Grade 11 – Book A TG

(CAPS Edition)

CONTENTS:

This book was compiled and processed by E.J. Du Toit in 2008.
Caps edition in 2012.

Contact number: 086 618 3709 (Fax!)

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ISBN 978-1-919957-76-0

Chapter A1

Number systems and exponents

A1.1 Number systems:

Exercise 1: Date:______________

(1) Complete:

* Natural numbers: $\mathbb{N} = \{1; 2; 3; ...\}$
* Whole numbers: $\mathbb{N}_0 = \{0; 1; 2; 3; ...\}$
* Integers: $\mathbb{Z} = \{...; -2; -1; 0; 1; 2; ...\}$
* Rational numbers: $\mathbb{Q} = \{\frac{a}{b} / a, b \in \mathbb{Z}; b \neq 0\}$
* Real numbers: $\mathbb{R} = $ {rational numbers} ∪ {irrational numbers}

(2) Write three examples of Irrational numbers: $\sqrt{7}$; $\sqrt[3]{4}$; 2,14... ; π etc.

(3) Consider: $x(x - 6)(x^2 - 5)(2x^2 + x - 3) = 0$.
Solve for x and write the value(s) of x for which the solution of the equation will have:
(a) irrational roots.
(b) natural roots.
(c) integral roots.

$x(x-6)(x^2-5)(2x^2+x-3)=0$

$x=0$ or $x=6$ or $x^2=5$ or $(2x+3)(x-1)=0$

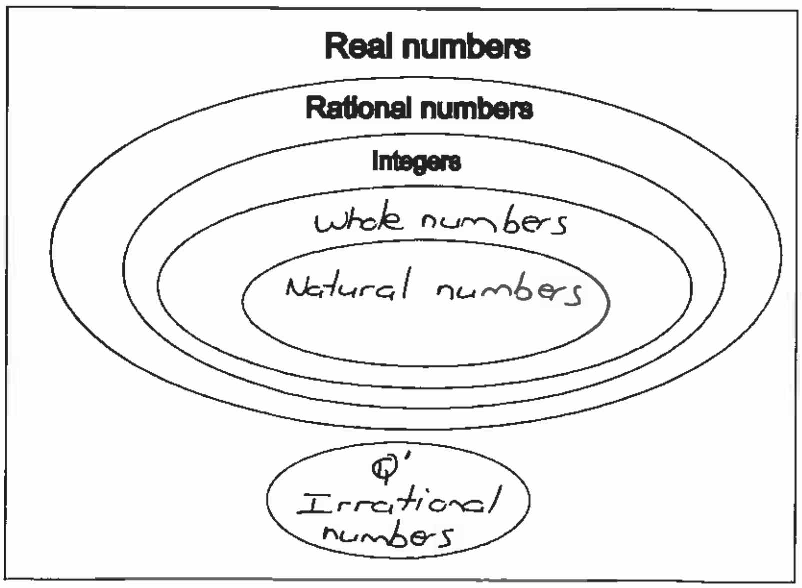
$x=\pm\sqrt{5}$ $x=-\frac{3}{2}$ or $x=1$

(a) $x = \pm\sqrt{5}$

(b) $x=6$ or $x=1$

(c) $x=0$ or $x=6$ or $x=1$

(4) Complete the following diagram which presents the system of real numbers:



A1.2 Non-Real numbers:

Examples of non-real numbers: $\sqrt{-2}$: $\sqrt{-9}$ or $\sqrt[4]{-5}$

But not $\sqrt[3]{-8}$, because -2 × -2 × -2 = -8 ∴ $\sqrt[3]{-8}$ = -2

Exercise 2: Date:____________

(1) Determine whether the following numbers are real or non-real. If real, indicate whether the number is rational or irrational:

(a) 7 Real → rational

(b) $-\sqrt{3}$ Real → irrational

(c) π Real → irrational

(d) $\sqrt{-16}$ Non-real

(e) $0,\dot{3}$ Real → rational

(f) $\frac{12}{36}$ Real → rational

(g) $\sqrt[3]{-125} = -5$ Real →

(h) $1 + \sqrt{9}$ $= 1 + 3 = 4$ Real → rational

(i) $\sqrt{(-2)^3}$ $= \sqrt{-8}$ Non-Real

(j) 0 Real → rational

(2) State whether the following statements are true or false:

(a) The product of two integers is always an integer again. True

(b) The product of two irrational numbers is always an irrational number. False
E.g. $\sqrt{2} \times \sqrt{2} = 2$

(c) If m is a natural number, $\sqrt{4}\,m$ will also be a natural number. True
$= 2m$

(d) The difference between two rational numbers is always a rational number. True

(e) The quotient of a rational number and an irrational number will always be rational. False E.g. $\frac{2}{\sqrt{2}}$ → irrational

(3) For which values of x will the following statements be: (i) undefined (ii) non-real

(a) $\frac{x+3}{x}$: (i) $x = 0$ (ii) None

(b) $\sqrt{x-1}$: (i) None (ii) $x - 1 < 0$ ∴ $x < 1$

(c) $\frac{\sqrt{x}}{x+2}$: (i) $x = -2$ (ii) $x < 0$

(4) Given: $P = \sqrt[4]{3y} - 1$. To which of the following numbers system(s) will P belong if:
[Number systems: $\mathbb{N}$; $\mathbb{N}_0$; $\mathbb{Z}$; $\mathbb{Q}$; $\mathbb{Q}'$; $\mathbb{R}$ or $\mathbb{R}'$]

(a) $y = \frac{1}{3}$

$P = \sqrt[4]{3 \times \frac{1}{3}} - 1$
$= \sqrt[4]{1} - 1$
$= 1 - 1 = 0$
$\mathbb{N}_0$; $\mathbb{Z}$; $\mathbb{Q}$; $\mathbb{R}$

(b) $y = -1$

$P = \sqrt[4]{3 \times -1} - 1$
$P = \sqrt[4]{-3} - 1$
$\mathbb{R}'$

(c) $y = 5$

$P = \sqrt[4]{3 \times 5} - 1$
$= \sqrt[4]{15} - 1$
$\mathbb{Q}'$; $\mathbb{R}$

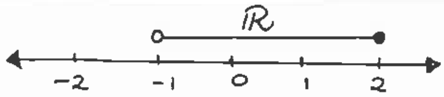
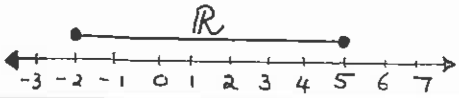
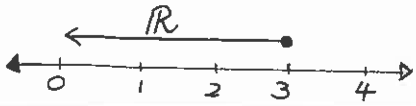
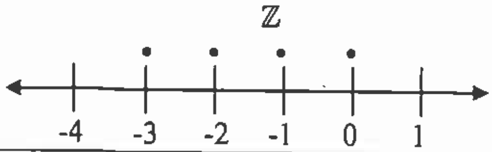
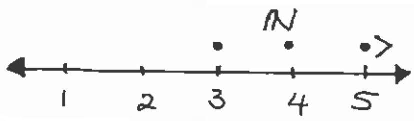
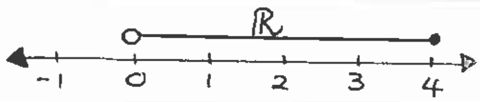
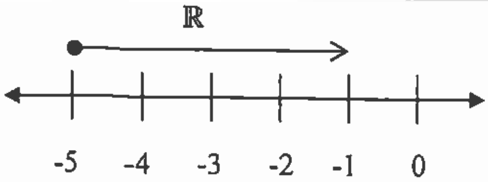
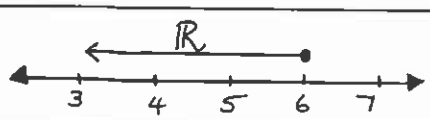
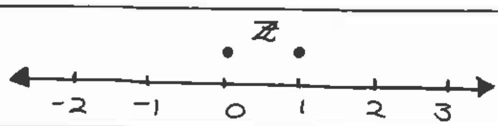
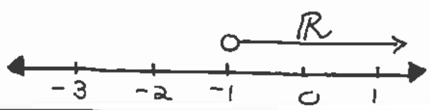
A1.3 Representation of real numbers:

As already seen in the previous grades, the real numbers can be represented by using on of the following ways:

(a) Interval notation.
(b) On a number line.
(c) As an inequality in set builder notation. Remember the following symbols:
  $\cup$ → the union of two or more intervals or sets.
  $\cap$ → the intersection of two or more intervals or sets.

Exercise 3: Date:______________

Complete the following table:

	Set builder notation:	Interval notation:	Number line:
(1)	$\{x / -1 < x \le 2 ; x \in \mathbb{R}\}$	$x \in (-1;2]$	
(2)	$\{x / -2 \le x \le 5; x \in \mathbb{R}\}$	$x \in [-2 ; 5]$	
(3)	$\{y / y \le 3; y \in \mathbb{R}\}$	$y \in (-\infty ; 3]$	
(4)	$\{x / -3 \le x \le 0; x \in \mathbb{Z}\}$ or $\{x / -4 < x < 1; x \in \mathbb{Z}\}$	N/a	
(5)	$\{y / y \ge 3 ; y \in \mathbb{N}\}$	N/a	
(6)	$\{m / 0 < m \le 4; m \in \mathbb{R}\}$	$m \in (0 ; 4]$	
(7)	$\{x / x \ge -5; x \in \mathbb{R}\}$	$x \in [-5; \infty)$	
(8)	$\{m : m \le 6 ; m \in \mathbb{R}\}$	$m \in (-\infty; 6]$	
(9)	$\{x / -1 < x < 2 ; x \in \mathbb{Z}\}$	N/a	
(10)	$\{x / x > -1; x \in \mathbb{R}\}$	$x \in (-1 ; \infty)$	

☺ **(1) Give a synonym for "non-real numbers":** Imagenary

(2) Do some investigation regarding complex numbers and the numbers it contain:
Represented by $\mathbb{C}$. Numbers as $\sqrt{-1}$ or $\sqrt{-7}$. Can be written as sum or difference of a real and a non-real number. e.g. $1+\sqrt{-1}$ or $3-2i$

A1.4 Exponents and surds:

A1.4.1 Exponents:

Basic exponential laws and properties:

(1) $x^m \times x^n = x^{m+n}$ (2) $x^m \div x^n = x^{m-n}$

(3) $(x^m)^n = x^{mn}$ (4) $(xy)^m = x^m y^m$ or $\left(\dfrac{x}{y}\right)^m = \dfrac{x^m}{y^m}$

(5) $x^0 = 1$ (6) $x^{-m} = \dfrac{1}{x^m}$

(7) $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ $(\,m, n \in \mathbb{Z}$ and $n > 0$ with $n \neq 1\,)$

E.g.1 *Simplify and write your answer as a positive exponent:*

(a) $$\frac{(x^3\ .\ y^{-2})^2}{x^3(xy)^3} = \frac{x^6\ .\ y^{-4}}{x^3\ .\ x^3\ .\ y^3} = \frac{x^6\ .\ y^{-4}}{x^6\ .\ y^3} = x^{6-6}\ .\ y^{-4-3} = x^0\ .\ y^{-7} = \frac{x^0}{y^7} = \underline{\frac{1}{y^7}}$$

(b) $$\sqrt{x} \times x^{\frac{1}{3}} \div x^{\frac{1}{4}} = x^{\frac{1}{2}} \times x^{\frac{1}{3}} \div x^{\frac{1}{4}} = x^{\frac{1}{2}+\frac{1}{3}-\frac{1}{4}} = x^{\frac{6+4-3}{12}} = \underline{x^{\frac{7}{12}}}$$

(c) $$\frac{25^{n+1}\ .\ 10^n}{8^{n-1}\ .\ 5^{3n}\ .\ (2^{-1})^{2n}} = \frac{(5^2)^{n+1}\ .\ (2\times 5)^n}{(2^3)^{n-1}\ .\ 5^{3n}\ .\ 2^{-2n}}$$
$$= \frac{5^{2n+2}\ .\ 2^n\ .\ 5^n}{2^{3n-3}\ .\ 5^{3n}\ .\ 2^{-2n}} = \frac{5^{3n+2}\ .\ 2^n}{2^{n-3}\ .\ 5^{3n}}$$
$$= 5^{3n+2-3n}\ .\ 2^{n-(n-3)} = 5^2\ .\ 2^{n-n+3} = \underline{5^2\ .\ 2^3} = \underline{200}$$

(d) $$\frac{2^x - 2^{x+1}}{2^{x-1} + 2^x}$$
$$= \frac{2^x - 2^x\ .\ 2^1}{2^x\ .\ 2^{-1} + 2^x}$$
$$= \frac{\cancel{2^x}(1-2^1)}{\cancel{2^x}(2^{-1}+1)} = \frac{1-2}{\frac{1}{2}+1} = -1 \div \left(\frac{1+2}{2}\right) = -1 \div \frac{3}{2} = -1 \times \frac{2}{3} = \underline{\frac{-2}{3}}$$

(e) $$\frac{x - x^{\frac{1}{2}} - 6}{x - 4} = \frac{(x^{\frac{1}{2}} - 3)\cancel{(x^{\frac{1}{2}} + 2)}}{(x^{\frac{1}{2}} - 2)\cancel{(x^{\frac{1}{2}} + 2)}} = \underline{\frac{(x^{\frac{1}{2}} - 3)}{(x^{\frac{1}{2}} - 2)}}$$

Exercise 4: Date:______________

Simplify, without using a calculator: (Write answers as positive exponents!)

(1) $(125x^6)^{\frac{1}{3}}$

$= (5^3 x^6)^{\frac{1}{3}}$

$= (5^3)^{\frac{1}{3}} (x^6)^{\frac{1}{3}}$

$= 5x^2$

(2) $\left(x^{\frac{1}{2}} - 2\right)^2$

$= (x^{\frac{1}{2}} - 2)(x^{\frac{1}{2}} - 2)$

$= (x^{\frac{1}{2}})^2 - 2x^{\frac{1}{2}} - 2x^{\frac{1}{2}} + 4$

$= x - 4x^{\frac{1}{2}} + 4$

(3) $\sqrt[3]{-8x^9y^{-3}}$

$= \sqrt[3]{(-2)^3 x^9 y^{-3}}$

$= (-2)^{\frac{3}{3}} x^{\frac{9}{3}} y^{-\frac{3}{3}}$

$= -2x^3 y^{-1}$

$= \frac{-2x^3}{y}$

(4) $3y^{\frac{1}{2}} \div (3y)^{\frac{-1}{2}}$

$= 3^1 y^{\frac{1}{2}} \div 3^{-\frac{1}{2}} y^{-\frac{1}{2}}$

$= 3^{1-(-\frac{1}{2})} y^{\frac{1}{2}-(-\frac{1}{2})}$

$= 3^{1\frac{1}{2}} y^{\frac{1}{2}+\frac{1}{2}}$

$= 3^{1\frac{1}{2}} y^1$

$x^{\frac{1}{4}} \times x^{\frac{1}{4}} = x^{\frac{2}{4}} = x^{\frac{1}{2}}$

(5) $(0{,}25m^{\frac{1}{4}})^2$

$= \left(\frac{25}{100} m^{\frac{1}{4}}\right)^2$

$= \left(\frac{1}{4} m^{\frac{1}{4}}\right)^2$

$= \left(\frac{1}{4}\right)^2 \left(m^{\frac{1}{4}}\right)^2 = \frac{1}{16} m^{\frac{1}{2}} = \frac{m^{\frac{1}{2}}}{16}$

(6) $\left(x^{\frac{1}{2}} + 4\right)\left(x^{\frac{1}{4}} - 2\right)\left(x^{\frac{1}{4}} + 2\right)$

$= (x^{\frac{1}{2}} + 4)(x^{\frac{1}{2}} - 4)$

$= (x^{\frac{1}{2}})^2 - 16$

$= x - 16$

(7) $\frac{x^{\frac{1}{2}} . \sqrt{x^3}}{x^{\frac{1}{3}}}$

$= \frac{x^{\frac{1}{2}} . x^{\frac{3}{2}}}{x^{\frac{1}{3}}}$

$= x^{\frac{1}{2}+\frac{3}{2}-\frac{1}{3}}$

$= x^{2-\frac{1}{3}}$

$= x^{1\frac{2}{3}}$

(8) $\left(\frac{-12x^4y^4z}{-3x^2z^3}\right)^{\frac{1}{2}}$

$= (4x^2y^4z^{-2})^{\frac{1}{2}}$

$= (2^2)^{\frac{1}{2}} (x^2)^{\frac{1}{2}} (y^4)^{\frac{1}{2}} (z^{-2})^{\frac{1}{2}}$

$= 2^1 x^1 y^2 z^{-1}$

$= \frac{2xy^2}{z}$

(9) $\frac{m^{-2} - 3}{m^{-3} - 3m^{-1}}$

$= \frac{(m^{2}-3)}{m^{-1}(m^{-2}-3)}$

$= \frac{1}{m^{-1}}$

$= m^1$

(10) $\frac{\left(9x^{\frac{2}{3}}y^{-4}\right)^{\frac{-3}{2}}}{3xy}$

$= \frac{(3^2)^{-\frac{3}{2}} (x^{\frac{2}{3}})^{-\frac{3}{2}} (y^{-4})^{-\frac{3}{2}}}{3xy}$

$= \frac{3^{-3} x^{-1} y^6}{3^1 x^1 y^1}$

$= 3^{-4} x^{-2} y^5$

$= \frac{y^5}{3^4 x^2}$

(11) $\dfrac{(x+y)^{-1}}{x^{-1}-y^{-1}}$

$= \dfrac{1}{(x+y)} \div \left(\dfrac{1}{x} - \dfrac{1}{y}\right)$

$= \dfrac{1}{(x+y)} \div \left(\dfrac{y-x}{xy}\right)$

$= \dfrac{1}{(x+y)} \times \dfrac{xy}{(y-x)} = \dfrac{xy}{y^2-x^2}$

(12) $\dfrac{2^{2n} - 3.2^n + 2}{2^n - 2}$

$= \dfrac{(2^n)^2 - 3.(2^n) + 2}{2^n - 2}$

$= \dfrac{(2^n-2)(2^n-1)}{(2^n-2)}$

$= 2^n - 1$

(13) $\left(m^{\frac{2}{3}} + n^{\frac{1}{3}}\right)^2$

$= \left(m^{\frac{2}{3}} + n^{\frac{1}{3}}\right)\left(m^{\frac{2}{3}} + n^{\frac{1}{3}}\right)$

$= \left(m^{\frac{2}{3}}\right)^2 + m^{\frac{2}{3}}n^{\frac{1}{3}} + m^{\frac{2}{3}}n^{\frac{1}{3}} + \left(n^{\frac{1}{3}}\right)^2$

$= m^{\frac{4}{3}} + 2m^{\frac{2}{3}}n^{\frac{1}{3}} + n^{\frac{2}{3}}$

(14) $\left(a^{\frac{1}{3}} - 5\right)\left(5 + a^{\frac{1}{3}}\right)$

$= \left(a^{\frac{1}{3}} - 5\right)\left(a^{\frac{1}{3}} + 5\right)$

$= \left(a^{\frac{1}{3}}\right)^2 - 25$

$= a^{\frac{2}{3}} - 25$

(15) $\sqrt[3]{(0{,}125)^{-2}} + \left(125^2\right)^{\frac{1}{3}}$

$= \sqrt[3]{\left(\frac{1}{8}\right)^{-2}} + \left(\left(5^3\right)^2\right)^{\frac{1}{3}}$

$= \sqrt[3]{\left(\frac{1}{2^3}\right)^{-2}} + \left(5^6\right)^{\frac{1}{3}}$

$= \sqrt[3]{\frac{1}{2^{-6}}} + 5^{\frac{6}{1}\times\frac{1}{3}}$

$= \sqrt[3]{2^6} + 5^2$

$= 2^{\frac{6}{3}} + 5^2$

$= 2^2 + 25$

$= 4 + 25 = 29$

(16) $\dfrac{12^{n+1} \,.\, 9^{n-2}}{18^{2n-1}.3^{-n}}$

$= \dfrac{(2^2\times 3^1)^{n+1} \,.\, (3^2)^{n-2}}{(2^1\times 3^2)^{2n-1} \,.\, 3^{-n}}$

$= \dfrac{2^{2n+2}\times 3^{n+1} \,.\, 3^{2n-4}}{2^{2n-1}\times 3^{4n-2} \,.\, 3^{-n}}$

$= \dfrac{2^{2n+2}\times 3^{3n-3}}{2^{2n-1}\times 3^{3n-2}}$

$= 2^{2n+2-(2n-1)} \times 3^{3n-3-(3n-2)}$

$= 2^{2n+2-2n+1} \times 3^{3n-3-3n+2}$

$= 2^3 \times 3^{-1}$

$= \dfrac{8}{3}$

(17) $\dfrac{5^{n+1} \,.\, 25^{n-1}}{125^{n-2}}$

$= \dfrac{5^{n+1} \,.\, (5^2)^{n-1}}{(5^3)^{n-2}}$

$= \dfrac{5^{n+1} \,.\, 5^{2n-2}}{5^{3n-6}}$

$= 5^{n+1+2n-2-(3n-6)}$

$= 5^{3n-1-3n+6} = 5^5$

$= 3\ 125$

(18) $\dfrac{3^{2n} - 9^{n+1}}{3^{2n}}$

$= \dfrac{3^{2n} - (3^2)^{n+1}}{3^{2n}}$

$= \dfrac{3^{2n} - 3^{2n}.3^2}{3^{2n}}$

$= \dfrac{3^{2n}(1 - 3^2)}{3^{2n}}$

$= 1 - 9 = -8$

(19) $\dfrac{3\times 2^x + 2^{x+1}}{5\times 2^x}$

$= \dfrac{3\times 2^x + 2^x \cdot 2^1}{5\times 2^x}$

$= \dfrac{2^x(3+2^1)}{5\times 2^x}$

$= \dfrac{2^x(5)}{5\times 2^x}$

$= \underline{1}$

(20) $\dfrac{3^2 . 5^0 . 4^{n-1}}{2^{2n+1} - 2^{2n}}$

$= \dfrac{3^2 . 1 . (2^2)^{n-1}}{2^{2n}.2^1 - 2^{2n}}$

$= \dfrac{9 . 2^{2n} . 2^{-2}}{2^{2n}(2^1 - 1)}$

$= \dfrac{9 . 2^{-2}}{2-1}$

$= \dfrac{9 . 2^{-2}}{1}$

$= 9 \times \dfrac{1}{2^2}$

$= \underline{\dfrac{9}{4}}$

(21) $\dfrac{3^{-2x} . 36^{x+1} . 3}{4^{x-1} . (0,5)^2}$

$= \dfrac{3^{-2x} . (2^2\times 3^2)^{x+1} . 3^1}{(2^2)^{x-1} . (\frac{1}{2})^2}$

$= \dfrac{3^{-2x} . 2^{2x+2} . 3^{2x+2} . 3^1}{2^{2x-2} . (2^{-1})^2}$

$= \dfrac{3^{-2x+2x+2+1} . 2^{2x+2}}{2^{2x-2} . 2^{-2}}$

$= \dfrac{3^3 . 2^{2x+2}}{2^{2x-2-2}}$

$= 3^3 . 2^{2x+2-(2x-4)}$

$= 3^3 . 2^{2x+2-2x+4}$

$= \underline{3^3 . 2^6}$

(22) $\dfrac{5.5^{-y-1} + 5^{-2y}.5^y}{3.5^{-y} - 5^{1-y}}$

$= \dfrac{5.5^{-y}.5^{-1} + 5^{-2y}.5^y}{3.5^{-y} - 5^{1-y}}$

$= \dfrac{5^{1-1}.5^{-y} + 5^{-2y+y}}{3.5^{-y} - 5^1.5^{-y}}$

$= \dfrac{5^0.5^{-y} + 5^{-y}}{3.5^{-y} - 5.5^{-y}}$

$= \dfrac{\cancel{5^{-y}}(5^0 + 1)}{\cancel{5^{-y}}(3 - 5)}$

$= \dfrac{1+1}{-2}$

$= \underline{-1}$

A1.4.2 Surds:

Remember: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ $(m,n \in \mathbb{Z}$ and $n \geq 2)$

E.g.2 Simplify: (a) $3\sqrt{2} + 7\sqrt{2} = \underline{10\sqrt{2}}$

(b) $\sqrt{8} \times \sqrt{2} = \sqrt{8\times 2} = \sqrt{16} = \underline{4}$

(c) $(\sqrt{3}+1)^2 = (\sqrt{3}+1)(\sqrt{3}+1) = 3 + 1\sqrt{3} + 1\sqrt{3} + 1$

$= \underline{4 + 2\sqrt{3}}$

(d) $\sqrt{18} + \sqrt{50} - 2\sqrt{8}$

$= \sqrt{9 \times 2} + \sqrt{25 \times 2} - 2\sqrt{4 \times 2}$

$= \sqrt{9} \times \sqrt{2} + \sqrt{25} \times \sqrt{2} - 2\sqrt{4} \times \sqrt{2}$

$= 3\sqrt{2} + 5\sqrt{2} - 2 \times 2\sqrt{2}$

$= 8\sqrt{2} - 4\sqrt{2}$

$= \underline{4\sqrt{2}}$

E.g.3 Rationalize the denominator: $\dfrac{2 + \sqrt{8}}{\sqrt{2}}$

$$= \frac{2 + \sqrt{8}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2 \times \sqrt{2} + \sqrt{8} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$

$$= \frac{2\sqrt{2} + \sqrt{16}}{\sqrt{4}} = \frac{2\sqrt{2} + 4}{2} = \frac{2\sqrt{2}}{2} + \frac{4}{2} = \underline{\sqrt{2} + 2}$$

Exercise 5: Date:______________

(1) Simplify, without using a calculator:

(a) $(\sqrt{3} - 2)(\sqrt{3} + 2)$

$= (\sqrt{3})^2 - 4$

$= 3 - 4$

$= \underline{-1}$

(b) $\sqrt{8} + \sqrt{50} - \sqrt{18}$

$= \sqrt{4 \times 2} + \sqrt{25 \times 2} - \sqrt{9 \times 2}$

$= \sqrt{4}\sqrt{2} + \sqrt{25}\sqrt{2} - \sqrt{9}\sqrt{2}$

$= 2\sqrt{2} + 5\sqrt{2} - 3\sqrt{2}$

$= \underline{4\sqrt{2}}$

(c) $(\sqrt{8} - 2^{\frac{1}{2}})^2$

$= (\sqrt{4 \times 2} - \sqrt{2})^2$

$= (2\sqrt{2} - \sqrt{2})^2$

$= (1\sqrt{2})^2$

$= \underline{2}$

(d) $\sqrt[3]{27x^6} + \sqrt[5]{32x^{10}}$

$= \sqrt[3]{3^3 x^6} + \sqrt[5]{2^5 x^{10}}$

$= 3^{\frac{3}{3}} x^{\frac{6}{3}} + 2^{\frac{5}{5}} x^{\frac{10}{5}}$

$= 3^1 x^2 + 2^1 x^2$

$= 3x^2 + 2x^2$

$= \underline{5x^2}$

(e) $(4\sqrt{2} - 3)^2$

$= (4\sqrt{2} - 3)(4\sqrt{2} - 3)$

$= (4\sqrt{2})^2 - 12\sqrt{2} - 12\sqrt{2} + 9$

$= 16 \times 2 - 24\sqrt{2} + 9$

$= 32 - 24\sqrt{2} + 9$

$= \underline{41 - 24\sqrt{2}}$

(f) $m \times \sqrt{27m^6} - \sqrt{12m^8}$

$= m \times \sqrt{9 \times 3 m^6} - \sqrt{4 \times 3 m^8}$

$= m \times 3\sqrt{3}\, m^{\frac{6}{2}} - 2\sqrt{3}\, m^{\frac{8}{2}}$

$= m^1 \times 3\sqrt{3}\, m^3 - 2\sqrt{3}\, m^4$

$= 3\sqrt{3}\, m^4 - 2\sqrt{3}\, m^4$

$= 1\sqrt{3}\, m^4$

(g) $\sqrt{3}\left(\sqrt{48} - 3\sqrt{75} + 2\sqrt{108}\right)$

$= \sqrt{3}\left(\sqrt{16 \times 3} - 3\sqrt{25 \times 3} + 2\sqrt{36 \times 3}\right)$

$= \sqrt{3}\left(4\sqrt{3} - 3 \times 5\sqrt{3} + 2 \times 6\sqrt{3}\right)$

$= \sqrt{3}\left(4\sqrt{3} - 15\sqrt{3} + 12\sqrt{3}\right)$

$= \sqrt{3}\left(1\sqrt{3}\right)$

$= 3$

(h) $\dfrac{\sqrt{18} - \sqrt{98}}{\sqrt{200}}$

$= \dfrac{\sqrt{9 \times 2} - \sqrt{49 \times 2}}{\sqrt{100 \times 2}}$

$= \dfrac{3\sqrt{2} - 7\sqrt{2}}{10\sqrt{2}}$

$= \dfrac{-4\cancel{\sqrt{2}}}{10\cancel{\sqrt{2}}}$

$= -\dfrac{2}{5}$

(i) $\dfrac{\sqrt{\sqrt{64}} - \sqrt{12}}{\sqrt{18} - \sqrt{27}}$

$= \dfrac{\sqrt{8} - \sqrt{12}}{\sqrt{18} - \sqrt{27}}$

$= \dfrac{\sqrt{4 \times 2} - \sqrt{4 \times 3}}{\sqrt{9 \times 2} - \sqrt{9 \times 3}}$

$= \dfrac{2\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} - 3\sqrt{3}}$

$= \dfrac{2(\cancel{\sqrt{2} - \sqrt{3}})}{3(\cancel{\sqrt{2} - \sqrt{3}})} = \dfrac{2}{3}$

(j) $\dfrac{(2 + \sqrt{3})(4 - \sqrt{3})}{\sqrt{100} + \sqrt{48}}$

$= \dfrac{8 - 2\sqrt{3} + 4\sqrt{3} - (\sqrt{3})^2}{10 + \sqrt{16 \times 3}}$

$= \dfrac{8 + 2\sqrt{3} - 3}{10 + 4\sqrt{3}}$

$= \dfrac{5 + 2\sqrt{3}}{10 + 4\sqrt{3}}$

$= \dfrac{\cancel{(5 + 2\sqrt{3})}}{2\cancel{(5 + 2\sqrt{3})}}$

$= \dfrac{1}{2}$

(k) $\dfrac{\sqrt[3]{27x^6} + \sqrt[4]{16x^8}}{\sqrt[3]{125x^{27}}}$

$= \dfrac{\sqrt[3]{3^3}\sqrt[3]{x^6} + \sqrt[4]{2^4}\sqrt[4]{x^8}}{\sqrt[3]{5^3}\sqrt[3]{x^{27}}}$

$= \dfrac{3^{\frac{3}{3}} x^{\frac{6}{3}} + 2^{\frac{4}{4}} x^{\frac{8}{4}}}{5^{\frac{3}{3}} x^{\frac{27}{3}}}$

$= \dfrac{3x^2 + 2x^2}{5x^9}$

$= \dfrac{\cancel{5}x^2}{\cancel{5}x^9}$

$= \dfrac{1}{x^7}$