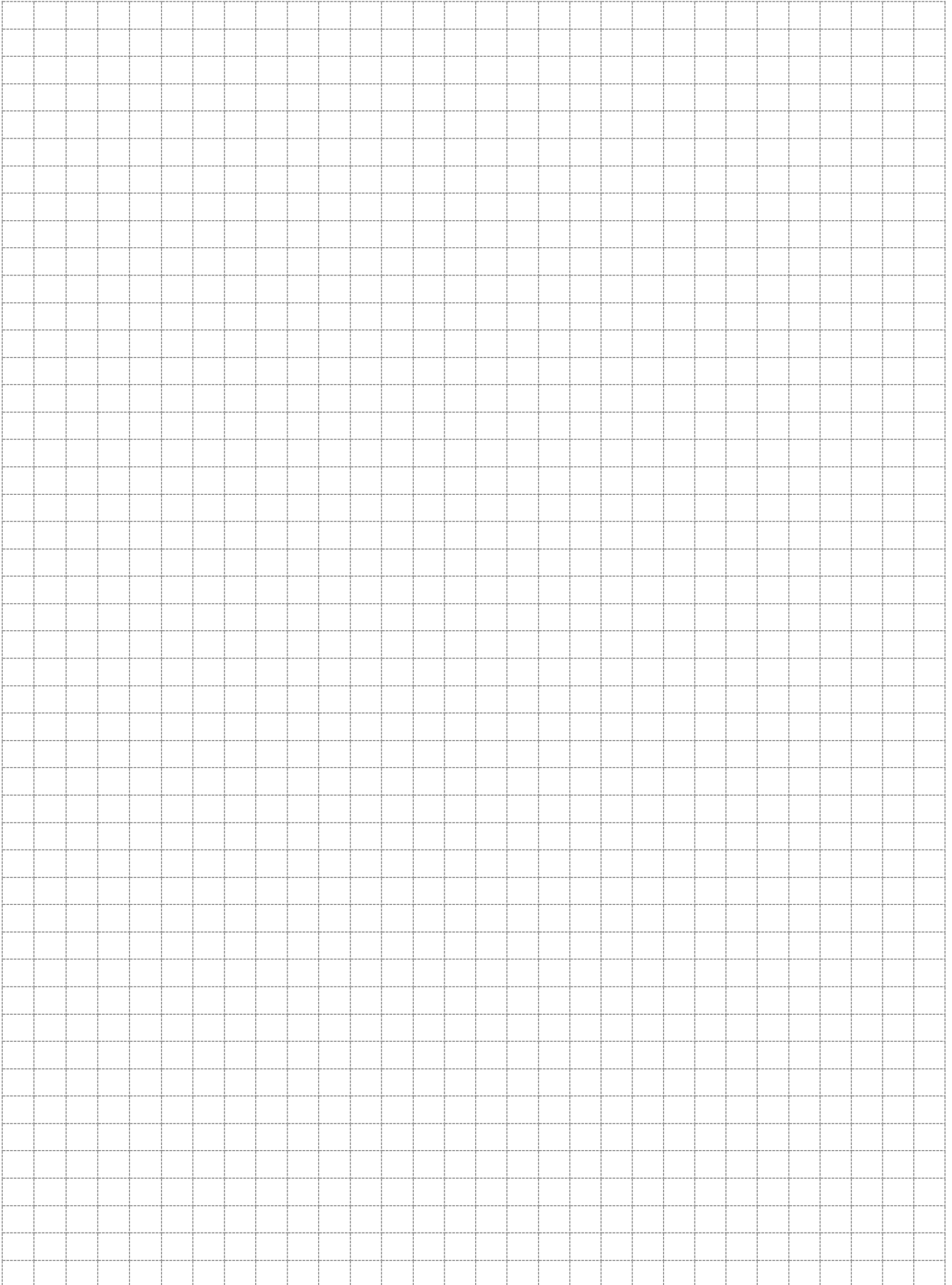
$$\therefore 2(0) - 4y = 8$$

$$0 - 4y = 8$$

$$-4y = 8$$

$$y = -2 \quad \therefore (0; -2)$$





E.g.3 (a) Sketch the following by using the gradient intercept method: $f(x) = -2x$
(b) Sketch on the same Cartesian plane: $g(x) = 3$

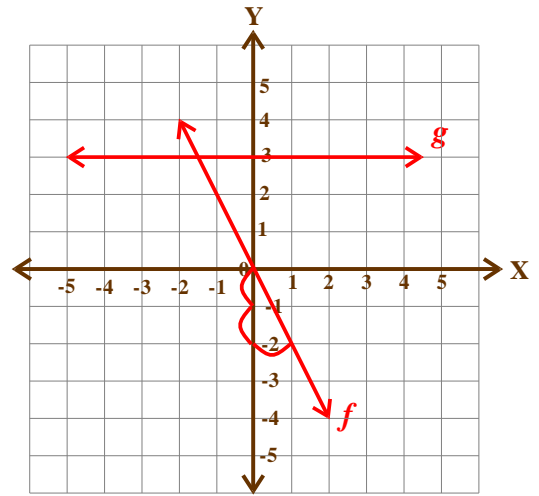
(a) \therefore Sketch $y = -2x + 0$

\therefore y- intercept: $c = 0$

and gradient: $m = -2$

$\therefore m = \frac{-2}{+1} = \frac{y\text{-difference}}{x\text{-difference}}$

(b) If $g(x) = 3$, then $y = 3$



Exercise 1:

Date: _____

(1) Sketch the following linear functions by using any method:

(a) $y = -x + 3$

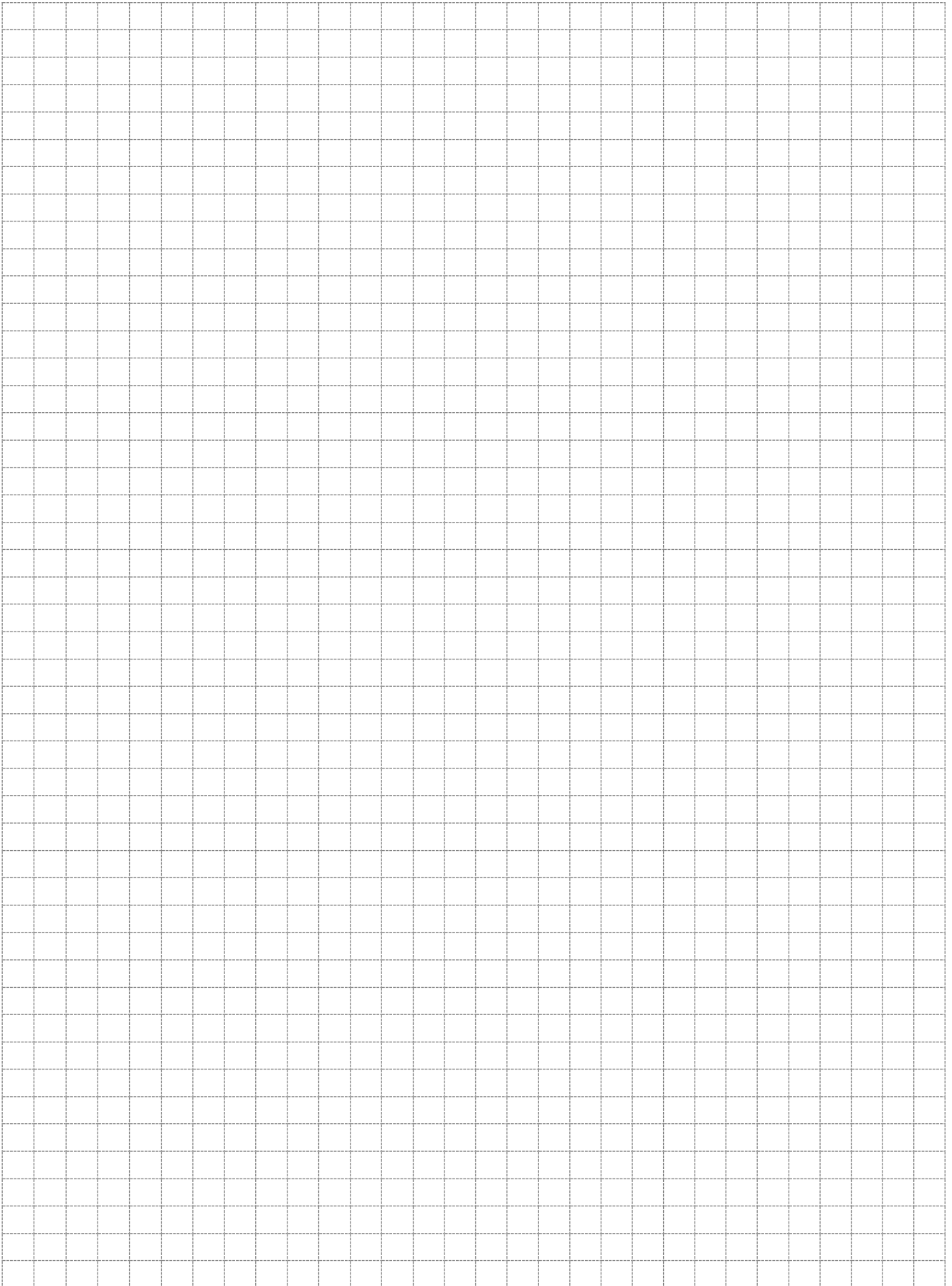
(b) $3x + 2y = -6$

(c) $3y = x$

(d) $f: x \rightarrow 4$

(e) $g(x) = -\frac{1}{2}x - 3$

(f) $h = \{(x; y) : y - 4 = 2x\}$



(2) Sketch the following linear functions by using any method and write down each function's domain and range:

(a) $f = \{(x; y) / 3y + 3 = x; x \in (-3; 6]\}$ (b) $g: x \rightarrow -2x$

(3) (a) Sketch the following linear functions on the same Cartesian plane. First write the functions in standard form.

$2y = 4x - 6$; $x = \frac{1}{2}y$ and $y - 2x = 1$

(b) What do you notice from the functions in (a)'s gradients? _____

(c) What do you notice from the line graphs in (a)? _____

(d) What deduction can you make from (b) and (c)? _____

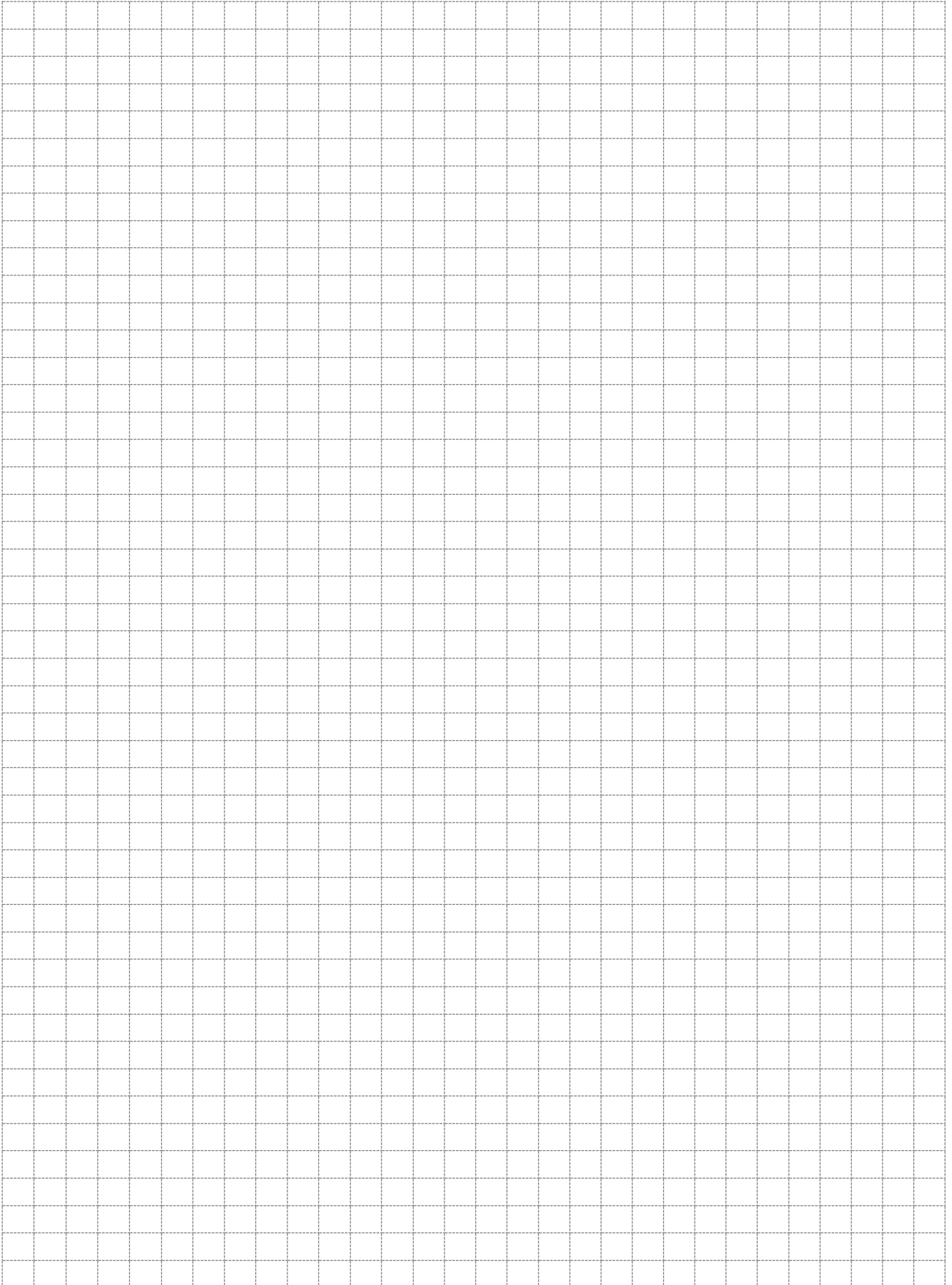
(4) (a) Sketch the following linear functions on the same Cartesian plane. First write the functions in standard form.

$2y = -x + 4$ and $y - 2x + 2 = 0$

(b) What do you notice from the functions in (a)'s gradients? _____

(c) What do you notice from the line graphs in (a)? _____

(d) What deduction can you make from (b) and (c)? _____



B2.3 Graphic solutions of simultaneous equations:

E.g.4 Determine the simultaneous solution to the following equations by first drawing the graphs:
 $2x - 1 = y$ and $8 - 2y = x$

Sketch $y = 2x - 1$:

$$m = 2 = \frac{2}{1} = \frac{y\text{-difference} \uparrow}{x\text{-difference} \leftrightarrow}$$

$$c = -1$$

Sketch $8 - 2y = x$:

x - intercept: ($y = 0$)

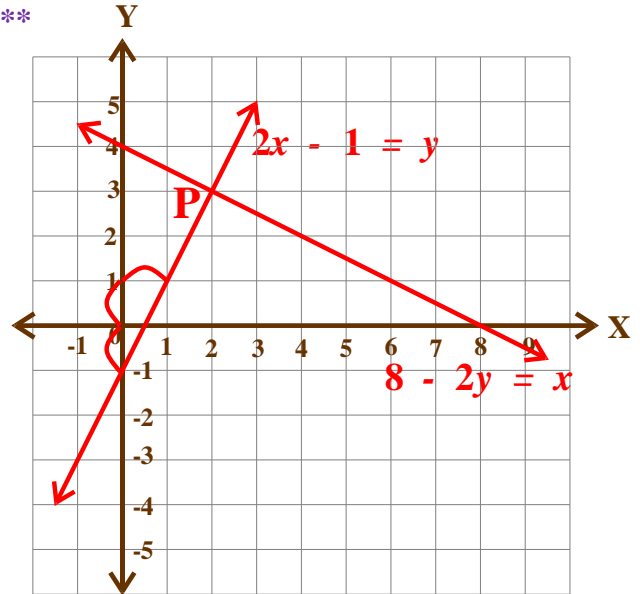
$$\therefore 8 - 2(0) = x \quad \therefore x = 8$$

y - intercept: ($x = 0$)

$$\therefore 8 - 2y = 0$$

$$8 = 2y \quad \therefore y = 4$$

\therefore Simultaneous solution is at $P(2 ; 3)$



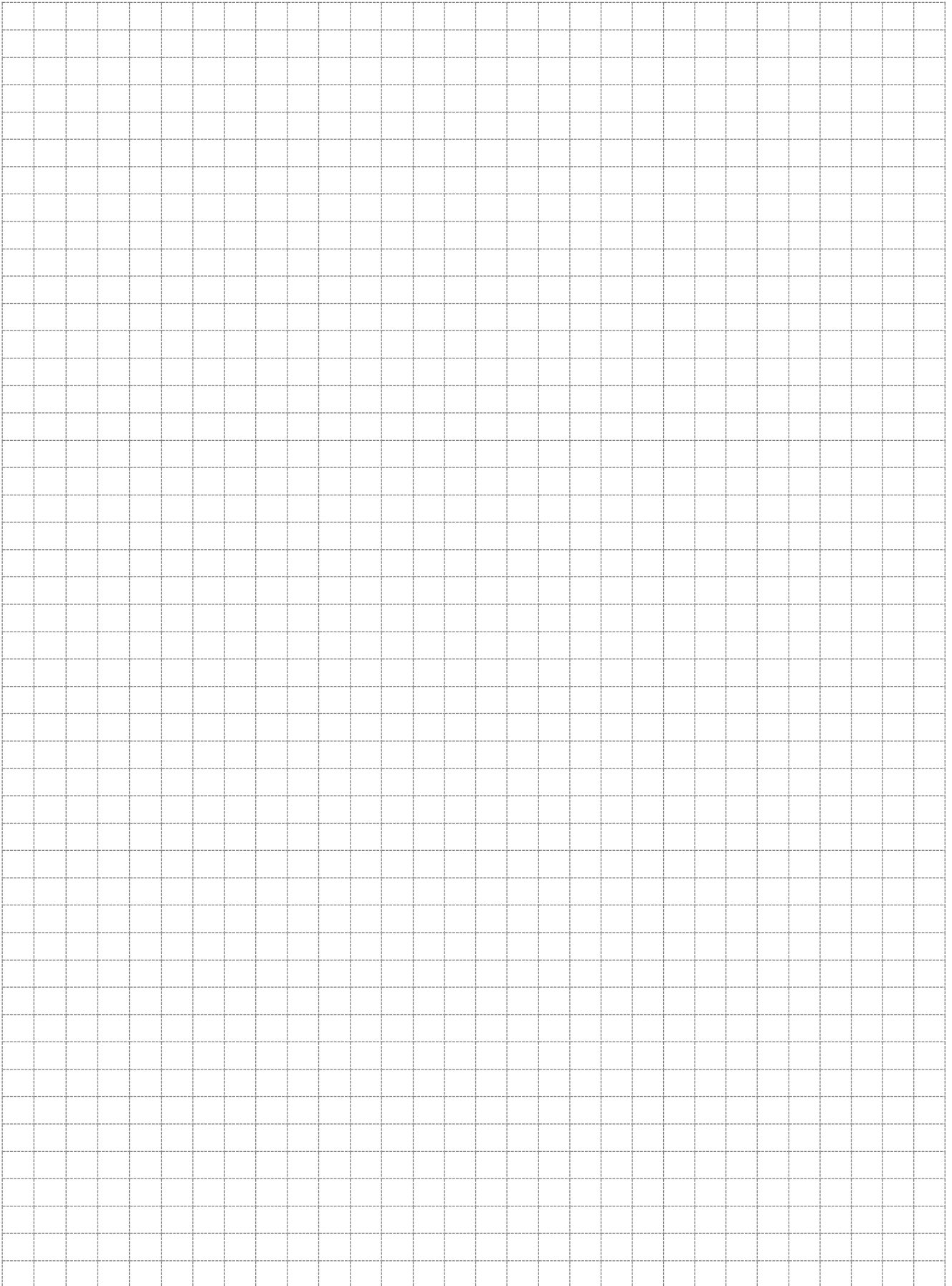
Exercise 2:

Date: _____

(1) Determine the simultaneous solution to the following equations by first drawing the graphs:

(a) $x + y = 1$ and $2x - y = 5$

(b) $3x - 4 = 2y$ and $9x = 6y$



(2) Determine the solution graphically: $\{(x; y) / 2x = -y - 1\} \cap \{(x; y) / 2y = 4x + 6\}$

B2.4 Parallel and perpendicular lines:

Therefore we can deduce the following from exercise 1:

From no. (3): **Parallel lines have the same gradients.** $\therefore m_1 = m_2$

From no. (4): **Perpendicular lines have reversed gradients.** $\therefore m_1 \times m_2 = -1$

E.g.5 Determine whether the following lines are parallel to each other, perpendicular to each other or neither of the two: $3x + y = 7$ and $-2y = 6x$

$$y = -3x + 7$$

$$y = \frac{-6x}{2} = -3x$$

$$\therefore m_1 = -3$$

$$\therefore m_2 = -3$$

\therefore The lines are parallel, because $m_1 = m_2$.

E.g.6 Calculate the value(s) of p if $2y + 3x = 2$ and $y - 2px + 3 = 0$ are perpendicular to each other.

$$2y + 3x = 2$$

and

$$y - 2px + 3 = 0$$

$$\therefore 2y = -3x + 2$$

$$\therefore y = 2px - 3$$

$$y = \frac{-3}{2}x + 1$$

$$\therefore m_1 = \frac{-3}{2}$$

$$\therefore m_2 = 2p$$

But the lines are perpendicular to each other, $\therefore m_1 \times m_2 = -1$

$$\frac{-3}{2} \times \frac{2p}{1} = -1$$

$$\therefore -3p = -1$$

$$\therefore p = \frac{-1}{-3} = \frac{1}{3}$$

Exercise 3:

Date: _____

(1) Determine whether the following lines are parallel to each other, perpendicular to each other or neither of the two:

(a) $x + y + 1 = 0$ and $x - y - 3 = 0$

(b) $2x + y = 3$ and $4x + 6 = 2y$

(c) $\frac{1}{3}x = y$ and $x - 3y = 4$

(d) $6y + 4x = -3$ and $1 + 3x = 2y$

(2) Calculate the value(s) of k if:

(a) $3x + 4y = -2$ and $6kx + 3 + y = 0$ are parallel to each other.

(b) $y - 2kx = 6$ and $4x - y = 1$ are perpendicular to each other.
