

NUMBERS AND CALCULATION WITH NUMBERS

Number Formats and Conventions

A million has 6 zeroes: 1 000 000
A billion/milliard has 9 zeroes: 1 000 000 000
A trillion has 12 zeroes: 1 000 000 000 000

Sometimes one can become confused between the use of a comma and the point. In South Africa the decimal comma separates the whole number from the fraction.
e.g. 3 000 000,453

Note, however, that some calculators use a comma to separate the thousands and the point to separate the fractions, e.g. 3,000,000.453 while others use spaces e.g. 3 000 000.453; it can also be represented as 3'000'000,453

To indicate an amount of money, separate the Rand from the cents with a comma and use spaces to indicate thousands e.g. R123 345,45

Exercise 1: Large Numbers

- (a) The Gariep Dam is the largest water reservoir in South Africa. This dam has a total storage capacity of approximately 5,3 trillion litres. It has a surface area of more than 370 square kilometres.
- (i) The capacity of one Olympic standard swimming pool is 2,5 million litres. How many swimming pools will fill the Gariep Dam?



Ref: Wikipedia



Ref: Wikipedia

- (ii) A factory uses about 287 458 kl of water to manufacture a product. How many Olympic standard swimming pools will this factory empty in the process?

(iii) The wall of the Gariep Dam is 88 m high and contains approximately 1,73 million cubic metre concrete. One concrete truck takes 6 m^3 . How many trucks of concrete did they order to build this wall?

(iv) The surface area of South Africa is $1\,221\,037 \text{ km}^2$. What percentage of South Africa's surface area is taken by the Gariep Dam with a 370 square kilometres surface?



(b) South Africa's population is estimated at 48 million people. If the average household has more or less 6 members, how many households are in the country?

(c) The surface area of South Africa is $1\,221\,037 \text{ km}^2$ and its population is estimated at 48 million people. What is the average number of people per square kilometre?



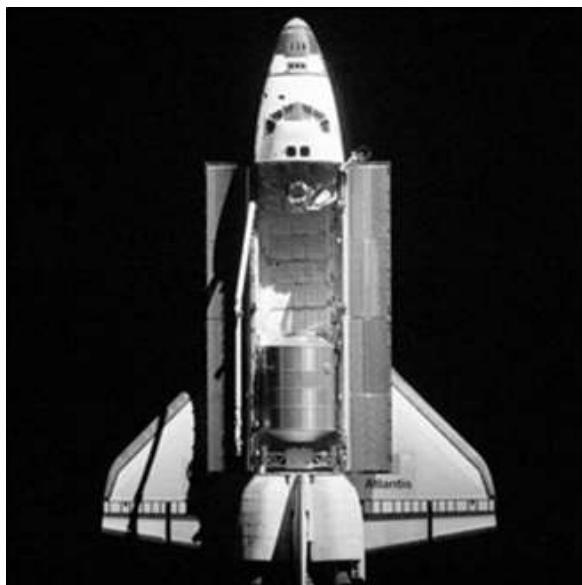
Ref: Wikipedia

(d) The sun is 148 million km from the earth and a space shuttle can reach a height of 207 thousand km above the earth. How many times more will it have to travel this distance in order to reach the sun?

(e) The average distance between the earth and the moon is 384 392 km. How much further is the sun from the moon?










(f) $300\,000\,000 \text{ m/s}$ is the speed of light. Write this speed in words.

(g) In 1996, researchers found that 5,6 million adults in England had taken illegal drugs in the previous year. By 2009, this number had fallen substantially, to 4,4 million. What was the difference in the number of people between 1996 and 2009?



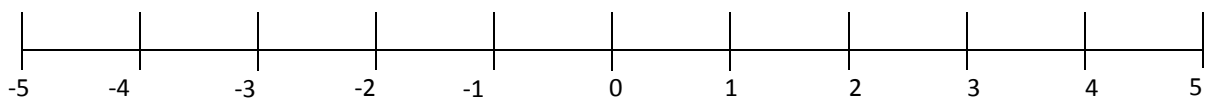
Exercise 2: Fractions.

(a) Complete the following table:

Common Fraction		Decimal Fraction
$\frac{13}{100}$		
		0,004
$\frac{1}{1\ 000}$		
		1,012
$\frac{32}{20\ 000}$		
		100,001
$2\frac{6}{10}$		
		1,03
$\frac{50}{100}$		

Exercise 3: Positive and Negative Numbers

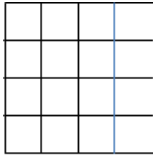
Make use of a number line to do the following without your calculator:



- The current temperature is 7°C . It drops with 8°C . What is the temperature now?
- The water level of the Gariiep Dam is normal in the beginning of the summer. The previous year was very dry and the water level was three cm below normal. During the rainy season the water level rises to 5 cm above normal. What is the difference between these two levels?
- The time difference between South Africa and the United States of America is 7 hours. It is 2 o'clock in the morning in South Africa. What is the time in the United States of America? They are behind us. (Make use of the number line)
- The current temperature is -7°C . It drops with 2°C . What is the temperature now?

Square numbers & roots

You are able to calculate the length of the square if the area is known.



You count 16 squares

The length of the side is 4 units.

Therefore: $\sqrt{16} = 4$

Examples of square numbers: (know these by heart)

1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2	13^2
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
1	4	9	16	25	36	49	64	81	100	121	144	169

Therefore: $\sqrt{1}=1$ and $\sqrt{4}=2$ etc.

1Exercise 4: Square Numbers & Roots

Complete the table below: (Make use of your calculator)

(a) $\sqrt{9} =$	(f) $\sqrt{25} \div \sqrt{16} =$
(b) $\sqrt{9 + 16} =$	(g) $\sqrt{2^2} =$
(c) $\sqrt{2\left(\frac{1}{2}\right)} =$	(h) $\sqrt{144} =$
(d) $\sqrt{2\frac{1}{2}} =$	(i) $\sqrt{32 - 4} =$
(e) $\sqrt{\frac{16}{4}} =$	(j) $\sqrt{9} + \sqrt{16} =$

(k) The area of a patio is in a square shape. It measures 25 m^2 . What is the length of its sides? _____

(l) The area of a square table is 1 m^2 . What is the perimeter of the table?

(m) The area of a square room measures 9 m^2 . What is the length of one wall?

Exercise 5: Mathematical Language and Estimation.

(a) Complete the table:

Sentence	Number sentence
The difference between six and twelve.	
Four times ten	
Two more than three	
Thirty thousand divided by ten.	
The total of 10, 200 and 23 is.	
Add up seventy and one hundred and ten.	
The product of twenty five and hundred	
Reduce R87 by R12,50.	
Subtract sixty from twelve.	
Half of $24m^2$	
Reduce six by ten	

(b)

CRAZY DAYS SALE!!!!	
Hats	R25,95
Socks	R13,45
T-Shirts	R49,35
Jeans	R75,59

Study the advertisement above and estimate the following answers:

Will 4 hats cost more or less than R100,00?	
Estimate the price of 2 pairs of socks	
Will 3 hats cost more or less than 1 jean?	
Will 4 pairs of socks cost more or less than 2 hats?	

Operations Using Numbers and Calculator Skills

Order of operations (BODMAS)

B	Brackets - Square numbers and roots
O	Of (Means Multiply)
DM	Multiplication and Division (From left to right)
AS	Add and Subtract (From left to right)

EXAMPLE:

$$\begin{aligned}
 & 2 \times 3 + 4 \div 2 + (9 - 1) - \frac{1}{2} \text{ van } 8 && \text{First the brackets} \\
 & = 2 \times 3 + 4 \div 2 + 8 - \frac{1}{2} \times 8 && \text{of (this is multiplication)} \\
 & = 2 \times 3 + 4 \div 2 + 8 - 4 && \text{then multiply and divide from left to right} \\
 & = 6 + 2 + 8 - 4 && \text{then add and subtract from left to right} \\
 & = \underline{12}
 \end{aligned}$$

Exercise 6: BODMAS

Determine the following: (You may use your calculator):

(a)	$2(2 - 3)^2 - 6 \div 2$	(i)	$6 \times 8 \div 2 + 3$
(b)	$5^2 - \sqrt{42 - 6}$	(j)	$983,5 - 100 - 10$
(c)	$R450 - R32,50 \times 10$	(k)	$325 - 36,3 \div 0,3 + 100$
(d)	$58 \div 2 + 2 \times 4 - \frac{2}{3} \text{ of } 30$	(l)	$3 \times 7 - 11 \div 2 \times 6 + 1$
(e)	$10\,000 \times 100 - 10 \times 10 + 10$	(m)	$1 \times 1 - 1 + 1 \div 1 + 1^2$
(f)	$10\,000(1,01)^2 - 1 \times 1$	(n)	$3,6(2,01 + 102,5)$
(g)	$20 - \frac{3}{5} \text{ of } 205$	(o)	$\frac{2}{5} \left(1 \frac{4}{9} \right) - 1$
(h)	$\frac{\sqrt{160 - 16}}{12} - 32 \div 8$	(p)	$10^2 + \frac{1}{2}$

Rounding

Exercise 7: Several Exercises on Rounding

Round off to two decimal places: (complete the table)

E.g. $354,7899 \approx 354,79$

but

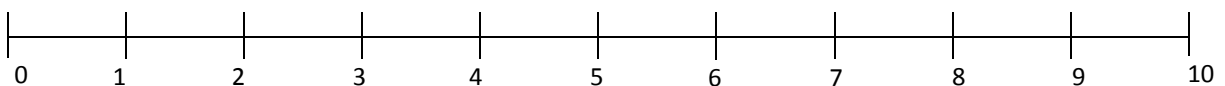
$354,7824 \approx 354,78$

(a)	123,2225	(g)	12,504
(b)	325,4567	(h)	11,406
(c)	341,455	(i)	0,008
(d)	19,999	(j)	0,00009
(e)	34,354999	(k)	0,005
(f)	67,899	(l)	1239,95443

Round off to the nearest 10: (complete the table)

(a)	15	(f)	457,345
(b)	343,35	(g)	568,224
(c)	169,991	(h)	299,201
(d)	22,09	(i)	342,456
(e)	936,789	(j)	11299,67

Round off to the nearest 5: (complete the table)



(a)	20	(e)	431
(b)	77	(f)	438,45
(c)	334	(g)	432,89
(d)	23	(h)	79

Round off to the nearest cent / Rand: (complete the table)

Round off to the nearest cent		Round off to the nearest Rand	
(a)	R134,905	(a)	R23,99
(b)	R23,544	(b)	R24,21324
(c)	R13,222222	(c)	R999,999
(d)	R45,2899	(d)	R345,578
(e)	R999,999	(e)	R13,46 ≈

Implications of Rounding Up or Down

Rounding up or down can have significant implications in real life.

- (a) If you work in a bank which holds accounts of 1 million people and you make a rounding mistake of one cent on each account in favour of the clients, how much money will the bank lose?

- (b) Complete the table: (Make sure you know when you must **round up** or **down**. This is determined by the context.)

The government printer can only print in units of one million. How many ballot papers will you need to print if you expect 2,3 million voters for the election?	
You prepare food for a big event and you double your recipe. You now need 2,24 kg rice. How much rice will you buy to the nearest kg?	
You need 4,3 ℓ paint for your room. They sell the paint in one litre containers. How many containers do you need to buy?	
144 boxes need to be packed onto the shelves. Each shelf can hold 13 boxes. Calculate how many shelves will be needed.	
A recipe states that a single portion of a particular dish requires 250 g of flour. Using this recipe, how many people can be fed with 5,55kg of flour?	
A CD costs R145,99. Calculate the cost of 13 CD's	
How many bricks do you need to buy if you calculated that you need 234,3 bricks?	
Susan buys fabric to sew a dress. She needs 2,3 m ² . They sell the fabric in 1 m ² . How much must she buy?	
Peter calculated that he needed 12,4 ℓ petrol to do his trip. They only sell petrol in 5ℓ cans. How many cans must he buy?	
One piece of gum costs 22c. How many pieces of gum can you buy for R4,25?	

Ratios

A **ratio** is the quotient obtained when comparing quantities of **the same kind** and the same units through division, e.g. there are 27 boys and 21 girls in a class. Therefore the ratio boys to girls is $\frac{27}{21}$ and is simplified (divide each part of the ratio with the same number) to $\frac{9}{7}$. We write the ratio of boys to girls as 27:21 = 9:7. (Or girls : boys = 7 : 9) Note that in this ratio notation **no units** are written down.

The ratio π is often used in Maths and is the **circumference** of a circle and its **diameter**.

$$\pi = \frac{\text{Circumference of a circle}}{\text{Its diameter}} = \frac{22}{7} = 3,14159 \approx 3,142$$

Exercise 8: Ratios

Answer the following questions: (Answer in full sentences and show all your calculations)

- (a) Sipho and Thandi own a business together and divide their profits in the ratio 4 : 3.
- Who receives the greater share of the profit?
 - What fraction of the profit will each one get?
 - If they make a profit of R3 416 in one month, how much will each one receive?
- (b) Mrs Ahmad prepares oats porridge for her children. For each bowl of oats porridge, she always uses 3 cups of water for every 2 cups of oats.
- Find the ratio of the number of cups of water to the number of cups of oats used.
 - If she wants to prepare 5 bowls of oats porridge, how many cups of water and how many cups of oats does she need?
 - If she uses 18 cups of oats, how many cups of water does she need?
- (c) John, Andre and Peter are business partners, and share in the profit proportional to their original capital contributions. John contributed R85 000, Andre R120 000 and Peter R105 000. After a profit of R156 500 was declared, they gave 7% of the profit to a welfare organization and then divided the balance of the profit. Calculate the amount each partner received
- (d) The instructions on a packet of crack filler state that one part of water must be added to two parts of crack filler and then mixed into a paste.
- How much crack filler must you add to 150 ml of water?
 - How much water must be used with 550 ml crack filler?
- (e) A professional hair stylist wants to colour her hair. The manufacturer's instructions of how to mix the tint, recommend the ratio of peroxide to hair colour as 1:2. She measures 10ml of hair colour. How many ml of peroxide must she mix it with?

- (f) A traffic officer finds that 18 567 vehicles pass over the freeway during one week. The ratio of cars to trucks is given as 6:1. Find the number of trucks which used the freeway during that week.
- (g) Elna and Susan are paid R440 for a task on which they worked $2\frac{1}{2}$ hours and $1\frac{1}{2}$ hours respectively. How much will each receive?

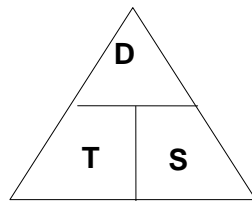
Rate

Exercise 9: Rate

Rate is when comparing **two different** quantities or different kinds or units through division. Rate is always expressed as "...per..." The symbol used is ".../..." Examples of rates include: Speed, distance and time relationships (e.g. km/h), product pricing (e.g. R/kg) etc.

Answer the following questions: (Show all your calculations)

SPEED, DISTANCE AND TIME



Know this triangle by heart!

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Distance} = \text{time} \times \text{speed}$$

- (a) Mr Klumper asked his driver to undertake a trip of 650 km. He drives at an average speed of 120 km/h. How long will it take him to do the trip? Write your answer down in hours and minutes.
- (b) Which car drives the fastest? Motorcar A drives 570 km in 4,75 hours; motorcar B drives 275 km in $2\frac{1}{2}$ hours and motorcar C drives 640 km in 5 hours 20 minutes.
- (c) A car drives at a constant speed and the distance covered is presented in the following table:
(Complete the table)

Hours:	1	2	3	6	7		10			
Km:		160	240			680		880	1640	2400

- (d) Determine the distance in meter if a vehicle travels at an average speed of 27,95 m/s for 1,36 seconds.
- (e) You walk 21 km in 4 hours. What is your average speed?
- (f) An aero plane flies 512 km in 40 minutes. What was its speed?

- (g) It takes Mr. Gouws 30 minutes to cover a distance of 132 km.
- Determine his average speed in km/h.
 - Did he drive according to the speed limit on the South African roads?
 - What should his speed have been to keep within the speed limit?
 - If he kept the speed limit, how long would it take him then?
- (h) The Gautrain departs at 07:42 from a station and reaches the next station at 8:04. How far are the stations apart if the train moves at 130 km/h?

- Ernst runs 400 meters in 58 seconds.
- What is his average speed in m/s?
- What is his average speed in km/h?
- Felix Baumgartner set the record for the highest manned balloon flight and fastest speed of free fall at 1357,64 km/h, making him the first human to break the sound barrier (343,2m/s) outside of a vehicle. The mission took place on 14 October 2012 when Baumgartner landed in eastern New Mexico after jumping from a world record height of 38 969,3 m above the earth
- Calculate his free fall speed in m/s
- How much faster was this than the sound barrier?
- What was his height in kilometres above the earth?
- How long ago did this event take place? (Round your answer to the nearest month)



PRODUCT PRICING

Best Buy

 300ml	$300\text{ml} + 10 = 30\text{ml}$ $R17,95 + 10 = \underline{R1,79}$
 600ml	$600\text{ml} + 20 = 30\text{ml}$ $R34,99 + 20 = \underline{R1,70} \checkmark$

- Consider the advertisement on the left, work out the rate per 100 ml in both cases and determine which one is the cheaper to buy.
- Sandra bought 7 meters of material for R84. What is the price of the material per meter? What is the rate in this case?
- A wholesaler buys a container with 60 light bulbs for R261. How much did he pay per bulb?
- 500g margarine costs R7,35 and 350g of the same kind costs R5,50. Which size is the cheaper to buy?

Proportion

Exercise 10: Proportion

Answer the following questions:

- (a) A truck, 3,5 m high, casts a shadow of 10,5 m long. How long is the shadow of a building, 20 m high, at the same moment?
- (b) 150 one litre bottles of cooldrink are needed to fill up a big tank. How many 1,5 litre bottles are needed to fill up the same tank?
- (c) An aeroplane, flying at a speed of 450 km/h, covers a certain distance in 3 hours and 15 min. At what speed must it fly to cover the same distance in 2 hours and 30 min?

- (d) A distance of 10 km is represented on a map by 1,5 cm.

How is a distance of 50 km represented on this map?

The distance between 2 towns on this map is 10 cm.

What is the real distance?

- (e) A strip of metal, 16 cm long, has a mass of 60 g.

Calculate the mass of a strip which is 8 cm long.

How long is a strip with a mass of 120 g?

- (f) When eight check-out points at a supermarket are open, it takes an average of 48 minutes to deal with 100 customers. If 12 check-out points were open, how long would it take to deal with the same 100 hundred customers? Is this an example of direct or indirect proportion?



Complete the table below:

Amount of check-out points	2	6	B	12
×	×	×	×	×
Amount of minutes for 100 customers	24	A	6	C
Total minutes	48	48	48	48

Percentages

A percentage expresses a part of 100. E.g. 45% means $\frac{45}{100}$

How to convert percentage to ordinary fractions	E.g. 45% means $\frac{45}{100} = \frac{9}{20}$
How to convert percentage to decimal fractions	E.g. $33,3\% = \frac{33,3}{100} = 0,333$
How to convert ordinary fractions to percentage:	E.g. $\frac{1}{4} \rightarrow \frac{1}{4} \times 100 = \frac{100}{4} = 25\%$ Use a calculator: $(1 \div 4 \times 100=)$ Don't use the %-sign on the calculator!
How to convert decimal fractions to percentage	E.g. $0,257 \rightarrow 0,257 \times 100 = 25,7\%$

Exercise 11: Percentages

Answer the following questions:

- (a) A man bequeaths 50% of his estate to his wife, 20 % of the balance to each of his 2 children and the rest to welfare. What amount goes to welfare if the estate is worth

R3 000 000?

- (b) A salesman receives a commission of 5% on his sales, as well as a weekly salary of R1 500. In one week he had sales of R13 000. How much commission did he earn?

Express his commission as a percentage of his weekly salary.

- (c) 18% of a farmer's wheat crop is 1 260 tons. What is the weight of the total crop?
- (d) A lottery win of R36 000 was split up between three people so that Mary received 25% of the win, Bill got 13% and Sam got the remainder. Find the percentage of the win that Sam received and the amount that Mary received.

- (e) How much will I save on buying these items in the 15%-off sale:

15% off everything SALE	
Tennis racquet - normally	R420,54
Pair of shorts - normally	R69,67
Picture frame - normally	R43,90
Alarm clock - normally	R38,96
Stereo radio - normally	R109,78