Grade 10 – Book C

(Revised CAPS edition)

CONTENTS:

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Chapter C1

Trigonometry

C1.1 Introduction to Trigonometry:

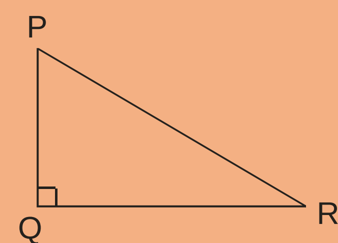
Trigonometry is the study of the relationship between the angles and sides of triangles. In grade 9 we already studied similarity. Similar triangles are triangles of which all three pairs of corresponding angles are equal or if the corresponding pairs of sides are proportional (in the same relation). Similar triangles therefore have the same shape, but not necessarily the same size!

Terminology: In a right-angled triangle the sides and angles are named as follows:

PR is the hypotenuse (h).

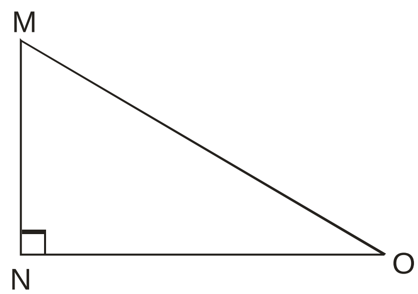
PQ is the opposite (o) side of $\hat{R}$.

QR is the adjacent (a) side of $\hat{R}$.

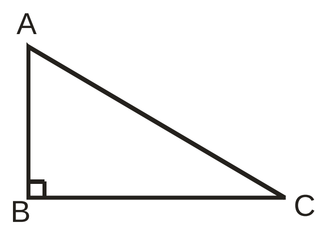


Exercise 1: Date: ________________

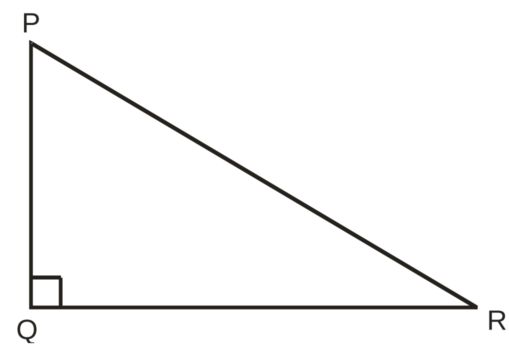
(1) Triangle 1:



Triangle 2:



Triangle 3:



Measure the length of each side and the size of each angle of all three triangles and substitute it as follows: (Round off to 2 dec.)

	Triangle 1:	Triangle 2:	Triangle 3:
(a)	$\frac{MN}{OM} = \frac{\ }{\ } \approx$ ______	$\frac{AB}{AC} = \frac{\ }{\ } \approx$ ______	$\frac{PQ}{PR} = \frac{\ }{\ } \approx$ ______
(b)	$\frac{ON}{OM} = \frac{\ }{\ } \approx$ ______	$\frac{BC}{AC} = \frac{\ }{\ } \approx$ ______	$\frac{QR}{PR} = \frac{\ }{\ } \approx$ ______
(c)	$\frac{MN}{NO} = \frac{\ }{\ } \approx$ ______	$\frac{AB}{BC} = \frac{\ }{\ } \approx$ ______	$\frac{PQ}{QR} = \frac{\ }{\ } \approx$ ______

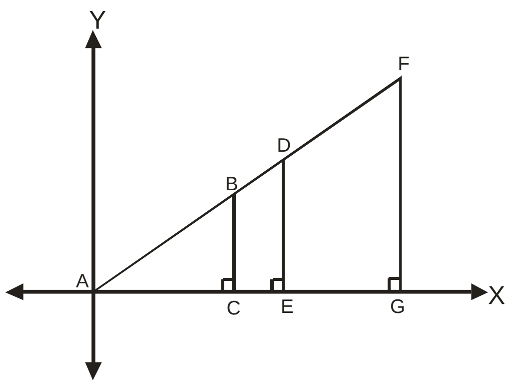
(2) (a) What do you notice from the angles in the 3 triangles in (1)? ____________________________

(b) What is the relationship between Δ MNO, Δ ABC and Δ PQR? __________________________

(c) What do you notice in terms of the ratios of the corresponding sides as measured in no.1 a - c?

____

(3) Use the figure on the right and complete the following:



(a) In Δ ABC and Δ ADE and Δ AFG:

Reason:

* $\hat{A}$ = ________ = ________ [__________________]

* $\hat{C}$ = ________ = ________ [__________________]

* $\hat{B}$ = ________ = ________ [__________________]

∴ Δ ABC /// Δ _______ /// Δ _______ [__________________]

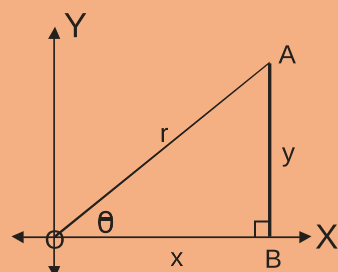
(b) From (a) we can deduce that: $\frac{AB}{AC} = \frac{AD}{AE} = \frac{AF}{AG}$ [Similar triangles]

Just like that: $\frac{AB}{BC} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$ and $\frac{BC}{AC} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

From exercise 1 we saw that the ratios of the sides of similar triangles are the same. This ratio of the sides therefore depends on the size of the triangle's angles.

Each of the different pairs of corresponding sides is named as follow:

θ is called the inclination angle and each of the following ratios are therefore dependent on θ!



The sine relationship:

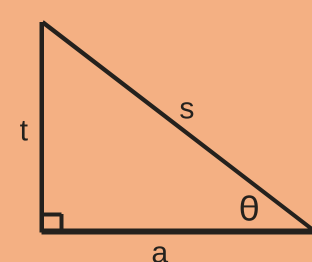
$$\sin\theta = \frac{AB}{OA} \left[\frac{\text{opposite side of }\theta}{\text{hypotenuse}}\right] = \frac{y}{r} = \frac{o}{h}$$

The cosine relationship:

$$\cos\theta = \frac{OB}{OA} \left[\frac{\text{adjacent side of }\theta}{\text{hypotenuse}}\right] = \frac{x}{r} = \frac{a}{h}$$

The tangent relationship:

$$\tan\theta = \frac{AB}{OB} \left[\frac{\text{opposite side of }\theta}{\text{adjacent side of }\theta}\right] = \frac{y}{x} = \frac{o}{a}$$

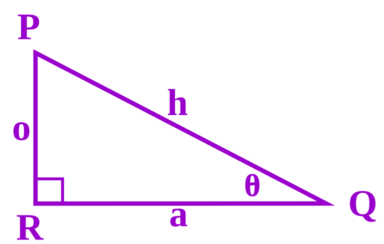


E.g.1 Write the following in terms of the sides of the triangle:

(a) $\sin\theta = \frac{o}{h} = \frac{PR}{PQ}$

(b) $\cos\theta = \frac{a}{h} = \frac{RQ}{PQ}$

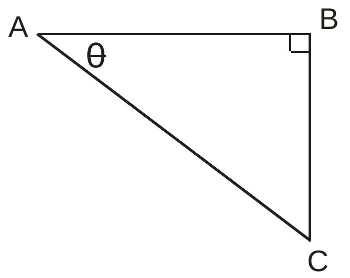
(c) $\tan\theta = \frac{o}{a} = \frac{PR}{RQ}$



Exercise 2: Date: ______________

Write the following in terms of the sides of the given triangle:

(1)

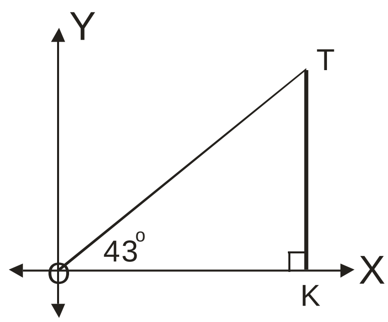


(a) $\sin\theta =$ __________

(b) $\cos\theta =$ __________

(c) $\tan\theta =$ __________

(2)

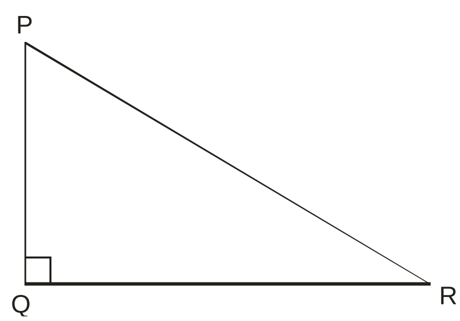


(a) $\cos 43^\circ =$ __________

(b) $\tan 43^\circ =$ __________

(c) $\sin 43^\circ =$ __________

(3)



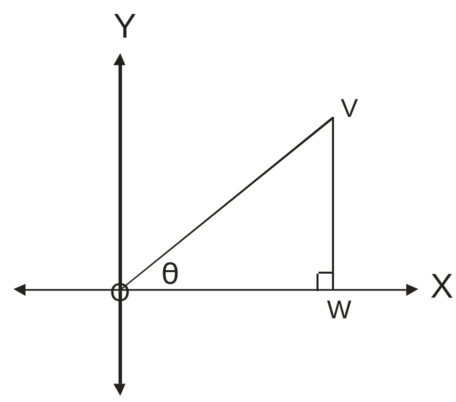
(a) $\sin\hat{R} =$ __________

(b) $\cos\hat{R} =$ __________

(c) $\tan\hat{R} =$ __________

(d) $\sin\hat{P} =$ __________

(4)

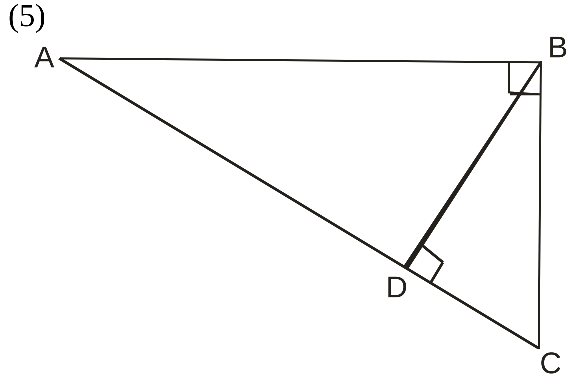


(a) $\cos\theta =$ __________

(b) $\tan\theta =$ __________

(c) $\sin\theta =$ __________

(5)



(a) sin Ĉ in Δ ABC = _ ____________

(b) cos Â in Δ ABD = ____________

(c) tan B̂ in Δ ABD = ____________

(d) cos B̂ in Δ BDC = ____________

(e) sin Ĉ in Δ BDC = ____________

(f) tan Â in Δ ABC = ____________

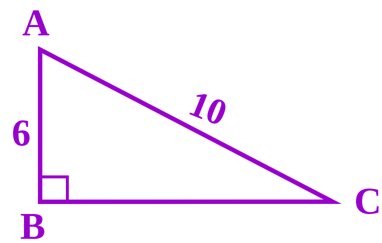
C1.2 Use of Pythagoras:

E.g.2 Calculate the following ratios:

(a) sin Â

(b) tan Â

(c) cos Ĉ



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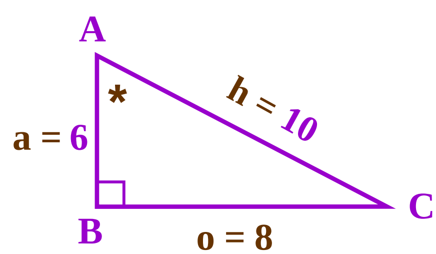
First calculate the length of BC by using the theorem of Pythagoras.

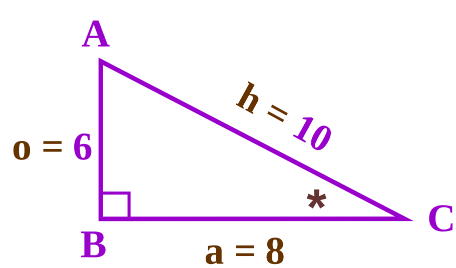
$$AC^2 = AB^2 + BC^2$$
$$10^2 = 6^2 + BC^2$$
$$100 = 36 + BC^2$$
$$100 - 36 = BC^2$$
$$\therefore BC^2 = 64$$
$$\therefore BC = 8$$



$\therefore$ (a) $\sin \hat{A} = \frac{o}{h} = \frac{8}{10} = 0,8$

$\therefore$ (b) $\cos \hat{A} = \frac{a}{h} = \frac{6}{10} = 0,6$

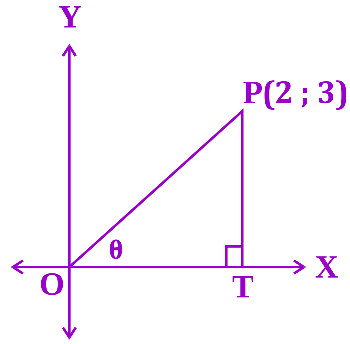
$\therefore$ (c) $\tan \hat{C} = \frac{o}{a} = \frac{6}{8} = 0,75$



E.g.3 Calculate the following ratios:

(a) cos θ

(b) tan θ



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First calculate the length of OP by using the theorem of Pythagoras.

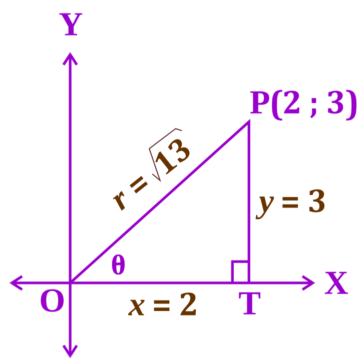
$\therefore \quad OP^2 = OT^2 + PT^2$

$OP^2 = 2^2 + 3^2$

$OP^2 = 4 + 9$

$OP^2 = 13$

$OP = \sqrt{13}$



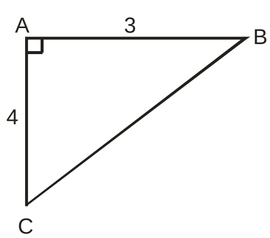
$\therefore$ **(a) cos** $\theta = \frac{x}{r} = \frac{2}{\sqrt{13}}$

$\therefore$ **(b) tan** $\theta = \frac{y}{x} = \frac{3}{2}$

Exercise 3: Date: ________________

Calculate:

(1)



(a) the length of BC.

(b) sin $\hat{B}$

(c) tan $\hat{B}$

(d) cos $\hat{B}$

(e) cos $\hat{C}$

(f) sin $\hat{C}$

____

____

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____

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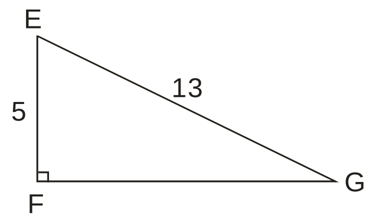
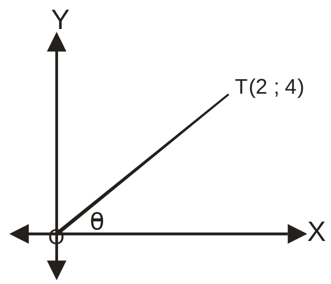
(2)



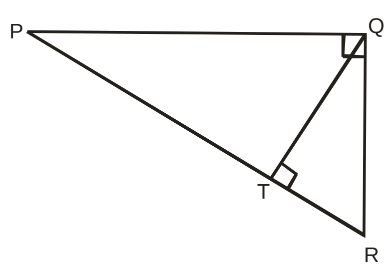
(a) $\cos \hat{E}$
(b) $\sin \hat{G}$
(c) $\tan \hat{E}$
(d) $\tan \hat{G}$

(3)



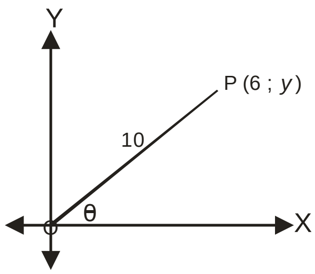
(a) the length of OT.
(b) $\cos \theta$
(c) $\sin \theta$
(d) $\tan \theta$

(4)



If PQ = 8 , PT = 6,4 and TR = 3,6 ; calculate:
(a) the lengths of QR and QT.
(b) $\cos R$ in Δ QTR
(c) $\tan P$ in Δ PQR
(d) $\sin Q$ in Δ PQT
(e) $\sin Q$ in Δ QTR

(5)

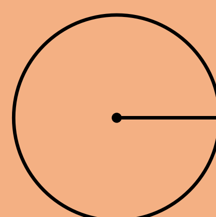


(a) y
(b) $\sin\theta$
(c) $\cos\theta$
(d) $\tan\theta$

C1.3 Use of the calculator:

C1.3.1 Degrees, minutes and seconds:

If measurement is done, use is made of distances and angle sizes.
We already know that a revolution, in other words the full turn of a circle, is 360 degrees (360^o).
Each degree is the angle at the centre of a circle describing the size of the arc which then represents a fraction of the circumference of the circle.
$\therefore 1^o$ is equal to $\frac{1}{360}$ of the circumference of the circle.
One minute (1') is equal to $\frac{1}{60}$ th of a degree.
One second (1") is equal to $\frac{1}{60}$ th of a minute.



E.g.4 (a) Describe the following angle size: $13° 24' 36''$

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13 degrees, 24 minutes and 36 seconds.

(b) Convert the following to degrees only: $13° 24' 36''$

****

$$13° 24' 36'' = 13° + 24' + \frac{36}{60}' = 13° + 24' + 0,6'$$
$$= 13° + 24,6' = 13° + \frac{24,6}{60}°$$
$$= 13° + 0,41° = 13,41°$$

(c) Convert the following to degrees and minutes: $64,3°$

****

$$64,3° = 64° + 0,3° = 64° + (0,3 \times 60)' = 64°\ 18'$$

Exercise 4: Date: ________________

(1) Convert the following to degrees only:

(a) $72° \ 24'$ (b) $88° \ 33'$ (c) $324° \ 48'$

(d) $25° \ 12' \ 36''$ (e) $112° \ 36' \ 54''$ (f) $7° \ 6' \ 18''$

(2) Convert the following to degrees and minutes:

(a) 38,5° (b) 101,7° (c) 16,45°

C1.3.2 The calculator:

C1.3.2.1 Trigonometric expressions:

> **REMEMBER: The calculator must be on "deg"!**
> Make use of a non-programmable, scientific calculator!

E.g.5 Calculate the following, correct to two decimals:

	Expression:		Display:		2 dec. places:	Keys:
(a)	$\sin 12°$	=	$0,2079\ldots$	≈	$0,21$	$\sin 12 \ =$
(b)	$\cos 42° \ 12'$	=	$0,7408\ldots$	≈	$0,74$	$\cos 42 \ ° \, ' \, '' \ 12 \ ° \, ' \, '' \ =$
(c)	$2 \tan 77°$	=	$8,6629\ldots$	≈	$8,66$	$2 \ \tan \ 77 \ =$
(d)	$\cos^2 44°$	=	$0,5174\ldots$	≈	$0,52$	$\cos 44) \ x^2$ or $(\cos 44)x^2 \ =$
(e)	$4 - \tan 220°$	=	$3,1609\ldots$	≈	$3,16$	$4 \ - \ \tan \ 220 \ =$
(f)	$\frac{\sin 67°}{3}$	=	$0,3068\ldots$	≈	$0,31$	$\sin 67 \ = \div \ 3 \ =$

Exercise 5: Date: ________________

(1) Calculate the following, correct to two decimals:

(a) $\sin 33° =$ ____________________ (b) $\cos 56° =$ ____________________

(c) $\tan 11{,}5° =$ ____________________ (d) $\sin 145° =$ ____________________

(e) $\sin 301° =$ ____________________ (f) $\cos 201° 24' =$ ____________________

(g) $\tan 88° 56' =$ ____________________ (h) $\cos 345° =$ ____________________

(i) $\sin 23{,}4° =$ ____________________ (j) $\tan 66° 34' =$ ____________________

(k) $\cos 64{,}1° =$ ____________________ (l) $\tan 6{,}6° =$ ____________________

(m) $\sin 12° 12' =$ ____________________ (n) $\cos 0{,}5° =$ ____________________

(2) Calculate the following, correct to 1 decimal: (Write down your keys!)

(a) $2 \sin 34° =$ ____________________ (b) $3{,}5 + \cos 200° =$ ____________________

(c) $\tan^2 130° =$ ____________________ (d) $\frac{\cos 71°}{2} =$ ____________________

(e) $\sin(32° + 12°) =$ ____________________ (f) $\cos 176° - \cos 76° =$ ____________________

(g) $\sqrt{\sin 16°} =$ ____________________ (h) $\tan 100° + 7{,}1 =$ ____________________

(i) $4 \div \sin 133° 24' =$ ____________________ (j) $\sin^3 72{,}12° =$ ____________________

(k) $7 + \frac{\tan 100°}{2} =$ ____________________ (l) $\cos(4 \times 31{,}3°) =$ ____________________

(m) $\sqrt{10 \cos 300°} =$ ____________________ (n) $\sin 30° \times \cos 30° =$ ____________________

(o) $-7{,}1 - \sin 304° =$ ____________________ (p) $1{,}6 - 2 \times \cos^2 123° =$ ____________________

C1.3.2.2 Trigonometric equations:

We have seen previously that for example $\sin 30° = 0{,}5$
$\therefore$ if $\sin x = 0{,}5$ then we can deduct that $x = 30°$ if $x \in [0° ; 90°]$

E.g.6 Solve x if $x \in [0° ; 90°]$; correct to 1 decimal:

(a) $\cos x = 0{,}34$ (b) $2\tan x = 4{,}64$ (c) $\sin 3x = 0{,}7$

****

(a) $\cos x = 0{,}34$

$\therefore \quad x \approx 70{,}1°$

[Keys: Shift $\cos^{-1}$ 0.34 =]

(b) $\tan x = \frac{4{,}64}{2}$

$\tan x = 2{,}32$

$\therefore \quad x \approx 66{,}7°$

[Keys: Shift $\tan^{-1}$ 2.32 =]

(c) $\sin 3x = 0{,}7$

$\therefore \quad 3x = 44{,}427\ldots$

$\therefore \quad x \approx 14{,}8°$ [Keys: Shift $\sin^{-1}$ 0.7 $= \div 3 =$]

Exercise 6: Date: ________________

Solve x if $x \in [0° ; 90°]$; correct to 1 decimal:

(1) $\sin x = 0{,}34$
____

(2) $\cos x = 0{,}551$
____

(3) $\tan x = 6{,}9$
____

(4) $\cos x = \frac{1}{2}$
____

(5) $\tan x = 44{,}4$
____

(6) $\sin x = 0{,}881$
____

(7) $\cos x = 0{,}401$
____
____

(8) $\sin x - 0{,}2 = 0$
____
____

(9) $\tan x = 2 \times 3$
____
____

(10) $4\sin x = 0{,}1$
____
____
____

(11) $\tan 3x = 6$
____
____
____

(12) $\cos(x + 10°) = 0{,}9$
____
____
____

(13) $\cos x + 2 = 2{,}444$

(14) $\tan^2 x = 0{,}64$

(15) $\frac{\sin x}{2} = 0{,}1$

(16) $\tan(x - 10°) = 20$

(17) $\cos 3x = 0{,}688$

(18) $\cos x - 3 = -2{,}445$

(19) $-2{,}3 \tan x = -3{,}2$

(20) $\sin\frac{x}{2} = 0{,}5$

(21) $\frac{2}{3}\sin x = \frac{1}{2}$

(22) $\tan(2x - 15°) = 2$

(23) $\frac{\cos 2x}{2} = 0{,}2$

(24) $\sin x = \tan 25°$

C1.3.2.3 Combinations:

E.g.7 Calculate $5 \sin 2A$ if $3 + \tan A = 4{,}2$ and $A \in [0° ; 90°]$. Round off A, correct to 1 decimal and the function value correct to 3 decimals.

****

If $3 + \tan A = 4{,}2$

$\therefore \tan A = 4{,}2 - 3$

$\tan A = 1{,}2$

$\therefore A = 50{,}2°$

$\therefore 5 \sin 2A$

$= 5 \sin(2 \times 50{,}2°)$

$= 5 \sin 100{,}4°$

$\approx 4{,}918$

Exercise 7: Date: ________________

Round off all angles to 1 decimal and each function value to three decimals!

(1) Calculate $\sin^2\theta$ if $2\cos\theta = 0{,}31$ and $\theta \in [0°\,;\,90°]$.

(2) If $-2\tan A = -2$ and $0° \leq A \leq 90°$, calculate $\cos(A + 12°)$.

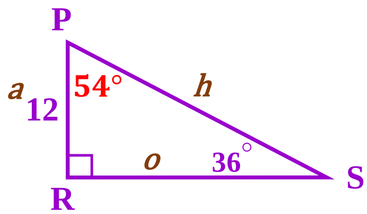
(3) If $\cos 2x = 0{,}4$ and $x \in [0°\,;\,90°]$, calculate $\cos^2 x + 3\tan x$.

(4) Calculate $\frac{\sin\theta + \cos\theta}{-3{,}1}$ if $0° \leq \theta \leq 90°$ and $\tan(\theta - 25°) = 2{,}1$.

C1.4 Solving right angled triangles:

**E.g.8 Calculate the unknown angles and sides in each of the following triangles:
Round off correct to one decimal!**

(a)



* $\hat{P} = 90° - 36° = 54°$

* $\tan \hat{P} = \frac{o}{a}$

$\tan 54° = \frac{RS}{12}$

$12 \tan 54° = RS$

$\therefore 16,5 = RS$

* $PS^2 = PR^2 + RS^2$

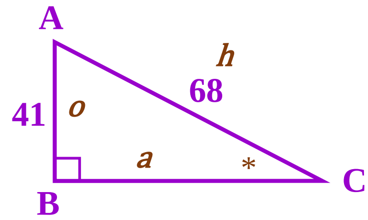
$PS^2 = 12^2 + 16,5^2$

$PS^2 = 144 + 272,25$

$PS^2 = 416,25$

$PS = 20,4$

(b)



* $\sin \hat{C} = \frac{o}{h}$

$\sin \hat{C} = \frac{41}{68}$

$\sin \hat{C} = 0,602 \ldots \ldots$

$\therefore \hat{C} = 37,080 \ldots \approx 37,1°$

* $AC^2 = AB^2 + BC^2$

$68^2 = 41^2 + BC^2$

$4\,624 = 1\,681 + BC^2$

$4\,624 - 1\,681 = BC^2$

$2\,943 = BC^2$

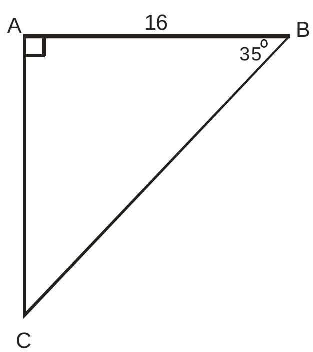
$54,2 = BC$

* $\hat{A} = 90° - 37,1° = 52,9°$

Exercise 8: Date: ________________

Solve the following triangles, correct to one decimal:

(1)



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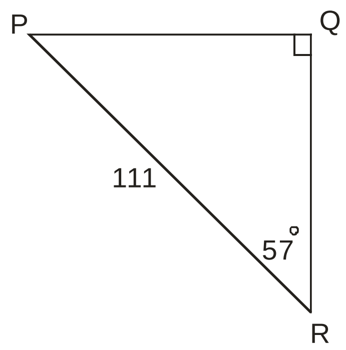
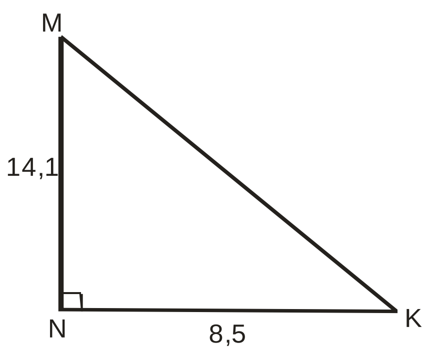
(2)



(3)



(4)