Grade 8 – Book A

(Revised CAPS edition)

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Chapter A1

Integers

A1.1 Number systems and properties of integers:

Exercise 1: Date: _____

Complete: * Natural numbers: $\mathbb{N} = \{$_____$\}$

* Whole numbers $\mathbb{N}_0 = \{$_____$\}$

* Integers: $\mathbb{Z} = \{$_____$\}$

The integers are expanded to include the fractions:

Rational numbers ($\mathbb{Q}$): Include all fractions which can be written as $\frac{a}{b}$, with a and b as integers and $b \neq 0$. This includes all finite and recurring decimal fractions.
E.g. $\frac{1}{3}$; $0,\dot{7}$; $-3\frac{5}{8}$; 2,34; $\sqrt{25}$; 9; $\sqrt[3]{27}$ etc.

Irrational numbers ($\mathbb{Q}'$): Include all infinite and non-recurring decimal fractions.
E.g. 3,68463.....; π; $\sqrt{10}$; $\sqrt[3]{4}$ etc.

Real numbers ($\mathbb{R}$) consist of all rational and irrational numbers in union: $\mathbb{Q} \cup \mathbb{Q}'$

Non-real numbers for example are: $\sqrt{-4}$; $\sqrt{-12}$ etc.
$\sqrt[3]{-8}$ and $\sqrt[5]{-243}$ however, are real numbers, because $\sqrt[3]{-8} = -2$ and $\sqrt[5]{-243} = -3$.

Properties of 1 and 0:

* $m \times 0 = 0$
* $0 \div m = 0$
* $m \div 0$ = undefined
* $m \times 1 = m$
* $m \div 1 = m$

Identity elements:

* 0 is the identity element of addition, because $m + 0 = m$
* 1 is the identity element of multiplication, because $m \times 1 = m$

Inverses:

* The sum of a number and its additive inverse is 0.
  E.g. **3** is the additive inverse of **-3**, because $3 + (-3) = 3 - 3 = 0$
* The multiplicative inverse (reciprocal) is the number multiplied with a certain number with a result of 1. E.g. the multiplicative inverse of **3** is $\frac{1}{3}$, because $3 \times \frac{1}{3} = \frac{3}{1} \times \frac{1}{3} = 1$

Other properties:

* Commutative operation: $m \times n = n \times m$ or $m + n = n + m$
* Associative operation: $(m \times n) \times p = m \times (n \times p)$ or $(m + n) + p = n + (m + p)$
* Distributive operation: $p \times (m + n) = p \times m + p \times n$ or $p \times (m - n) = p \times m - p \times n$

A1.2 Rules for divisibility:

Divisor:	Rules for divisibility:
2	Last digit must be an even number or a 0.
3	Sum of all the digits must be divisible by 3.
4	Two last digits must be divisible by 4.
5	Last digit must be 5 or 0.
6	Rules for divisibility for 2 and 3 must apply.
8	Last three digits must be divisible by 8.
9	Sum of all the digits must be divisible by 9.
10	Last digit must be 0.
11	Calculate the sums of the alternate digits. The difference between these sums must be 0, or it must be divisible by 11.

E.g. 1 Determine whether 10 527 is divisible by the numbers in the above table:

2: NO, because the number (10 527) does not end on an even number.
3: YES, because the sum of the digits, 1+0+5+2+7=15 is divisible by 3.
4: NO, because 27(10 527) is not divisible by 4.
5: NO, because the number does not end on a 5 or a 0.
6: NO, because the rule of divisibility for 2 does not apply.
8: NO, because the last three digits, 527, are not divisible by 8.
9: NO, because the sum of the digits viz. 1+0+5+2+7=15 is not divisible by 9.
10: NO, because the last digit is not 0.
11: YES, because the difference between 1+5+7=13 and 0+2=2 with 13 - 2 = 11.

Exercise 2: Date: _____

Determine whether the following numbers are divisible by the numbers in the above table:

(1) 1 275: _____

(2) 2 772: _____

(3) 7 920: _____

_____

_____

_____

_____

_____

_____

☺ **A certain number is divisible by 2, 3, 5 and 11. This number is not divisible by 8 and 9, but it is divisible by 4. Determine the smallest number that meets these conditions.**

_____

_____

A1.3 Factors:

E.g. 2 The factors of 10 are: $F_{10} = \{1 ; 2 ; 5 ; 10\}$

Exercise 3: Date: _____

Complete:

(1) F_{20} = _____

(2) F_{16} = _____

(3) F_{5} = _____

(4) F_{32} = _____

(5) F_{15} = _____

(6) F_{28} = _____

(7) F_{12} = _____

(8) F_{7} = _____

(9) F_{36} = _____

(10) F_{11} = _____

A1.4 Multiples:

E.g. 3 The multiples of 10 are: $M_{10} = \{10 ; 20 ; 30 ; \ldots\ldots\ldots\ldots\}$

Exercise 4: Date: _____

Complete:

(1) M_6 = _____

(2) M_{20} = _____

(3) M_7 = _____

(4) M_{12} = _____

(5) M_{36} = _____

(6) M_0 = _____

(7) M_{35} = _____

(8) M_{16} = _____

(9) M_{11} = _____

(10) M_3 = _____

☺ **Determine the multiples of 6 which are also factors of 120.**

_____

A1.5 Prime numbers and compound numbers:

Exercise 5: Date: _____

Complete:

(1) The definition of a prime number is: _____

_____

(2) The smallest prime number is: _____

(3) The only even prime number is: _____

(4) The definition of a compound number is: _____

_____

(5) Which natural number is **neither** a prime number nor a compound number? _____

(6) Which natural numbers smaller than 50, are prime numbers?
(Do the following: Encircle 2 ; 3 ; 5 and 7 and cross out all the multiples of 2 ; 3 ; 5 ; and 7.
The numbers which are left will be the prime numbers. Remember to cross out 1 as well!)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

∴ The prime numbers smaller than 50 are: {_____}

A1.6 Prime factors:

E.g. 4 The factors of 6 are: $F_6 = \{1 ; 2 ; 3 ; 6\}$

∴ *The prime factors of 6 are:* ***2 and 3****. (In other words they are factors which are prime numbers)*

E.g. 5 The factors of 20 are: $F_{20} = \{1 ; 2 ; 4 ; 5 ; 10 ; 20\}$

∴ *The prime factors of 20 are:* ***2 and 5****.*

E.g. 6 Determine the prime factors of 60:

2	60
2	30
3	15
5	5
	1

∴ $60 = 2 \times 2 \times 3 \times 5$

$= 2^2 \times 3 \times 5$

Exercise 6: Date: _____

Determine the prime factors of:

(1) 12 (2) 35 (3) 32 (4) 44

(5) 48 (6) 27 (7) 56 (8) 100

(9) 18 (10) 168 (11) 588 (12) 450

A1.7 LCM and HCF:

LCM = Lowest common multiple.
HCF = Highest common factor.

E.g.7 Determine the LCM of 8 ; 12 and 20
[First determine the prime factors!]

$8 = \boxed{2 \times 2} \times 2$
$12 = \boxed{2 \times 2} \times 3$
$20 = \boxed{2 \times 2} \times 5$

$\therefore LCM = \boxed{2 \times 2} \times 2 \times 3 \times 5 = \underline{\mathbf{120}}$

E.g.8 Determine the HCF of 36 and 60.
[First determine the prime factors!]

$36 = \boxed{2 \times 2 \times 3} \times 3$
$60 = \boxed{2 \times 2 \times 3} \times 5$

$\therefore HCF = \boxed{2 \times 2 \times 3} = \underline{\mathbf{12}}$

Exercise 7: Date: _____

(1) Determine the HCF of the following by finding the prime factors first:

(a) 14 = _____
21 = _____ ∴HCF = _____
35 = _____ = _____

(b) 27 = _____
45 = _____ ∴HCF = _____
72 = _____ = _____

(c) 12 = _____ ∴HCF = _____
168 = _____ = _____

(d) 38 = _____
57 = _____ ∴HCF = _____
95 = _____ = _____

(e) 10 = _____
15 = _____ ∴HCF = _____
105 = _____ = _____

(2) Determine the LCM of the following by finding the prime factors first:

(a) 6 = _____

12 = _____ ∴LCM = _____

18 = _____ = _____

(b) 8 = _____ ∴LCM = _____

20 = _____ = _____

(c) 2 = _____

6 = _____ ∴LCM = _____

11 = _____ = _____

(d) 21 = _____ ∴LCM = _____

49 = _____ = _____

(e) 3 = _____

9 = _____

12 = _____ ∴LCM = _____

60 = _____ = _____

(f) 15 = _____

45 = _____ ∴LCM = _____

270 = _____ = _____

(3) Determine the LCM and the HCF:

(a) 16 = _____ ∴LCM = _____

48 = _____ = _____

56 = _____ ∴HCF = _____

= _____

(b) 5 = _____ ∴LCM = _____

24 = _____ = _____

∴HCF = _____

☺ **Mounting boards of (a) 24 cm², (b) 36 cm² and (c) 18 cm² have to be cut. Determine the area of the smallest mounting board panel that should be used so that any combination of (a), (b) and/or (c) can be cut from it, without wasting any board. [Make use of prime factors.]**

_____

_____

_____

_____

A1.8 Square roots and cube roots:

E.g.9 Determine the following by using prime factors:

(a) $\sqrt{784}$ *(b)* $\sqrt[3]{3\,375}$

****

2	784
2	392
2	196
2	98
7	49
7	7
	1

$\therefore\ 784 = 2\times2\times2\times2\times7\times7$

$= 2^2 \times 2^2 \times 7^2$

$\therefore \sqrt{784} = 2 \times 2 \times 7$

$= \underline{\underline{28}}$

3	3 375
3	1 125
3	375
5	125
5	25
5	5
	1

$\therefore\ 3\,375 = 3\times3\times3\times5\times5\times5$

$= 3^3 \times 5^3$

$\therefore \sqrt[3]{3\,375} = 3 \times 5$

$= \underline{\underline{15}}$

Exercise 8: Date: _____

Calculate: (by using prime factors)

(1) $\sqrt{576}$ = _____

_____

_____

_____