Grade 11 – Book A

(CAPS Edition)

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Chapter A1

Number systems and exponents

A1.1 Number systems:

Exercise 1: Date:_______________

(1) Complete:

* Natural numbers: $\mathbb{N}$ = ______________________________
* Whole numbers: $\mathbb{N}_0$ = ______________________________
* Integers: $\mathbb{Z}$ = ______________________________
* Rational numbers: $\mathbb{Q}$ = ______________________________
* Real numbers: $\mathbb{R}$ = ______________________________

(2) Write three examples of Irrational numbers: ______________________________

(3) Consider: $x(x - 6)(x^2 - 5)(2x^2 + x - 3) = 0$.
Solve for x and write the value(s) of x for which the solution of the equation will have:
(a) irrational roots.
(b) natural roots.
(c) integral roots.

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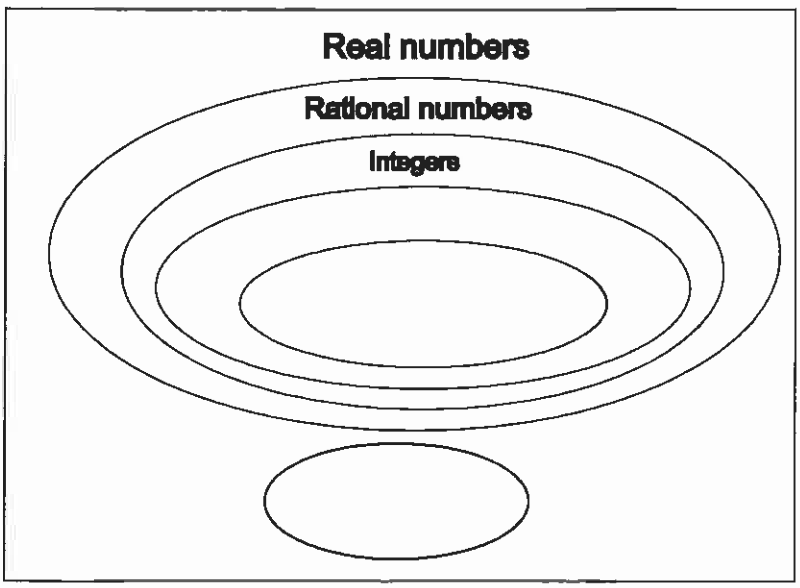
(4) Complete the following diagram which presents the system of real numbers:



A1.2 Non-Real numbers:

Examples of non-real numbers: $\sqrt{-2}$; $\sqrt{-9}$ or $\sqrt[4]{-5}$

But not $\sqrt[3]{-8}$, because -2 × -2 × -2 = -8 $\therefore \sqrt[3]{-8}$ = -2

Exercise 2: Date:______________

(1) Determine whether the following numbers are real or non-real. If real, indicate whether the number is rational or irrational:

(a) 7 ______________ (b) $-\sqrt{3}$ ______________

(c) π ______________ (d) $\sqrt{-16}$ ______________

(e) $0,\dot{3}$ ______________ (f) $\frac{12}{36}$ ______________

(g) $\sqrt[3]{-125}$ ______________ (h) $1+\sqrt{9}$ ______________

(i) $\sqrt{(-2)^3}$ ______________ (j) 0 ______________

(2) State whether the following statements are true or false:

(a) The product of two integers is always an integer again. ______________

(b) The product of two irrational numbers is always an irrational number. ______________

(c) If m is a natural number, $\sqrt{4}\,m$ will also be a natural number. ______________

(d) The difference between two rational numbers is always a rational number.______________

(e) The quotient of a rational number and an irrational number will always be rational. ______________

(3) For which values of x will the following statements be: (i) undefined (ii) non-real

(a) $\dfrac{x+3}{x}$: ______________________________

(b) $\sqrt{x-1}$: ______________________________

(c) $\dfrac{\sqrt{x}}{x+2}$: ______________________________

(4) Given: P = $\sqrt[4]{3y}$ - 1. To which of the following numbers system(s) will P belong if:

[Number systems: $\mathbb{N}$; $\mathbb{N}_0$; $\mathbb{Z}$; $\mathbb{Q}$; $\mathbb{Q}'$; $\mathbb{R}$ or $\mathbb{R}'$]

(a) $y = \frac{1}{3}$ (b) $y = -1$ (c) $y = 5$

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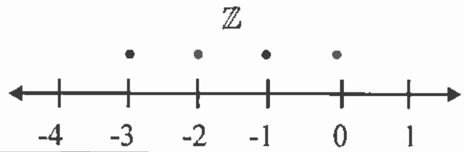
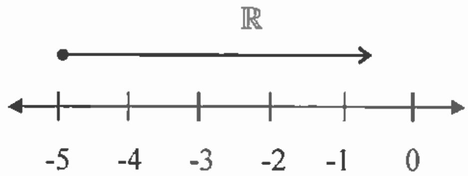
A1.3 **Representation of real numbers**:

As already seen in the previous grades, the real numbers can be represented by using on of the following ways:

(a) Interval notation.
(b) On a number line.
(c) As an inequality in set builder notation. Remember the following symbols:
   $\cup$ → the union of two or more intervals or sets.
   $\cap$ → the intersection of two or more intervals or sets.

Exercise 3: Date:_______________

Complete the following table:

	Set builder notation:	Interval notation:	Number line:
(1)	$\{x\,/\,-1 < x \leq 2\,;\, x \in \mathbb{R}\}$		
(2)		$x \in [-2\,;\,5]$	
(3)		$y \in (-\infty\,;\,3]$	
(4)			
(5)	$\{y\,/\;\, y \geq 3\,;\, y \in \mathbb{N}\}$		
(6)		$m \in (0\,;\,4]$	
(7)			
(8)	$\{m : m \leq 6\,;\, m \in \mathbb{R}\}$		
(9)	$\{x\,/\,-1 < x < 2\,;\, x \in \mathbb{Z}\}$		
(10)		$x \in (-1\,;\,\infty)$	

☺ **(1) Give a synonym for "non-real numbers": ______________________________**

(2) Do some investigation regarding complex numbers and the numbers it contain:

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A1.4 Exponents and surds:
A1.4.1 Exponents:

Basic exponential laws and properties:

(1) $x^m \times x^n = x^{m+n}$ (2) $x^m \div x^n = x^{m-n}$

(3) $(x^m)^n = x^{mn}$ (4) $(xy)^m = x^m y^m$ or $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

(5) $x^0 = 1$ (6) $x^{-m} = \frac{1}{x^m}$

(7) $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ $(m, n \in \mathbb{Z}$ *and* $n > 0$ *with* $n \neq 1)$

E.g.1 *Simplify and write your answer as a positive exponent:*

(a) $$\frac{(x^3 \cdot y^{-2})^2}{x^3(xy)^3} = \frac{x^6 \cdot y^{-4}}{x^3 \cdot x^3 \cdot y^3} = \frac{x^6 \cdot y^{-4}}{x^6 \cdot y^3} = x^{6-6} \cdot y^{-4-3} = x^0 \cdot y^{-7} = \frac{x^0}{y^7} = \underline{\frac{1}{y^7}}$$

(b) $$\sqrt{x} \times x^{\frac{1}{3}} \div x^{\frac{1}{4}} = x^{\frac{1}{2}} \times x^{\frac{1}{3}} \div x^{\frac{1}{4}} = x^{\frac{1}{2}+\frac{1}{3}-\frac{1}{4}} = x^{\frac{6+4-3}{12}} = \underline{x^{\frac{7}{12}}}$$

(c) $$\frac{25^{n+1} \cdot 10^n}{8^{n-1} \cdot 5^{3n} \cdot (2^{-1})^{2n}} = \frac{(5^2)^{n+1} \cdot (2 \times 5)^n}{(2^3)^{n-1} \cdot 5^{3n} \cdot 2^{-2n}}$$

$$= \frac{5^{2n+2} \cdot 2^n \cdot 5^n}{2^{3n-3} \cdot 5^{3n} \cdot 2^{-2n}} = \frac{5^{3n+2} \cdot 2^n}{2^{n-3} \cdot 5^{3n}}$$

$$= 5^{3n+2-3n} \cdot 2^{n-(n-3)} = 5^2 \cdot 2^{n-n+3} = \underline{5^2 \cdot 2^3} = \underline{200}$$

(d) $$\frac{2^x - 2^{x+1}}{2^{x-1} + 2^x}$$

$$= \frac{2^x - 2^x \cdot 2^1}{2^x \cdot 2^{-1} + 2^x}$$

$$= \frac{2^x(1 - 2^1)}{2^x(2^{-1} + 1)} = \frac{1-2}{\frac{1}{2}+1} = -1 \div \left(\frac{1+2}{2}\right) = -1 \div \frac{3}{2} = -1 \times \frac{2}{3} = \underline{\frac{-2}{3}}$$

(e) $$\frac{x - x^{\frac{1}{2}} - 6}{x - 4} = \frac{(x^{\frac{1}{2}} - 3)(x^{\frac{1}{2}} + 2)}{(x^{\frac{1}{2}} - 2)(x^{\frac{1}{2}} + 2)} = \underline{\frac{(x^{\frac{1}{2}} - 3)}{(x^{\frac{1}{2}} - 2)}}$$

Exercise 4: Date:_______________

Simplify, without using a calculator: (Write answers as positive exponents!)

(1) $\left(125x^{6}\right)^{\frac{1}{3}}$

(2) $\left(x^{\frac{1}{2}} - 2\right)^{2}$

(3) $\sqrt[3]{-8x^{9}y^{-3}}$

(4) $3y^{\frac{1}{2}} \div (3y)^{\frac{-1}{2}}$

(5) $(0{,}25m^{\frac{1}{4}})^{2}$

(6) $\left(x^{\frac{1}{2}} + 4\right)\left(x^{\frac{1}{4}} - 2\right)\left(x^{\frac{1}{4}} + 2\right)$

(7) $\dfrac{x^{\frac{1}{2}} . \sqrt{x^{3}}}{x^{\frac{1}{3}}}$

(8) $\left(\dfrac{-12\,x^{4}\,y^{4}\,z}{-3\,x^{2}\,z^{3}}\right)^{\frac{1}{2}}$

(9) $\dfrac{m^{-2} - 3}{m^{-3} - 3m^{-1}}$

(10) $\dfrac{\left(9x^{\frac{2}{3}}y^{-4}\right)^{\frac{-3}{2}}}{3xy}$

(11) $\dfrac{(x + y)^{-1}}{x^{-1} - y^{-1}}$

(12) $\dfrac{2^{2n} - 3.2^{n} + 2}{2^{n} - 2}$

(13) $\left(m^{\frac{2}{3}} + n^{\frac{1}{3}}\right)^{2}$

(14) $\left(a^{\frac{1}{3}} - 5\right)\left(5 + a^{\frac{1}{3}}\right)$

(15) $\sqrt[3]{(0{,}125)^{-2}} + \left(125^{2}\right)^{\frac{1}{3}}$

(16) $\dfrac{12^{n+1} \, . \, 9^{n-2}}{18^{2n-1} . 3^{-n}}$

(17) $\dfrac{5^{n+1} \, . \, 25^{n-1}}{125^{n-2}}$

(18) $\dfrac{3^{2n} - 9^{n+1}}{3^{2n}}$

(19) $\dfrac{3 \times 2^x + 2^{x+1}}{5 \times 2^x}$

(20) $\dfrac{3^2 . 5^0 . 4^{n-1}}{2^{2n+1} - 2^{2n}}$

(21) $\dfrac{3^{-2x} . 36^{x+1} . 3}{4^{x-1} . (0,5)^2}$

(22) $\dfrac{5 . 5^{-y-1} + 5^{-2y} . 5^y}{3 . 5^{-y} - 5^{1-y}}$

A1.4.2 <u>Surds</u>:

Remember: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ $(m, n \in \mathbb{Z}$ and $n \geq 2)$

E.g.2 Simplify:

(a) $3\sqrt{2} + 7\sqrt{2} = \underline{10\sqrt{2}}$

(b) $\sqrt{8} \times \sqrt{2} = \sqrt{8 \times 2} = \sqrt{16} = \underline{4}$

(c) $(\sqrt{3} + 1)^2 = (\sqrt{3} + 1)(\sqrt{3} + 1) = 3 + 1\sqrt{3} + 1\sqrt{3} + 1$

$= \underline{4 + 2\sqrt{3}}$

(d) $\sqrt{18} + \sqrt{50} - 2\sqrt{8}$

$= \sqrt{9 \times 2} + \sqrt{25 \times 2} - 2\sqrt{4 \times 2}$

$= \sqrt{9} \times \sqrt{2} + \sqrt{25} \times \sqrt{2} - 2\sqrt{4} \times \sqrt{2}$

$= 3\sqrt{2} + 5\sqrt{2} - 2 \times 2\sqrt{2}$

$= 8\sqrt{2} - 4\sqrt{2}$

$= \underline{4\sqrt{2}}$

E.g.3 Rationalize the denominator: $\dfrac{2 + \sqrt{8}}{\sqrt{2}}$

$= \dfrac{2 + \sqrt{8}}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{2 \times \sqrt{2} + \sqrt{8} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

$= \dfrac{2\sqrt{2} + \sqrt{16}}{\sqrt{4}} = \dfrac{2\sqrt{2} + 4}{2} = \dfrac{2\sqrt{2}}{2} + \dfrac{4}{2} = \underline{\sqrt{2} + 2}$

Exercise 5: Date:______________

(1) Simplify, without using a calculator:

(a) $(\sqrt{3} - 2)(\sqrt{3} + 2)$ (b) $\sqrt{8} + \sqrt{50} - \sqrt{18}$ (c) $(\sqrt{8} - 2^{\frac{1}{2}})^2$

(d) $\sqrt[3]{27x^6} + \sqrt[5]{32x^{10}}$ (e) $(4\sqrt{2} - 3)^2$

(f) $m \times \sqrt{27m^6} - \sqrt{12m^8}$

(g) $\sqrt{3}\left(\sqrt{48} - 3\sqrt{75} + 2\sqrt{108}\right)$

(h) $\dfrac{\sqrt{18} - \sqrt{98}}{\sqrt{200}}$

(i) $\dfrac{\sqrt{\sqrt{64}} - \sqrt{12}}{\sqrt{18} - \sqrt{27}}$

(j) $\dfrac{\left(2 + \sqrt{3}\right)\left(4 - \sqrt{3}\right)}{\sqrt{100} + \sqrt{48}}$

(k) $\dfrac{\sqrt[3]{27x^6} + \sqrt[4]{16x^8}}{\sqrt[3]{125x^{27}}}$