

Graad 10 – Boek C **(Hersiene KABV uitgawe)**

ONDERWYSERS HANDLEIDING

INHOUD:

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Besoek ook www.abcmathsandscience.co.za vir ekstra oefeninge, toetse en vraestelle.

Hoofstuk C1

Trigonometrie

C1.1 Inleiding tot Trigonometrie:

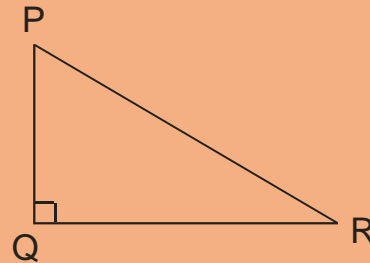
Trigonometrie is die studie van die verhouding tussen die hoeke en sye van driehoeke. In graad 9 het ons reeds gelykvormigheid bestudeer. Gelykvormige driehoeke is driehoeke waarvan al drie pare ooreenstemmende hoeke gelyk is aan mekaar óf as die ooreenstemmende pare sye eweredig is (in dieselfde verhouding is.) Gelykvormige driehoeke het dus dieselfde vorm, maar nie noodwendig dieselfde grootte nie!

Terme: In 'n reghoekige driehoek word die sye en hoeke as volg benoem:

PR is die skuinssy (s).

PQ is die teenoorstaande (t) sy van \hat{R} .

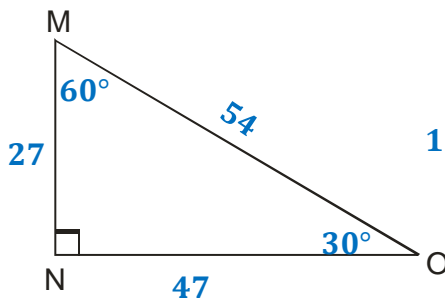
QR is \hat{R} se aangrensende (a) sy.



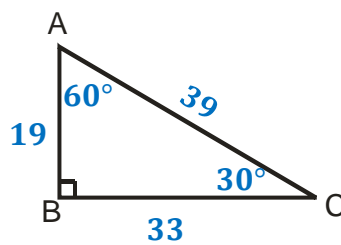
Oefening 1:

Datum: _____

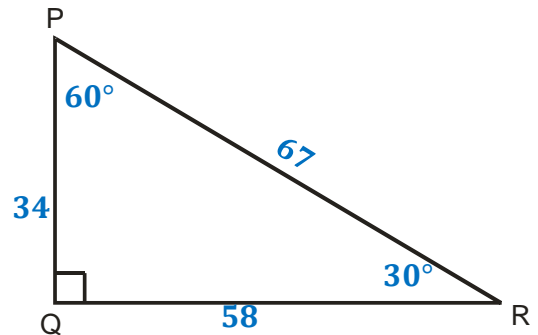
(1) Driehoek 1:



Driehoek 2:



Driehoek 3:



Meet die lengte van elke sy en die grootte van elke hoek van al die driehoeke en vervang dit as volg: (Rond af tot 2 des.) **In mm.**

Driehoek 1:

(a) $\frac{MN}{OM} = \frac{27}{54} = 0,50$

(b) $\frac{ON}{OM} = \frac{47}{54} \approx 0,87$

(c) $\frac{MN}{NO} = \frac{27}{47} \approx 0,57$

Driehoek 2:

(a) $\frac{AB}{AC} = \frac{19}{39} \approx 0,49$

(b) $\frac{BC}{AC} = \frac{33}{39} \approx 0,85$

(c) $\frac{AB}{BC} = \frac{19}{33} \approx 0,58$

Driehoek 3:

(a) $\frac{PQ}{PR} = \frac{34}{67} \approx 0,51$

(b) $\frac{QR}{PR} = \frac{58}{67} \approx 0,87$

(c) $\frac{PQ}{QR} = \frac{34}{58} \approx 0,59$

- (2) (a) Wat merk jy van die hoeke van die 3 driehoeke in (1)? **Alle ooreenst \angle^e is gelyk.**
 (b) Wat is die verband tussen ΔMNO , ΔABC en ΔPQR ? **$\Delta MNO \sim \Delta ABC \sim \Delta PQR$**
 (c) Wat merk jy i.v.m. die verhoudings van die ooreenstemmende sye soos gemeet in nr.1 a – c?
Die ooreenstemmende verhoudings in a, b en c is baie naby aan mekaar vir al drie die Δ^e .

(3) Gebruik die figuur regs en voltooi die volgende:

(a) In ΔABC en ΔADE en ΔAFG :

Bewering:

$$* \hat{A} = \hat{A} = \hat{A}$$

$$* \hat{C} = \hat{E} = \hat{G}$$

$$* \hat{B} = \hat{D} = \hat{F}$$

$$\therefore \Delta ABC \sim \Delta ADE \sim \Delta AFG$$

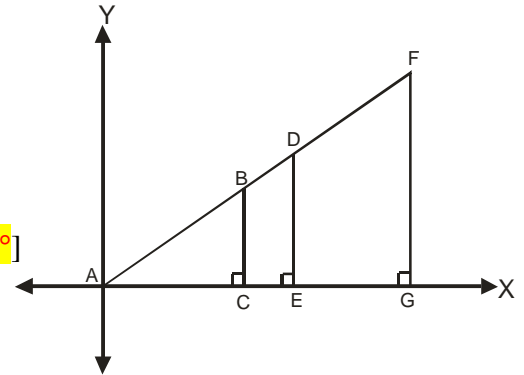
Rede:

[Gemeenskaplike \angle]

[Alle \angle^e gelyk aan 90°]

[Binne \angle^e van Δ]

[$\angle\angle\angle$]



(b) Uit (a) kan ons aflei dat: $\frac{AB}{AC} = \frac{AD}{AE} = \frac{AF}{AG}$ [Gelykvormige driehoeke]

$$\text{Net so is: } \frac{AB}{BC} = \frac{AD}{DE} = \frac{AF}{FG} \quad \text{en} \quad \frac{BC}{AC} = \frac{DE}{AE} = \frac{FG}{AG}$$

Uit oefening 1 het ons gesien dat die verhoudings van die sye van gelykvormige driehoeke dus dieselfde is. Hierdie verhouding van die sye hang dus af van die grootte van die driehoeke se hoeke.

Elk van die verskillende pare ooreenstemmende sye word as volg benoem:
 θ word die inklinasiehoek genoem en elk van die volgende verhoudings is dus afhanklik van θ !

Die sinusverhouding:

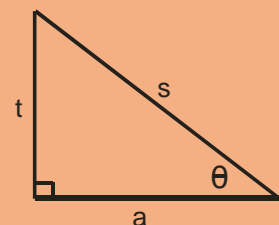
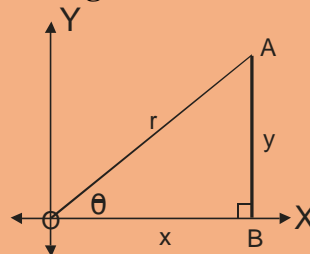
$$\sin \theta = \frac{AB}{OA} \left[\frac{\theta \text{ se teenoorstaande sy}}{\text{die skuinssy}} \right] = \frac{y}{r} = \frac{t}{s}$$

Die cosinusverhouding:

$$\cos \theta = \frac{OB}{OA} \left[\frac{\theta \text{ se aangrensende sy}}{\text{die skuinssy}} \right] = \frac{x}{r} = \frac{a}{s}$$

Die tangensverhouding:

$$\tan \theta = \frac{AB}{OB} \left[\frac{\theta \text{ se teenoorstaande sy}}{\theta \text{ se aangrensende sy}} \right] = \frac{y}{x} = \frac{t}{a}$$

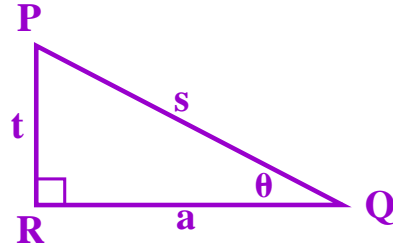


Vb.1 Skryf die volgende in terme van die sye van die driehoek:

$$(a) \quad \sin \theta = \frac{t}{s} = \frac{PR}{PQ}$$

$$(b) \quad \cos \theta = \frac{a}{s} = \frac{RQ}{PQ}$$

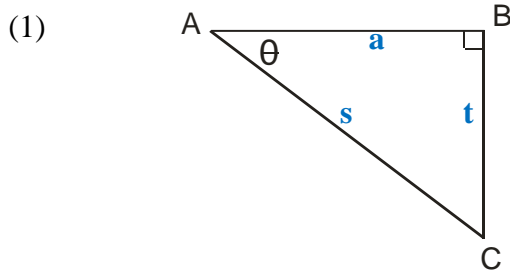
$$(c) \quad \tan \theta = \frac{t}{a} = \frac{PR}{RQ}$$



Oefening 2:

Datum: _____

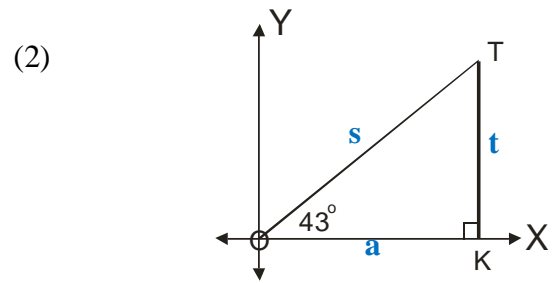
Skryf die volgende in terme van die sye van die gegewe driehoek:



$$(a) \quad \sin \theta = \frac{t}{s} = \frac{BC}{AC}$$

$$(b) \quad \cos \theta = \frac{a}{s} = \frac{AB}{AC}$$

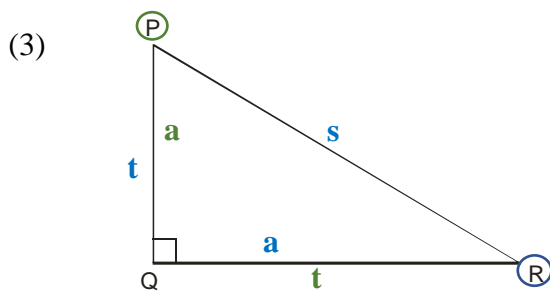
$$(c) \quad \tan \theta = \frac{t}{a} = \frac{BC}{AB}$$



$$(a) \quad \cos 43^\circ = \frac{a}{s} = \frac{OK}{OT}$$

$$(b) \quad \tan 43^\circ = \frac{t}{a} = \frac{TK}{OK}$$

$$(c) \quad \sin 43^\circ = \frac{t}{s} = \frac{TK}{OT}$$

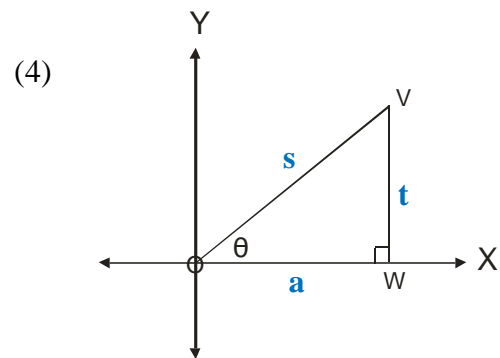


$$(a) \quad \sin \hat{R} = \frac{t}{s} = \frac{PQ}{PR}$$

$$(b) \quad \cos \hat{R} = \frac{a}{s} = \frac{QR}{PR}$$

$$(c) \quad \tan \hat{R} = \frac{t}{a} = \frac{PQ}{QR}$$

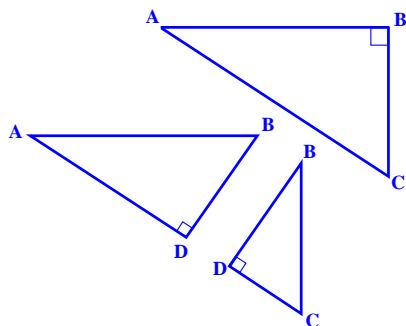
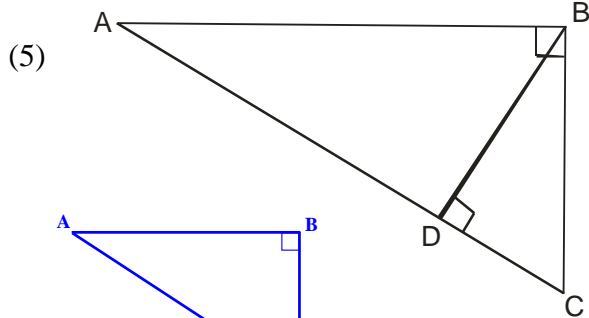
$$(d) \quad \sin \hat{P} = \frac{t}{s} = \frac{QR}{PR}$$



$$(a) \quad \cos \theta = \frac{a}{s} = \frac{OW}{OV}$$

$$(b) \quad \tan \theta = \frac{t}{a} = \frac{VW}{OW}$$

$$(c) \quad \sin \theta = \frac{t}{s} = \frac{VW}{OV}$$



$$(a) \sin \hat{C} \text{ in } \Delta ABC = \frac{t}{s} = \frac{AB}{AC}$$

$$(b) \cos \hat{A} \text{ in } \Delta ABD = \frac{a}{s} = \frac{AD}{AB}$$

$$(c) \tan \hat{B} \text{ in } \Delta ABD = \frac{t}{a} = \frac{AD}{BD}$$

$$(d) \cos \hat{B} \text{ in } \Delta BDC = \frac{a}{s} = \frac{BD}{BC}$$

$$(e) \sin \hat{C} \text{ in } \Delta BDC = \frac{t}{s} = \frac{BD}{BC}$$

$$(f) \tan \hat{A} \text{ in } \Delta ABC = \frac{t}{a} = \frac{BC}{AB}$$

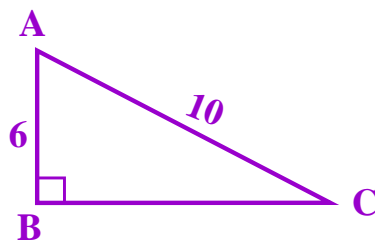
C1.2 Gebruik van Pythagoras:

Vb.2 Bereken die volgende verhoudings:

(a) $\sin \hat{A}$

(b) $\tan \hat{A}$

(c) $\cos \hat{C}$



Bereken eers die lengte van BC deur gebruik te maak van die stelling van Pythagoras.

$$AC^2 = AB^2 + BC^2$$

$$10^2 = 6^2 + BC^2$$

$$100 = 36 + BC^2$$

$$100 - 36 = BC^2$$

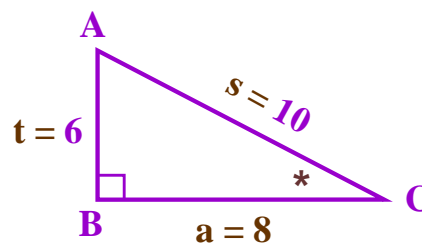
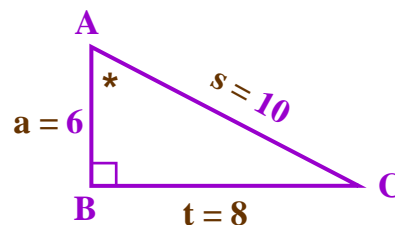
$$\therefore BC^2 = 64$$

$$\therefore BC = 8$$

$$\therefore (a) \sin \hat{A} = \frac{t}{s} = \frac{8}{10} = 0,8$$

$$\therefore (b) \cos \hat{A} = \frac{a}{s} = \frac{6}{10} = 0,6$$

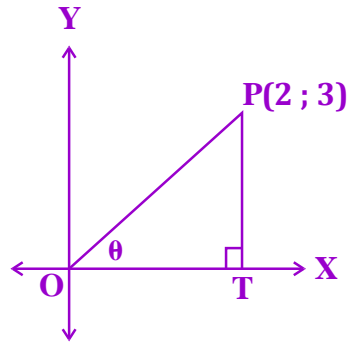
$$\therefore (c) \tan \hat{C} = \frac{t}{a} = \frac{6}{8} = 0,75$$



Vb.3 Bereken die volgende verhoudings:

(a) $\cos \theta$

(b) $\tan \theta$



Bereken eers weer die lengte van OP met behulp van die stelling van Pythagoras.

$$\therefore OP^2 = OT^2 + PT^2$$

$$OP^2 = 2^2 + 3^2$$

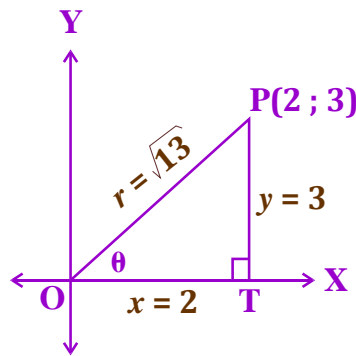
$$OP^2 = 4 + 9$$

$$OP^2 = 13$$

$$OP = \sqrt{13}$$

$$\therefore \text{(a) } \cos \theta = \frac{x}{r} = \frac{2}{\sqrt{13}}$$

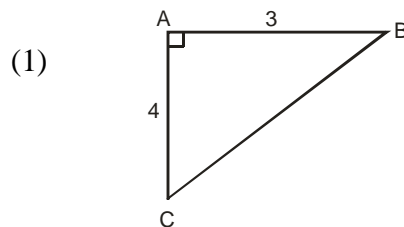
$$\therefore \text{(b) } \tan \theta = \frac{y}{x} = \frac{3}{2}$$



Oefening 3:

Datum: _____

Bereken:



(a) $BC^2 = AB^2 + AC^2$

$$BC^2 = 3^2 + 4^2$$

$$BC^2 = 9 + 16$$

$$BC^2 = 25$$

$$\therefore BC = 5$$

(a) die lengte van BC.

(b) $\sin \hat{B}$

(c) $\tan \hat{B}$

(d) $\cos \hat{B}$

(e) $\cos \hat{C}$

(f) $\sin \hat{C}$

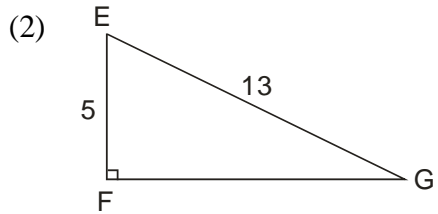
(b) $\sin \hat{B} = \frac{t}{s} = \frac{4}{5}$

(c) $\tan \hat{B} = \frac{t}{a} = \frac{4}{3}$

(d) $\cos \hat{B} = \frac{a}{s} = \frac{3}{5}$

(e) $\cos \hat{C} = \frac{a}{s} = \frac{4}{5}$

(f) $\sin \hat{C} = \frac{t}{s} = \frac{3}{5}$



$$EG^2 = EF^2 + FG^2$$

$$13^2 = 5^2 + FG^2$$

$$169 = 25 + FG^2$$

$$FG^2 = 169 - 25 = 144$$

$$\therefore FG = 12$$

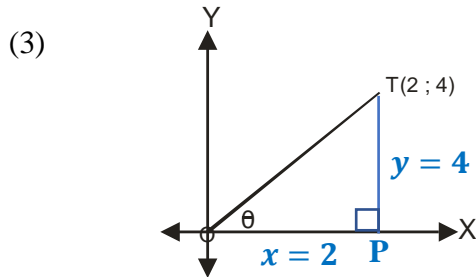
- (a) $\cos \hat{E}$
 (b) $\sin \hat{G}$
 (c) $\tan \hat{E}$
 (d) $\tan \hat{G}$

$$(a) \cos \hat{E} = \frac{a}{s} = \frac{5}{13}$$

$$(b) \sin \hat{G} = \frac{t}{s} = \frac{5}{13}$$

$$(c) \tan \hat{E} = \frac{t}{a} = \frac{12}{5}$$

$$(d) \tan \hat{G} = \frac{t}{a} = \frac{5}{12}$$



- (a) die lengte van OT.
 (b) $\cos \theta$
 (c) $\sin \theta$
 (d) $\tan \theta$

$$(a) OT^2 = OP^2 + TP^2$$

$$OT^2 = 2^2 + 4^2$$

$$OT^2 = 4 + 16$$

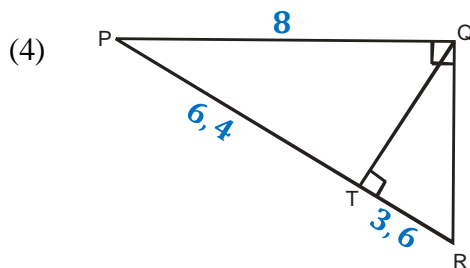
$$OT^2 = 20$$

$$\therefore OT = \sqrt{20}$$

$$(b) \cos \theta = \frac{x}{r} = \frac{2}{\sqrt{20}}$$

$$(c) \sin \theta = \frac{y}{r} = \frac{4}{\sqrt{20}}$$

$$(d) \tan \theta = \frac{y}{x} = \frac{4}{2} = 2$$



As $PQ = 8$, $PT = 6,4$ en $TR = 3,6$; bereken:

- (a) die lengtes van QR en QT.
 (b) $\cos R$ in ΔQTR
 (c) $\tan P$ in ΔPQR
 (d) $\sin Q$ in ΔPQT
 (e) $\sin Q$ in ΔQTR

(a) In ΔPQR :

$$PR^2 = PQ^2 + QR^2$$

$$QR^2 = 10^2 - 8^2$$

$$QR^2 = 100 - 64$$

$$QR^2 = 36$$

$$\therefore QR = 6$$

In ΔPQT :

$$PQ^2 = PT^2 + TQ^2$$

$$TQ^2 = 8^2 - 6,4^2$$

$$TQ^2 = 64 - 40,96$$

$$TQ^2 = 23,04$$

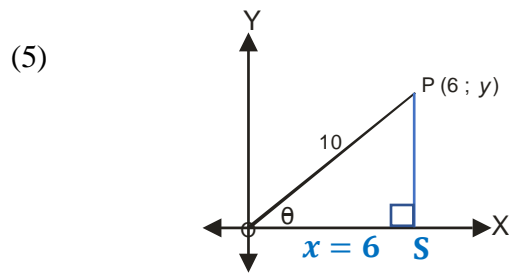
$$\therefore TQ = 4,8$$

$$(b) \cos \hat{R} = \frac{a}{s} = \frac{3,6}{6} = \frac{3}{5}$$

$$(c) \tan \hat{P} = \frac{t}{a} = \frac{6}{8} = \frac{3}{4}$$

$$(d) \sin \hat{Q} = \frac{t}{s} = \frac{6,4}{8} = \frac{4}{5}$$

$$(e) \sin Q = \frac{t}{s} = \frac{3,6}{6} = \frac{3}{5}$$



- (a) y
 (b) $\sin \theta$
 (c) $\cos \theta$
 (d) $\tan \theta$

(a) $OP^2 = PS^2 + OS^2$

$$10^2 = y^2 + 6^2$$

$$y^2 = 100 - 36$$

$$y^2 = 64$$

$$\therefore y = 8$$

(b) $\sin \theta = \frac{y}{r} = \frac{8}{10} = \frac{4}{5}$

(c) $\cos \theta = \frac{x}{r} = \frac{6}{10} = \frac{3}{5}$

(d) $\tan \theta = \frac{y}{x} = \frac{8}{6} = \frac{4}{3}$

C1.3 Gebruik van die sakrekenaar:

C1.3.1 Grade, minute en sekondes:

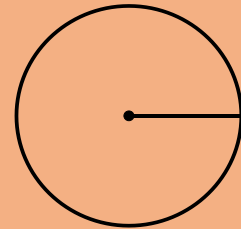
Indien opmeting gedoen word, word gebruik gemaak van afstande en hoekgroottes. Ons weet reeds dat 'n omwenteling, m.a.w. die volle draai van 'n sirkel, 360 grade (360°) is.

Elke graad is die hoek by die middelpunt van 'n sirkel wat die grootte van die boog beskryf wat dan 'n breukdeel van die omtrek van die sirkel voorstel.

$\therefore 1^\circ$ is gelykstaande aan $\frac{1}{360}$ ste van die omtrek van die sirkel.

Een minuut ($1'$) is gelykstaande aan $\frac{1}{60}$ ste van 'n graad.

Een sekonde ($1''$) is gelykstaande aan $\frac{1}{60}$ ste van 'n minuut.



Vb.4 (a) **Beskryf die volgende hoekgrootte: $13^\circ 24' 36''$**

13 grade, 24 minute en 36 sekondes.

(b) **Skakel die volgende om na grade alleenlik: $13^\circ 24' 36''$**

$$\begin{aligned} 13^\circ 24' 36'' &= 13^\circ + 24' + \frac{36}{60}' = 13^\circ + 24' + 0,6' \\ &= 13^\circ + 24,6' = 13^\circ + \frac{24,6}{60}^\circ \\ &= 13^\circ + 0,41^\circ = \mathbf{13,41^\circ} \end{aligned}$$

(c) **Skakel die volgende om na grade en minute: $64,3^\circ$**

$$64,3^\circ = 64^\circ + 0,3^\circ = 64^\circ + (0,3 \times 60)' = \mathbf{64^\circ 18'}$$

Oefening 4:

Datum: _____

(1) Skakel die volgende om na grade alleenlik:

(a) $72^{\circ} 24'$

$$= 72^{\circ} + \frac{24}{60}^{\circ}$$

$$= 72^{\circ} + 0,4^{\circ}$$

$$= 72,4^{\circ}$$

(b) $88^{\circ} 33'$

$$= 88^{\circ} + \frac{33}{60}^{\circ}$$

$$= 88^{\circ} + 0,55^{\circ}$$

$$= 88,55^{\circ}$$

(c) $324^{\circ} 48'$

$$= 324^{\circ} + \frac{48}{60}^{\circ}$$

$$= 324^{\circ} + 0,8^{\circ}$$

$$= 324,8^{\circ}$$

(d) $25^{\circ} 12' 36''$

$$= 25^{\circ} + 12' + \frac{36}{60}'$$

$$= 25^{\circ} + 12' + 0,6'$$

$$= 25^{\circ} + 12,6'$$

$$= 25^{\circ} + \frac{12,6}{60}^{\circ}$$

$$= 25,21^{\circ}$$

(e) $112^{\circ} 36' 54''$

$$= 112^{\circ} + 36' + \frac{54}{60}'$$

$$= 112^{\circ} + 36' + 0,9'$$

$$= 112^{\circ} + 36,9'$$

$$= 112^{\circ} + \frac{36,9}{60}^{\circ}$$

$$= 112,615^{\circ}$$

(f) $7^{\circ} 6' 18''$

$$= 7^{\circ} + 6' + \frac{18}{60}'$$

$$= 7^{\circ} + 6' + 0,3'$$

$$= 7^{\circ} + 6,3'$$

$$= 7^{\circ} + \frac{6,3}{60}^{\circ}$$

$$= 7,105^{\circ}$$

(2) Skakel die volgende om na grade en minute:

(a) $38,5^{\circ}$

$$= 38^{\circ} + 0,5^{\circ}$$

$$= 38^{\circ} + (0,5 \times 60)'$$

$$= 38^{\circ} 30'$$

(b) $101,7^{\circ}$

$$= 101^{\circ} + 0,7^{\circ}$$

$$= 101^{\circ} + (0,7 \times 60)'$$

$$= 101^{\circ} 42'$$

(c) $16,45^{\circ}$

$$= 16^{\circ} + 0,45^{\circ}$$

$$= 16^{\circ} + (0,45 \times 60)'$$

$$= 16^{\circ} 27'$$

C1.3.2 Die sakrekenaar:**C1.3.2.1 Trigonometriese uitdrukkings:****ONTHOU:** Die sakrekenaar moet op “deg” wees!

Maak gebruik van 'n nie-programmeerbare, wetenskaplike sakrekenaar!

Vb.5 Bereken die volgende, korrek tot twee desimale:

<u>Uitdrukking:</u>	<u>Vertoon:</u>	<u>2 des. plekke:</u>	<u>Sleutels:</u>
(a) $\sin 12^{\circ}$	$= 0,2079 \dots$	$\approx 0,21$	$\sin 12 =$
(b) $\cos 42^{\circ} 12'$	$= 0,7408 \dots$	$\approx 0,74$	$\cos 42^{\circ} 12' =$
(c) $2 \tan 77^{\circ}$	$= 8,6629 \dots$	$\approx 8,66$	$2 \tan 77 =$
(d) $\cos^2 44^{\circ}$	$= 0,5174 \dots$	$\approx 0,52$	$\cos 44)^2$ of $(\cos 44)x^2 =$
(e) $4 - \tan 220^{\circ}$	$= 3,1609 \dots$	$\approx 3,16$	$4 - \tan 220 =$
(f) $\frac{\sin 67^{\circ}}{3}$	$= 0,3068 \dots$	$\approx 0,31$	$\sin 67 \div 3 =$

Oefening 5:

Datum: _____

(1) Bereken die volgende, korrek tot twee desimale:

- (a) $\sin 33^\circ = 0,5446 \dots \approx 0,54$ (b) $\cos 56^\circ = 0,5591 \dots \approx 0,56$
 (c) $\tan 11,5^\circ = 0,2034 \dots \approx 0,20$ (d) $\sin 145^\circ = 0,5735 \dots \approx 0,57$
 (e) $\sin 301^\circ = -0,8571 \dots \approx -0,86$ (f) $\cos 201^\circ 24' = -0,9310 \dots \approx -0,93$
 (g) $\tan 88^\circ 56' = 53,7085 \dots \approx 53,71$ (h) $\cos 345^\circ = 0,9659 \dots \approx 0,97$
 (i) $\sin 23,4^\circ = 0,3971 \dots \approx 0,40$ (j) $\tan 66^\circ 34' = 2,3071 \dots \approx 2,31$
 (k) $\cos 64,1^\circ = 0,4368 \dots \approx 0,44$ (l) $\tan 6,6^\circ = 0,1157 \dots \approx 0,12$
 (m) $\sin 12^\circ 12' = 0,2113 \dots \approx 0,21$ (n) $\cos 0,5^\circ = 0,9999 \dots \approx 1,00$

(2) Bereken die volgende, korrek tot 1 desimaal: (Skryf jou sleutels neer!)

- (a) $2 \sin 34^\circ = 1,1183 \dots \approx 1,1$ (b) $3,5 + \cos 200^\circ = 2,5603 \dots \approx 2,6$
 $[2 \sin 34 =]$ $[3,5 + \cos 200 =]$
 (c) $\tan^2 130^\circ = 1,4202 \dots \approx 1,4$ (d) $\frac{\cos 71^\circ}{2} = 0,1627 \dots \approx 0,2$
 $[(\tan 130)x^2]$ of $[\tan 130 = x^2 =]$ $[\cos 71 = \div 2 =]$ of $[\frac{\cos (71)}{2} =]$
 (e) $\sin(32^\circ + 12^\circ) = 0,6945 \dots \approx 0,7$ (f) $\cos 176^\circ - \cos 76^\circ = -1,23 \dots \approx -1,2$
 $[\sin(32 + 12) =]$ $[\cos 176] - \cos 76 =]$
 (g) $\sqrt{\sin 16^\circ} = 0,525 \dots \approx 0,5$ (h) $\tan 100^\circ + 7,1 = 1,4287 \dots \approx 1,4$
 $[\sqrt{\sin 16} =]$ $[\tan 100] + 7.1 =]$
 (i) $4 \div \sin 133^\circ 24' = 5,5052 \dots \approx 5,5$ (j) $\sin^3 72,12^\circ = 0,8619 \dots \approx 0,9$
 $[4 \div \sin 133^\circ 24' =]$ $[(\sin 72.12)x^3 =]$ of $[\sin 72.12 = x^3 =]$
 (k) $7 + \frac{\tan 100^\circ}{2} = 4,1643 \dots \approx 4,2$ (l) $\cos(4 \times 31,3^\circ) = -0,576 \dots \approx -0,6$
 $[7 + \tan 100] \div 2 =]$ $[\cos(4 \times 31.3 =]$
 (m) $\sqrt{10 \cos 300^\circ} = 2,236 \dots \approx 2,2$ (n) $\sin 30^\circ \times \cos 30^\circ = 0,433 \dots \approx 0,4$
 $[\sqrt{10 \cos 300} = S \Leftrightarrow D]$ $[\sin 30) \times \cos 30 = S \Leftrightarrow D]$
 (o) $-7,1 - \sin 304^\circ = -6,270 \dots \approx -6,3$ (p) $1,6 - 2 \times \cos^2 123^\circ = 1,006 \dots \approx 1,0$
 $[(-)7.1 - \sin 304 =]$ $[1.6 - 2(\cos 123) x^2 =]$

C1.3.2.2 Trigonometriese vergelykings:

Ons het gesien dat bv. $\sin 30^\circ = 0,5$
 \therefore as $\sin x = 0,5$ kan ons aflei dat $x = 30^\circ$ as $x \in [0^\circ; 90^\circ]$

Vb.6 Los op vir x as $x \in [0^\circ; 90^\circ]$; korrek tot 1 desimaal:

(a) $\cos x = 0,34$

(b) $2 \tan x = 4,64$

(c) $\sin 3x = 0,7$

(a) $\cos x = 0,34$

$\therefore x \approx 70,1^\circ$

[Sleutels: Shift $\cos^{-1} 0.34 =]$

(b) $\tan x = \frac{4,64}{2}$

$\tan x = 2,32$

$\therefore x \approx 66,7^\circ$

[Sleutels: Shift $\tan^{-1} 2.32 =]$

(c) $\sin 3x = 0,7$

$\therefore 3x = 44,427 \dots$

$\therefore x \approx 14,8^\circ$

[Sleutels: Shift $\sin^{-1} 0.7 = \div 3 =]$

Oefening 6:

Datum: _____

Los op vir x as $x \in [0^\circ; 90^\circ]$; korrek tot 1 desimaal:

(1) $\sin x = 0,34$

$\therefore x \approx 19,9^\circ$

(2) $\cos x = 0,551$

$\therefore x \approx 56,6^\circ$

(3) $\tan x = 6,9$

$\therefore x \approx 81,8^\circ$

(4) $\cos x = \frac{1}{2}$

$\therefore x \approx 60,0^\circ$

(5) $\tan x = 44,4$

$\therefore x \approx 88,7^\circ$

(6) $\sin x = 0,881$

$\therefore x \approx 61,8^\circ$

(7) $\cos x = 0,401$

$\therefore x \approx 66,4^\circ$

(8) $\sin x - 0,2 = 0$

$\sin x = 0,2$

$\therefore x \approx 11,5^\circ$

(9) $\tan x = 2 \times 3$

$\tan x = 6$

$\therefore x \approx 80,5^\circ$

(10) $4 \sin x = 0,1$

$\sin x = \frac{0,1}{4}$

$\sin x = 0,025$

$\therefore x \approx 1,4^\circ$

(11) $\tan 3x = 6$

$\therefore 3x = 80,53 \dots^\circ$

$\therefore x \approx 26,8^\circ$

(12) $\cos(x + 10^\circ) = 0,9$

$\therefore x + 10^\circ = 25,84 \dots^\circ$

$\therefore x \approx 15,8^\circ$

(13)	$\cos x + 2 = 2,444$	(14)	$\tan^2 x = 0,64$	(15)	$\frac{\sin x}{2} = 0,1$
	$\cos x = 2,444 - 2$		$\tan x = \sqrt{0,64}$		$\sin x = 0,1 \times 2$
	$\cos x = 0,444$		$\tan x = 0,8$		$\sin x = 0,2$
	$\therefore x \approx 63,6^\circ$		$\therefore x \approx 38,7^\circ$		$\therefore x \approx 11,5^\circ$
(16)	$\tan(x - 10^\circ) = 20$	(17)	$\cos 3x = 0,688$	(18)	$\cos x - 3 = -2,445$
	$x - 10^\circ = 87,13..^\circ$		$3x = 46,52..^\circ$		$\cos x = -2,445 + 3$
	$x = 87,13..^\circ + 10^\circ$		$x = \frac{46,52..^\circ}{3}$		$\cos x = 0,555$
	$\therefore x \approx 97,1^\circ$		$\therefore x \approx 15,5^\circ$		$\therefore x \approx 56,3^\circ$
(19)	$-2,3 \tan x = -3,2$	(20)	$\sin \frac{x}{2} = 0,5$	(21)	$\frac{2}{3} \sin x = \frac{1}{2}$
	$\tan x = \frac{-3,2}{-2,3}$		$\frac{x}{2} = 30^\circ$		$\sin x = \frac{1}{2} \times \frac{3}{2}$
	$\tan x = 1,391..$		$x = 30^\circ \times 2$		$\sin x = \frac{3}{4} = 0,75$
	$\therefore x \approx 54,3^\circ$		$\therefore x = 60^\circ$		$\therefore x \approx 48,6^\circ$
(22)	$\tan(2x - 15^\circ) = 2$	(23)	$\frac{\cos 2x}{2} = 0,2$	(24)	$\sin x = \tan 25^\circ$
	$2x - 15^\circ = 63,43..^\circ$		$\cos 2x = 0,2 \times 2$		$\sin x = 0,466..$
	$2x = 63,43..^\circ + 15^\circ$		$\cos 2x = 0,4$		$\therefore x \approx 27,8^\circ$
	$2x = \frac{78,43..^\circ}{2}$		$2x = 66,42 ...^\circ$		
	$\therefore x \approx 39,2^\circ$		$\therefore x \approx 33,2^\circ$		

C1.3.2.3 Kombinasies:

Vb.7 Bereken $5 \sin 2A$ as $3 + \tan A = 4,2$ en $A \in [0^\circ; 90^\circ]$. Rond A af korrek tot 1 desimaal en die antwoord korrek af tot 3 desimale.

As	$3 + \tan A = 4,2$	\therefore	$5 \sin 2A$
\therefore	$\tan A = 4,2 - 3$		$= 5 \sin (2 \times 50,2^\circ)$
	$\tan A = 1,2$		$= 5 \sin 100,4^\circ$
\therefore	$A = 50,2^\circ$		$\approx 4,918$

Oefening 7:

Datum: _____

Rond alle hoeken af tot 1 desimaal en elke funktiewaarde af tot drie desimale!

(1) Bereken $\sin^2 \theta$ as $2 \cos \theta = 0,31$ en $\theta \in [0^\circ; 90^\circ]$.

$$\begin{aligned}
 2 \cos \theta &= 0,31 & \therefore \sin^2 \theta \\
 \cos \theta &= \frac{0,31}{2} & = (\sin 81,1^\circ)^2 \\
 \cos \theta &= 0,155 & = 0,97606 \dots \\
 \therefore \theta &\approx 81,1^\circ & \approx 0,976
 \end{aligned}$$

(2) As $-2 \tan A = -2$ en $0^\circ \leq A \leq 90^\circ$, bereken $\cos(A + 12^\circ)$.

$$\begin{aligned}
 -2 \tan A &= -2 & \therefore \cos(A + 12^\circ) \\
 \tan A &= \frac{-2}{-2} & = \cos(45^\circ + 12^\circ) \\
 \tan A &= 1 & = \cos(57^\circ) \\
 \therefore A &= 45^\circ & \approx 0,545
 \end{aligned}$$

(3) As $\cos 2x = 0,4$ en $x \in [0^\circ; 90^\circ]$, bereken $\cos^2 x + 3 \tan x$.

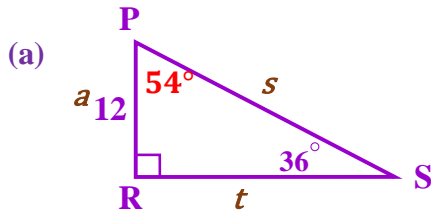
$$\begin{aligned}
 \cos 2x &= 0,4 & \therefore \cos^2 x + 3 \tan x \\
 2x &= 66,421 \dots^\circ & = (\cos 33,2^\circ)^2 + 3 \tan 33,2^\circ \\
 x &= \frac{66,421 \dots^\circ}{2} & = 2,6633 \dots \\
 \therefore x &\approx 33,2^\circ & \approx 2,663
 \end{aligned}$$

(4) Bereken $\frac{\sin \theta + \cos \theta}{-3,1}$ as $0^\circ \leq \theta \leq 90^\circ$ en $\tan(\theta - 25^\circ) = 2,1$.

$$\begin{aligned}
 \tan(\theta - 25^\circ) &= 2,1 & \therefore \frac{\sin \theta + \cos \theta}{-3,1} \\
 (\theta - 25^\circ) &= 64,536 \dots^\circ & = \frac{\sin 89,5^\circ + \cos 89,5^\circ}{-3,1} \\
 \theta &= 64,536 \dots^\circ + 25^\circ & = -0,3253 \dots \\
 \therefore \theta &\approx 89,5^\circ & \approx -0,325
 \end{aligned}$$

C1.4 Oplos van reghoekige driehoeke:

Vb.8 Bereken die onbekende hoek en sye in elk van die volgende driehoeke:
Rond korrek af tot een desimaal!



$$* \hat{P} = 90^\circ - 36^\circ = 54^\circ$$

$$* \tan \hat{P} = \frac{t}{a}$$

$$\tan 54^\circ = \frac{RS}{12}$$

$$12 \tan 54^\circ = RS$$

$$\therefore 16,5 = RS$$

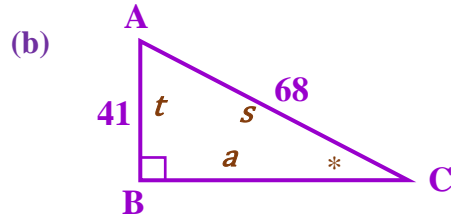
$$* PS^2 = PR^2 + RS^2$$

$$PS^2 = 12^2 + 16,5^2$$

$$PS^2 = 144 + 272,25$$

$$PS^2 = 416,25$$

$$PS = 20,4$$



$$* \sin \hat{C} = \frac{t}{s}$$

$$\sin \hat{C} = \frac{41}{68}$$

$$\sin \hat{C} = 0,602 \dots$$

$$\therefore \hat{C} = 37,080 \dots \approx 37,1^\circ$$

$$* AC^2 = AB^2 + BC^2$$

$$68^2 = 41^2 + BC^2$$

$$4624 = 1681 + BC^2$$

$$4624 - 1681 = BC^2$$

$$2943 = BC^2$$

$$54,2 = BC$$

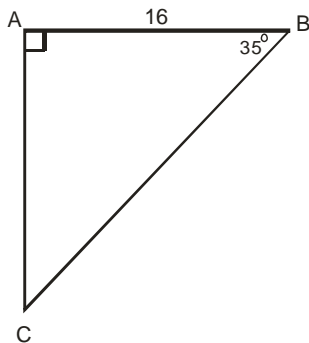
$$* \hat{A} = 90^\circ - 37,1^\circ = 52,9^\circ$$

Oefening 8:

Datum: _____

Los die volgende driehoeke op, korrek tot een desimaal:

(1)



$$* \hat{C} = 180^\circ - 90^\circ - 35^\circ = 55^\circ$$

[Binne \angle^e van Δ]

$$* \tan \hat{B} = \frac{t}{a} = \frac{AC}{AB}$$

$$\therefore \tan 35^\circ = \frac{AC}{16}$$

$$\therefore 16 \times \tan 35^\circ = AC$$

$$\therefore AC = 11,2$$

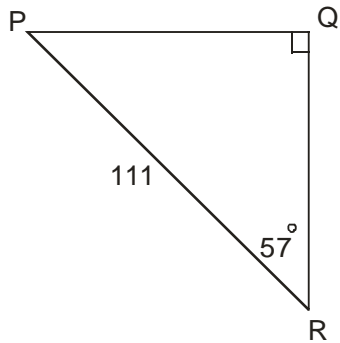
$$* BC^2 = AC^2 + AB^2$$

$$BC^2 = (11,2)^2 + (16)^2$$

$$BC^2 = 381,44$$

$$\therefore BC = 19,5$$

(2)



$$* \hat{P} = 180^\circ - 90^\circ - 57^\circ = 55^\circ$$

[Binne \angle^e van Δ]

$$* \sin \hat{R} = \frac{t}{s} = \frac{PQ}{PR}$$

$$\therefore \sin 57^\circ = \frac{PQ}{111}$$

$$\therefore 111 \times \sin 57^\circ = PQ$$

$$\therefore PQ = 93,1$$

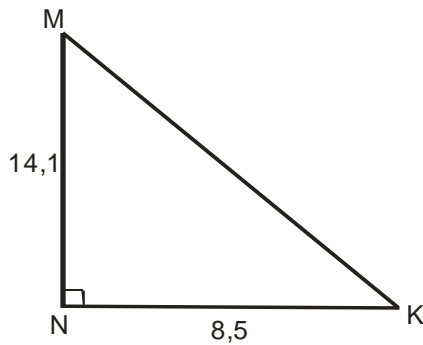
$$* QR^2 = PR^2 - PQ^2$$

$$QR^2 = (111)^2 - (93,1)^2$$

$$QR^2 = 3\,653,39$$

$$\therefore QR = 60,4$$

(3)



$$* \tan \hat{K} = \frac{t}{a} = \frac{MN}{NK}$$

$$\tan \hat{K} = \frac{14,1}{8,5} = 1,658 \dots$$

$$\therefore \hat{K} = 58,9^\circ$$

$$* \hat{M} = 90^\circ - 58,9^\circ = 31,1^\circ$$

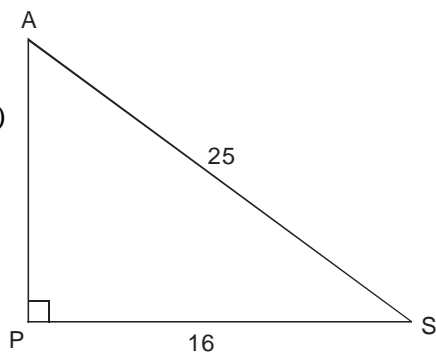
$$* MK^2 = MN^2 + NK^2$$

$$MK^2 = (14,1)^2 + (8,5)^2$$

$$MK^2 = 271,06$$

$$\therefore MK = 16,5$$

(4)



$$* \cos \hat{S} = \frac{a}{s} = \frac{PS}{AS} = \frac{16}{25}$$

$$\therefore \cos \hat{S} = 0,64$$

$$\therefore \hat{S} = 50,2^\circ$$

$$* \hat{A} = 90^\circ - 50,2^\circ = 39,8^\circ$$

$$* AP^2 = AS^2 - PS^2$$

$$AP^2 = (25)^2 - (16)^2$$

$$AP^2 = 369$$

$$\therefore AP = 19,2$$