Hoofstuk A1

Getallestelsels en eksponente

A1.1 Getallestelsels:

Oefening 1:

(1) Voltooi:
- Natuurlike getalle: $\mathbb{N}$ = $\{1; 2; 3; ...\}$
- Telgetalle: $\mathbb{N}_0$ = $\{0; 1; 2; 3; ...\}$
- Heelgetalle: $\mathbb{Z}$ = $\{...; -2; -1; 0; 1; 2; ...\}$
- Rasionale getalle: $\mathbb{Q}$ = $\{\frac{a}{b} \mid a, b \in \mathbb{Z}; b \neq 0\}$
- Reële getalle: $\mathbb{R}$ = {Rasionale getalle} ∪ {Irrasionale getalle}

(2) Gee drie voorbeelde van Irrasionale getalle: $\sqrt{3}$; $\sqrt[3]{4}$; $\sqrt{7}$

(3) Beskou: $x(x - 6)(x^2 - 5)(2x^2 + x - 3) = 0$.
Los op vir x en skryf die waarde(s) van x waarvoor die oplossing van die uitdrukking:
(a) irrasionale wortels het.
(b) natuurlike wortels het.
(c) heelgetallige wortels het.

$x(x-6)(x^2-5)(2x+3)(x-1) = 0$

$x = 0$ of $x = 6$ of $x^2 = 5$ of $x = \frac{-3}{2}$ of $x = 1$

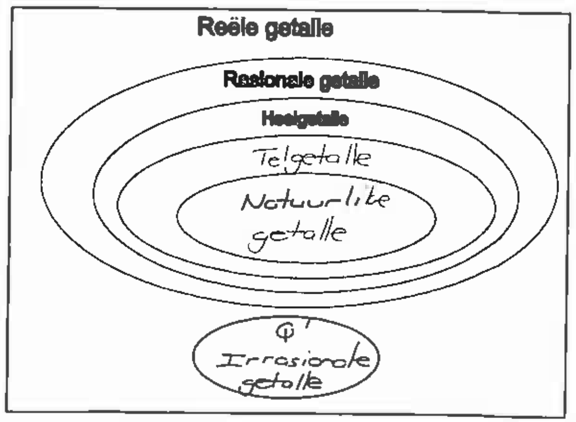
$x = \pm\sqrt{5}$

(a) $x = \pm\sqrt{5}$

(b) $x = 6$ of $x = 1$

(c) $x = 0$ of $x = 6$ of $x = 1$

(4) Voltooi die volgende figuur wat die samestelling van die stelsel reële getalle verteenwoordig:



A1.2 Nie-Reële getalle:

Oefening 2:

(1) Bepaal of die volgende getalle reël of nie-reëel is. Indien dit reëel is, dui aan of die getal rasionaal of irrasionaal sal wees.

(a) 7 Reëel → rasionaal
(b) $-\sqrt{3}$ Reëel → irrasionaal
(c) π Reëel → irrasionaal
(d) $\sqrt{-16}$ Nie-reëel
(e) $0,\dot{3}$ Reëel → rasionaal
(f) $\frac{12}{36}$ Reëel → rasionaal
(g) $\sqrt[3]{-125} = -5$ Reëel → rasionaal
(h) $1 + \sqrt{9} = 1 + 3 = 4$ Reëel → rasionaal
(i) $\sqrt{(-2)^3} = \sqrt{-8}$ Nie-reëel
(j) 0 Reëel → rasionaal

(2) Sê of die volgende bewerings waar of vals is:
(a) Die produk van twee heelgetalle is altyd weer 'n heelgetal. Waar
(b) Die produk van twee irrasionale getalle is altyd weer 'n irrasionale getal. Vals
Bv. $\sqrt{2} \times \sqrt{2} = 2$
(c) As m 'n natuurlike getal is, dan sal $\sqrt{4}\,m$ ook 'n natuurlike getal wees. Waar
$= 2m$
(d) Die verskil tussen twee rasionale getalle is altyd weer 'n rasionale getal. Waar
(e) Die kwosiënt van 'n rasionale getal en 'n irrasionale getal is altyd rasionaal. Vals
Bv. $\frac{2}{\sqrt{2}} \rightarrow$ irrasionaal

(3) Vir watter waardes van x is die volgende bewerings: (i) ongedefinieerd (ii) nie-reëel

(a) $\frac{x + 3}{x}$: (i) $x = 0$ (ii) Geen

(b) $\sqrt{x - 1}$: (i) Geen (ii) $x - 1 < 0$ ∴ $x < 1$

(c) $\frac{\sqrt{x}}{x + 2}$: (i) $x = -2$ (ii) $x < 0$

(4) Gegee: $P = \sqrt[4]{3y} - 1$. Tot watter van die volgende getallestelsel(s) sal P behoort indien:
[Getallestelsels: $\mathbb{N}$; $\mathbb{N}_0$; $\mathbb{Z}$; $\mathbb{Q}$; $\mathbb{Q}'$; $\mathbb{R}$ of $\mathbb{R}'$]

(a) $y = \frac{1}{3}$
$P = \sqrt[4]{3 \times \frac{1}{3}} - 1$
$= \sqrt[4]{1} - 1$
$= 1 - 1 = 0$
$\mathbb{N}_0$; $\mathbb{Z}$; $\mathbb{Q}$; $\mathbb{R}$

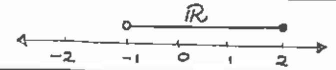
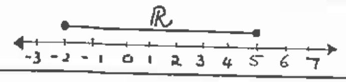
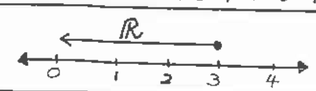
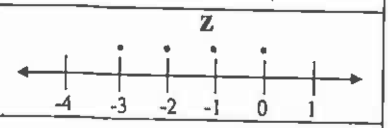
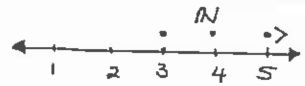
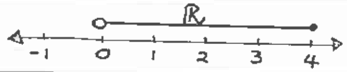
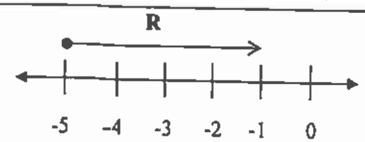
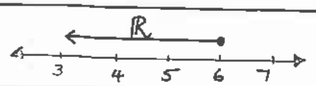
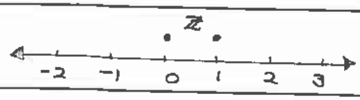
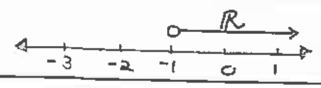
(b) $y = -1$
$P = \sqrt[4]{3 \times -1} - 1$
$= \sqrt[4]{-3} - 1$
$\mathbb{R}'$

(c) $y = 5$
$P = \sqrt[4]{3 \times 5} - 1$
$= \sqrt[4]{15} - 1$
$\mathbb{Q}'$; $\mathbb{R}$

A1.3 Voorstelling van die reële getalle:

Oefening 3:

Voltooi die volgende tabel:

	Versamelingskeurdernotasie:	Intervalnotasie:	Getallelyn:
(1)	$\{x / -1 < x \leq 2 ; x \in \mathbf{R}\}$	$x \in (-1;2]$	
(2)	$\{x / -2 \leq x \leq 5 ; x \in \mathbb{R}\}$	$x \in [-2;5]$	
(3)	$\{y / y \leq 3 ; y \in \mathbb{R}\}$	$y \in (-\infty;3]$	
(4)	$\{x / -3 \leq x \leq 0 ; x \in \mathbb{Z}\}$ of $\{x / -4 < x < 1 ; x \in \mathbb{Z}\}$	Geen	
(5)	$\{y / y \geq 3 ; y \in \mathbf{N}\}$	Geen	
(6)	$\{m / 0 < m \leq 4 ; m \in \mathbb{R}\}$	$m \in (0;4]$	
(7)	$\{x / x \geq -5 ; x \in \mathbb{R}\}$	$x \in [-5;\infty)$	
(8)	$\{m : m \leq 6 ; m \in \mathbf{R}\}$	$m \in (-\infty;6]$	
(9)	$\{x / -1 < x < 2 ; x \in \mathbf{Z}\}$	Geen	
(10)	$\{x / x > -1 ; x \in \mathbb{R}\}$	$x \in (-1;\infty)$	

☺ (1) **Gee 'n sinoniem vir "nie-reële getalle":** Denkbeeldig

(2) **Doen navorsing oor komplekse getalle en gee twee voorbeelde van komplekse getalle:** Word voorgestel deur $\mathbb{C}$. Getalle soos $\sqrt{-1}$ of $\sqrt{-7}$. Kan geskryf word as die som of verskil van reële en nie-reële getalle bv. $1 + \sqrt{-1}$ of $3 - 2i$

A1.4 Eksponente en wortelvorme:

A1.4.1 Eksponente:

Oefening 4:

Vereenvoudig, sonder 'n sakrekenaar: (Skryf antwoorde as positiewe eksponente!)

(1) $(125x^6)^{\frac{1}{3}}$

$= (5^3 x^6)^{\frac{1}{3}}$

$= (5^3)^{\frac{1}{3}} (x^6)^{\frac{1}{3}}$

$= 5x^2$

(2) $\left(x^{\frac{1}{2}} - 2\right)^2$

$= \left(x^{\frac{1}{2}} - 2\right)\left(x^{\frac{1}{2}} - 2\right)$

$= \left(x^{\frac{1}{2}}\right)^2 - 2x^{\frac{1}{2}} - 2x^{\frac{1}{2}} + 4$

$= x - 4x^{\frac{1}{2}} + 4$

(3) $\sqrt[3]{-8x^9y^{-3}}$

$= \sqrt[3]{(-2)^3 x^9 y^{-3}}$

$= (-2)^{\frac{3}{3}} x^{\frac{9}{3}} y^{-\frac{3}{3}}$

$= -2x^3 y^{-1}$

$= \frac{-2x^3}{y}$

(4) $3y^{\frac{1}{2}} \div (3y)^{-\frac{1}{2}}$

$= 3^1 y^{\frac{1}{2}} \div 3^{-\frac{1}{2}} y^{-\frac{1}{2}}$

$= 3^{1-(-\frac{1}{2})} y^{\frac{1}{2}-(-\frac{1}{2})}$

$= 3^{1\frac{1}{2}} y^{\frac{1}{2}+\frac{1}{2}}$

$= 3^{1\frac{1}{2}} y^1$

$x^{\frac{1}{4}} \times x^{\frac{1}{4}} = x^{\frac{2}{4}} = x^{\frac{1}{2}}$

(5) $(0{,}25m^{\frac{1}{4}})^2$

$= \left(\frac{25}{100} m^{\frac{1}{4}}\right)^2$

$= \left(\frac{1}{4} m^{\frac{1}{4}}\right)^2$

$= \left(\frac{1}{4}\right)^2 \left(m^{\frac{1}{4}}\right)^2 = \frac{1}{16} m^{\frac{1}{2}} = \frac{m^{\frac{1}{2}}}{16}$

(6) $\left(x^{\frac{1}{2}} + 4\right)\left(x^{\frac{1}{4}} - 2\right)\left(x^{\frac{1}{4}} + 2\right)$

$= \left(x^{\frac{1}{2}} + 4\right)\left(x^{\frac{1}{2}} - 4\right)$

$= \left(x^{\frac{1}{2}}\right)^2 - 16$

$= x - 16$

(7) $\frac{x^{\frac{1}{2}} . \sqrt{x^3}}{x^{\frac{1}{3}}}$

$= \frac{x^{\frac{1}{2}} . x^{\frac{3}{2}}}{x^{\frac{1}{3}}}$

$= x^{\frac{1}{2} + \frac{3}{2} - \frac{1}{3}}$

$= x^{2 - \frac{1}{3}}$

$= x^{1\frac{2}{3}}$

(8) $\left(\frac{-12x^4 y^4 z}{-3x^2 z^3}\right)^{\frac{1}{2}}$

$= \left(4x^2 y^4 z^{-2}\right)^{\frac{1}{2}}$

$= (2^2)^{\frac{1}{2}} (x^2)^{\frac{1}{2}} (y^4)^{\frac{1}{2}} (z^{-2})^{\frac{1}{2}}$

$= 2^1 x^1 y^2 z^{-1}$

$= \frac{2xy^2}{z}$

(9) $\frac{m^{-2} - 3}{m^{-1} - 3m^{-1}}$

$= \frac{\cancel{(m^{-2} - 3)}}{m^{-1}\cancel{(m^{-2} - 3)}}$

$= \frac{1}{m^{-1}}$

$= m^1$

(10) $\frac{\left(9x^{\frac{2}{3}}y^{-4}\right)^{-\frac{3}{2}}}{3xy}$

$= \frac{(3^2)^{-\frac{3}{2}} \left(x^{\frac{2}{3}}\right)^{-\frac{3}{2}} (y^{-4})^{-\frac{3}{2}}}{3xy}$

$= \frac{3^{-3} x^{-1} y^6}{3^1 x^1 y^1}$

$= 3^{-4} x^{-2} y^5$

$= \frac{y^5}{3^4 x^2}$

(11) $\dfrac{(x+y)^{-1}}{x^{-1}-y^{-1}}$

$= \dfrac{1}{(x+y)} \div \left(\dfrac{1}{x} - \dfrac{1}{y}\right)$

$= \dfrac{1}{(x+y)} \div \left(\dfrac{y-x}{xy}\right)$

$= \dfrac{1}{(x+y)} \times \dfrac{xy}{(y-x)} = \dfrac{xy}{y^2-x^2}$

(12) $\dfrac{2^{2n} - 3.2^n + 2}{2^n - 2}$

$= \dfrac{(2^n)^2 - 3.(2^n) + 2}{2^n - 2}$

$= \dfrac{\cancel{(2^n-2)}(2^n-1)}{\cancel{(2^n-2)}}$

$= 2^n - 1$

(13) $\left(m^{\frac{2}{3}} + n^{\frac{1}{3}}\right)^2$

$= \left(m^{\frac{2}{3}} + n^{\frac{1}{3}}\right)\left(m^{\frac{2}{3}} + n^{\frac{1}{3}}\right)$

$= \left(m^{\frac{2}{3}}\right)^2 + m^{\frac{2}{3}}n^{\frac{1}{3}} + m^{\frac{2}{3}}n^{\frac{1}{3}} + \left(n^{\frac{1}{3}}\right)^2$

$= m^{\frac{4}{3}} + 2m^{\frac{2}{3}}n^{\frac{1}{3}} + n^{\frac{2}{3}}$

(14) $\left(a^{\frac{1}{3}} - 5\right)\left(5 + a^{\frac{1}{3}}\right)$

$= \left(a^{\frac{1}{3}} - 5\right)\left(a^{\frac{1}{3}} + 5\right)$

$= \left(a^{\frac{1}{3}}\right)^2 - 25$

$= a^{\frac{2}{3}} - 25$

(15) $\sqrt[3]{(0,125)^{-2}} + (125^2)^{\frac{1}{3}}$

$= \sqrt[3]{\left(\frac{1}{8}\right)^{-2}} + \left(\left(5^3\right)^2\right)^{\frac{1}{3}}$

$= \sqrt[3]{\left(\frac{1}{2^3}\right)^{-2}} + \left(5^6\right)^{\frac{1}{3}}$

$= \sqrt[3]{\frac{1}{2^{-6}}} + 5^{6\times\frac{1}{3}}$

$= \sqrt[3]{2^6} + 5^2$

$= 2^{\frac{6}{3}} + 5^2$

$= 2^2 + 25$

$= 4 + 25 = 29$

(16) $\dfrac{12^{n+1} . 9^{n-2}}{18^{2n-1} . 3^{-n}}$

$= \dfrac{(2^2\times3^1)^{n+1} . (3^2)^{n-2}}{(2^1\times3^2)^{2n-1} . 3^{-n}}$

$= \dfrac{2^{2n+2}\times3^{n+1} . 3^{2n-4}}{2^{2n-1}\times3^{4n-2} . 3^{-n}}$

$= \dfrac{2^{2n+2}\times3^{3n-3}}{2^{2n-1}\times3^{3n-2}}$

$= 2^{2n+2-(2n-1)} \times 3^{3n-3-(3n-2)}$

$= 2^{2n+2-2n+1} \times 3^{3n-3-3n+2}$

$= 2^3 \times 3^{-1}$

$= \dfrac{8}{3}$

(17) $\dfrac{5^{n+1} . 25^{n-1}}{125^{n-2}}$

$= \dfrac{5^{n+1} . (5^2)^{n-1}}{(5^3)^{n-2}}$

$= \dfrac{5^{n+1} . 5^{2n-2}}{5^{3n-6}}$

$= 5^{n+1+2n-2-(3n-6)}$

$= 5^{3n-1-3n+6} = 5^5$

$= 3\,125$

(18) $\dfrac{3^{2n} - 9^{n+1}}{3^{2n}}$

$= \dfrac{3^{2n} - (3^2)^{n+1}}{3^{2n}}$

$= \dfrac{3^{2n} - 3^{2n}.3^2}{3^{2n}}$

$= \dfrac{\cancel{3^{2n}}(1-3^2)}{\cancel{3^{2n}}}$

$= 1 - 9 = -8$

(19) $\dfrac{3\times2^x + 2^{x+1}}{5\times2^x}$

$= \dfrac{3\times2^x + 2^x.2^1}{5\times2^x}$

$= \dfrac{2^x(3+2^1)}{5\times2^x}$

$= \dfrac{2^x(5)}{5\times2^x}$

$= 1$

(20) $\dfrac{3^2.5^0.4^{n-1}}{2^{2n+1} - 2^{2n}}$

$= \dfrac{3^2.1.(2^2)^{n-1}}{2^{2n}.2^1 - 2^{2n}}$

$= \dfrac{9.\cancel{2^{2n}}.2^{-2}}{\cancel{2^{2n}}(2^1-1)}$

$= \dfrac{9.2^{-2}}{2-1}$

$= \dfrac{9.2^{-2}}{1}$

$= 9\times\dfrac{1}{2^2}$

$= \dfrac{9}{4}$

(21) $\dfrac{3^{-2x}.36^{x+1}.3}{4^{x-1}.(0,5)^2}$

$= \dfrac{3^{-2x}.(2^2\times3^2)^{x+1}.3^1}{(2^2)^{x-1}.\left(\frac{1}{2}\right)^2}$

$= \dfrac{3^{-2x}.2^{2x+2}.3^{2x+2}.3^1}{2^{2x-2}.(2^{-1})^2}$

$= \dfrac{3^{-2x+2x+2+1}.2^{2x+2}}{2^{2x-2}.2^{-2}}$

$= \dfrac{3^3.2^{2x+2}}{2^{2x-2-2}}$

$= 3^3.2^{2x+2-(2x-4)}$

$= 3^3.2^{2x+2-2x+4}$

$= 3^3.2^6$

(22) $\dfrac{5.5^{-y-1} + 5^{-2y}.5^{y}}{3.5^{-y} - 5^{1-y}}$

$= \dfrac{5.5^{-y}.5^{-1} + 5^{-2y}.5^{y}}{3.5^{-y} - 5^{1-y}}$

$= \dfrac{5^{1-1}.5^{-y} + 5^{-2y+y}}{3.5^{-y} - 5^1.5^{-y}}$

$= \dfrac{5^0.5^{-y} + 5^{-y}}{3.5^{-y} - 5.5^{-y}}$

$= \dfrac{\cancel{5^{-y}}(5^0+1)}{\cancel{5^{-y}}(3-5)}$

$= \dfrac{1+1}{-2}$

$= -1$

<u>Oefening 5:</u>

(1) Vereenvoudig, sonder die gebruik van 'n sakrekenaar:

(a) $(\sqrt{3} - 2)(\sqrt{3} + 2)$

$= (\sqrt{3})^2 - 4$

$= 3 - 4$

$= -1$

(b) $\sqrt{8} + \sqrt{50} - \sqrt{18}$

$= \sqrt{4\times2} + \sqrt{25\times2} - \sqrt{9\times2}$

$= \sqrt{4}\sqrt{2} + \sqrt{25}\sqrt{2} - \sqrt{9}\sqrt{2}$

$= 2\sqrt{2} + 5\sqrt{2} - 3\sqrt{2}$

$= 4\sqrt{2}$

(c) $\left(\sqrt{8} - 2^{\frac{1}{2}}\right)^2$

$= \left(\sqrt{4\times2} - \sqrt{2}\right)^2$

$= \left(2\sqrt{2} - \sqrt{2}\right)^2$

$= \left(1\sqrt{2}\right)^2$

$= 2$

(d) $\sqrt[3]{27x^6} + \sqrt[5]{32x^{10}}$

$= \sqrt[3]{3^3x^6} + \sqrt[5]{2^5x^{10}}$

$= 3^{\frac{3}{3}}x^{\frac{6}{3}} + 2^{\frac{5}{5}}x^{\frac{10}{5}}$

$= 3^1x^2 + 2^1x^2$

$= 3x^2 + 2x^2$

$= 5x^2$

(e) $(4\sqrt{2} - 3)^2$

$= (4\sqrt{2} - 3)(4\sqrt{2} - 3)$

$= (4\sqrt{2})^2 - 12\sqrt{2} - 12\sqrt{2} + 9$

$= 16 \times 2 - 24\sqrt{2} + 9$

$= 32 - 24\sqrt{2} + 9$

$= 41 - 24\sqrt{2}$

(f) $m \times \sqrt{27m^6} - \sqrt{12m^8}$

$= m \times \sqrt{9 \times 3m^6} - \sqrt{4 \times 3m^8}$

$= m \times 3\sqrt{3}\, m^{\frac{6}{2}} - 2\sqrt{3}\, m^{\frac{8}{2}}$

$= m^1 \times 3\sqrt{3}\, m^3 - 2\sqrt{3}\, m^4$

$= 3\sqrt{3}\, m^4 - 2\sqrt{3}\, m^4$

$= 1\sqrt{3}\, m^4$

(g) $\sqrt{3}\left(\sqrt{48} - 3\sqrt{75} + 2\sqrt{108}\right)$

$= \sqrt{3}\left(\sqrt{16 \times 3} - 3\sqrt{25 \times 3} + 2\sqrt{36 \times 3}\right)$

$= \sqrt{3}\left(4\sqrt{3} - 3 \times 5\sqrt{3} + 2 \times 6\sqrt{3}\right)$

$= \sqrt{3}\left(4\sqrt{3} - 15\sqrt{3} + 12\sqrt{3}\right)$

$= \sqrt{3}\left(1\sqrt{3}\right)$

$= 3$

(h) $\dfrac{\sqrt{18} - \sqrt{98}}{\sqrt{200}}$

$= \dfrac{\sqrt{9 \times 2} - \sqrt{49 \times 2}}{\sqrt{100 \times 2}}$

$= \dfrac{3\sqrt{2} - 7\sqrt{2}}{10\sqrt{2}}$

$= \dfrac{-4\cancel{\sqrt{2}}}{10\cancel{\sqrt{2}}}$

$= -\dfrac{2}{5}$

(i) $\dfrac{\sqrt{\sqrt{64}} - \sqrt{12}}{\sqrt{18} - \sqrt{27}}$

$= \dfrac{\sqrt{8} - \sqrt{12}}{\sqrt{18} - \sqrt{27}}$

$= \dfrac{\sqrt{4 \times 2} - \sqrt{4 \times 3}}{\sqrt{9 \times 2} - \sqrt{9 \times 3}}$

$= \dfrac{2\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} - 3\sqrt{3}}$

$= \dfrac{2\cancel{(\sqrt{2} - \sqrt{3})}}{3\cancel{(\sqrt{2} - \sqrt{3})}} = \dfrac{2}{3}$

(j) $\dfrac{(2 + \sqrt{3})(4 - \sqrt{3})}{\sqrt{100} + \sqrt{48}}$

$= \dfrac{8 - 2\sqrt{3} + 4\sqrt{3} - (\sqrt{3})^2}{10 + \sqrt{16 \times 3}}$

$= \dfrac{8 + 2\sqrt{3} - 3}{10 + 4\sqrt{3}}$

$= \dfrac{5 + 2\sqrt{3}}{10 + 4\sqrt{3}}$

$= \dfrac{\cancel{(5 + 2\sqrt{3})}}{2\cancel{(5 + 2\sqrt{3})}}$

$= \dfrac{1}{2}$

(k) $\dfrac{\sqrt[3]{27x^6} + \sqrt[4]{16x^8}}{\sqrt[3]{125x^{27}}}$

$= \dfrac{\sqrt[3]{3^3}\sqrt[3]{x^6} + \sqrt[4]{2^4}\sqrt[4]{x^8}}{\sqrt[3]{5^3}\sqrt[3]{x^{27}}}$

$= \dfrac{3^{\frac{3}{3}}x^{\frac{6}{3}} + 2^{\frac{4}{4}}x^{\frac{8}{4}}}{5^{\frac{3}{3}}x^{\frac{27}{3}}}$

$= \dfrac{3x^2 + 2x^2}{5x^9}$

$= \dfrac{\cancel{5}x^2}{\cancel{5}x^9}$

$= \dfrac{1}{x^7}$

(2) Vereenvoudig, sonder die gebruik van 'n sakrekenaar. Waar nodig, rasionaliseer die noemer.

(a) $\dfrac{\sqrt{3} + \sqrt{5}}{\sqrt{3}}$

$= \dfrac{(\sqrt{3} + \sqrt{5})}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{\sqrt{3}(\sqrt{3} + \sqrt{5})}{(\sqrt{3})(\sqrt{3})}$

$= \dfrac{(\sqrt{3})(\sqrt{3}) + (\sqrt{3})(\sqrt{5})}{3}$

$= \dfrac{3 + \sqrt{15}}{3}$

(b) $\dfrac{\sqrt[3]{-8} - \sqrt[5]{32}}{\sqrt{72}}$

$= \dfrac{-2 - 2}{\sqrt{36 \times 2}}$

$= \dfrac{-4}{6\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{-4\sqrt{2}}{6 \times (\sqrt{2})^2}$

$= \dfrac{-4\sqrt{2}}{12} = \dfrac{-\sqrt{2}}{3}$

(c) $\dfrac{8\sqrt{5} - \sqrt{125}}{\sqrt{45}}$

$= \dfrac{8\sqrt{5} - \sqrt{25 \times 5}}{\sqrt{9 \times 5}}$

$= \dfrac{8\sqrt{5} - 5\sqrt{5}}{3\sqrt{5}}$

$= \dfrac{3\sqrt{5}}{3\sqrt{5}}$

$= 1$

A1.5 HERSIENINGSOEFENING:

(1) Beskou P = $\{-1\ ;\ \sqrt{8}\ ;\ \frac{1}{2}\ ;\ 101\ ;\ \underset{}{\overset{=5}{\sqrt[3]{125}}}\ ;\ 0{,}13526.....\ ;\ \frac{-3}{4}\ ;\ 5{,}6\dot{7}\ ;\ 0\ ;\ \frac{\sqrt{4}}{\sqrt{5}}\ ;\ \sqrt{-100}\ \}$

Skryf die getalle in versameling P neer wat tot die volgende getallestelsel behoort:

(a) Natuurlike getalle: $101\ ;\ \sqrt[3]{125} = 5\ ;$

(b) $\mathbb{Z}$: $-1\ ;\ 101\ ;\ \sqrt[3]{125} = 5\ ;\ 0$

(c) Irrasionale getalle: $\sqrt{8}\ ;\ 0{,}13526...\ ;\ \frac{\sqrt{4}}{\sqrt{5}}$

(d) $\mathbb{R}'$: $\sqrt{-100}$

(2) As $C = \sqrt{\dfrac{4 - x}{-10x}}$, beskryf die waarde van C as reeël / nie - reeël en rasionaal / irrasionaal indien x = -1:

$C = \sqrt{\dfrac{4-(-1)}{-10(-1)}} = \sqrt{\dfrac{4+1}{+10}} = \sqrt{\dfrac{5}{10}} = \sqrt{\dfrac{1}{2}}$

$\therefore$ Reëel en irrasionaal

(3) Is die volgende bewerings waar of vals?

(a) $(-3)^2 = -3^2$ Vals — $-3^2 = -(3)(3) = -9$; $(-3)^2 = (-3)(-3) = 9$

(b) $3^3 . 4^3 = 12^3$ Waar — $3^3 . 4^3 = (3 \times 4)^3 = 12^3$

(c) $8^x + 8^{2x} = 8^{3x}$ Vals — ongelyksoortige terme

(d) $2^3 \times 2^5 = 4^4$ Waar — $2^3 \times 2^5 = 2^8$ $4^4 = (2^2)^4 = 2^8$

(e) $2^{-p} = (\frac{1}{2})^p$ Waar — $2^{-p} = (2^{-1})^p = (\frac{1}{2})^p$

(4) Bewys dat $\sqrt{10}$ tussen 3 en 4 lê, sonder om van 'n sakrekenaar gebruik te maak.

$3 = \sqrt{9}$ en $4 = \sqrt{16}$

maar $\sqrt{9} < \sqrt{10} < \sqrt{16}$ $\therefore$ $3 < \sqrt{10} < 4$

(5) Vereenvoudig die volgende, sonder die gebruik van 'n sakrekenaar:

(a) $7m^{-2}n^1 \times -2m^3n^4$

$= -14\, m^{-2+3} \times n^{1+5}$

$= -14\, m^1 \times n^6$

$= -14mn^6$

(b) $36^{\frac{1}{2}} \times 8^{\frac{2}{3}} + 64^{\frac{-1}{6}} \div 7^0$

$= (6^2)^{\frac{1}{2}} \times (2^3)^{\frac{2}{3}} + (2^6)^{-\frac{1}{6}} \div 1$

$= 6^1 \times 2^2 + 2^{-1} \div 1$

$= 6 \times 4 + \frac{1}{2}$

$= 24\frac{1}{2}$

(c) $\sqrt{\sqrt[3]{27}} \times \sqrt{12}$

$= \sqrt{3} \times \sqrt{4 \times 3}$

$= \sqrt{3} \times 2\sqrt{3}$

$= 2 \times 3$

$= 6$

(d) $(2\frac{1}{4})^{\frac{1}{2}}$

$= \left(\frac{9}{4}\right)^{\frac{1}{2}}$

$= \left(\frac{3^2}{4^2}\right)^{\frac{1}{2}}$

$= \frac{(3^2)^{\frac{1}{2}}}{(4^2)^{\frac{1}{2}}}$

$= \frac{3}{4}$

(e) $\dfrac{15^{2n+1} \cdot 9^{-n}}{25^n \cdot 3^{1-n}}$

$= \dfrac{(3 \times 5)^{2n+1} \cdot (3^2)^{-n}}{(5^2)^n \cdot 3^{1-n}}$

$= \dfrac{3^{2n+1}\, 5^{2n+1} \cdot 3^{-2n}}{5^{2n} \cdot 3^{1-n}}$

$= 3^{2n+1-2n-1+n} \cdot 5^{2n+1-2n}$

$= 3^n \cdot 5^1$

$= 5 \cdot 3^n$

(f) $\left(\dfrac{9x^2y^{-4}}{z^6}\right)^{\frac{1}{2}} \div \dfrac{(8x^3)^{\frac{2}{3}}}{4(z^{-1}y^{-1})^3}$

$= \dfrac{(3^2)^{\frac{1}{2}}(x^2)^{\frac{1}{2}}(y^{-4})^{\frac{1}{2}}}{(z^6)^{\frac{1}{2}}} \div \dfrac{(2^3)^{\frac{2}{3}}(x^3)^{\frac{2}{3}}}{4(z^{-1})^3(y^{-1})^3}$

$= \dfrac{3 \cdot x \cdot y^{-2}}{z^3} \div \dfrac{2^2 \cdot x^2}{4 \cdot z^{-3} \cdot y^{-3}}$

$= \dfrac{3\, x^1 y^{-2}}{z^3} \times \dfrac{z^{-3}\, y^{-3}}{x^2}$

$= \dfrac{3\, y^{-5}}{z^6\, x^1} = \dfrac{3}{x y^5 z^6}$

(g) $\dfrac{2 \cdot 5^m - 100 \cdot 5^{m-2}}{5^m + 5^{m+1}}$

$= \dfrac{2 \cdot 5^m - 100 \cdot 5^m \cdot 5^{-2}}{5^m + 5^m \cdot 5^1}$

$= \dfrac{\cancel{5^m}(2 - 100 \times \frac{1}{25})}{\cancel{5^m}(1 + 5)}$

$= \dfrac{2 - 4}{6}$

$= \dfrac{-2}{6} = \dfrac{-1}{3}$

(h) $\dfrac{(0,25)^x \times 32^{x+1}}{24^x \times 3^{-x}}$

$= \dfrac{\left(\frac{1}{4}\right)^x \times (2^5)^{x+1}}{(2^3 \times 3^1)^x \times 3^{-x}}$

$= \dfrac{(2^{-2})^x \times 2^{5x+5}}{2^{3x} \times 3^x \times 3^{-x}}$

$= \dfrac{2^{-2x} \times 2^{5x+5}}{2^{3x} \times 3^0}$

$= 2^{-2x+5x+5-3x}$

$= 2^5 = 32$

(i) $\dfrac{\sqrt{48y^{16}} + \sqrt{27}\, y^{16}}{4 + 3y^8}$

$= \dfrac{\sqrt{16 \times 3}\,\sqrt{y^{16}} + \sqrt{9 \times 3}\, y^{16}}{4 + 3y^8}$

$= \dfrac{4\sqrt{3}\, y^8 + 3\sqrt{3}\, y^{16}}{4 + 3y^8}$

$= \dfrac{\sqrt{3}\, y^8 (\cancel{4 + 3y^8})}{(\cancel{4 + 3y^8})}$

$= \sqrt{3}\, y^8$

(j) $\dfrac{3m \cdot 3^{3x-1} + m \cdot 27^x}{m^4 \cdot 3^{4x} \cdot (3^{-1})^x \cdot m^{-3}}$

$= \dfrac{3^1 m \cdot 3^{3x} \cdot 3^{-1} + m \cdot 3^{3x}}{m^4 \cdot 3^{4x} \cdot 3^{-x} \cdot m^{-3}}$

$= \dfrac{3^{3x}(3^0 m + m)}{m^{4-3} \cdot 3^{4x-x}}$

$= \dfrac{\cancel{3^{3x}}(1m + m)}{m^1 \cdot \cancel{3^{3x}}}$

$= \dfrac{2\cancel{m}}{\cancel{m}} = 2$

(6) As $\sqrt{2} = m$ en $\sqrt[3]{3} = n$, bewys dat: $\sqrt{32} + \sqrt{18} + \sqrt[3]{6} \times \sqrt[3]{4} - \sqrt{72} = m + 2n$

$LK = \sqrt{32} + \sqrt{18} + \sqrt[3]{6} \times \sqrt[3]{4} - \sqrt{72}$

$= \sqrt{16 \times 2} + \sqrt{9 \times 2} + \sqrt[3]{6 \times 4} - \sqrt{36 \times 2}$

$= 4\sqrt{2} + 3\sqrt{2} + \sqrt[3]{24} - 6\sqrt{2}$

$= 7\sqrt{2} - 6\sqrt{2} + \sqrt[3]{8 \times 3}$

$= 1\sqrt{2} + \sqrt[3]{8}\,\sqrt[3]{3}$

$= \sqrt{2} + 2\sqrt[3]{3}$

$= m + 2n$

$= RK$

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Hoofstuk A2

Algebraïese uitdrukkings en vergelykings

A2.1 Vereenvoudiging van breuke:

Oefening 1:
Vereenvoudig: (Geen noemers is gelyk aan nul nie!)

(1) $\dfrac{x^2 - 5x + 6}{x^2 - 9}$

$= \dfrac{(x-2)\cancel{(x-3)}}{\cancel{(x-3)}(x+3)}$

$= \dfrac{x-2}{x+3}$

(2) $\dfrac{2}{x+1} - \dfrac{2}{x-1}$

$= \dfrac{2(x-1) - 2(x+1)}{(x+1)(x-1)}$

$= \dfrac{2x - 2 - 2x - 2}{(x+1)(x-1)}$

$= \dfrac{-4}{(x+1)(x-1)}$

(3) $\dfrac{m^2 + 10m + 25}{2m^2 + 10m} \times \dfrac{m^2 - m}{m^2 + 4m - 5}$

$= \dfrac{\cancel{(m+5)}\cancel{(m+5)}}{2\cancel{m}\cancel{(m+5)}} \times \dfrac{\cancel{m}\cancel{(m-1)}}{\cancel{(m+5)}\cancel{(m-1)}}$

$= \dfrac{1}{2}$

(4) $\dfrac{p}{p^2 - 1} + \dfrac{3}{p^2 - p - 2}$

$= \dfrac{p}{(p-1)(p+1)} + \dfrac{3}{(p-2)(p+1)}$

$= \dfrac{p(p-2) + 3(p-1)}{(p-1)(p+1)(p-2)}$

$= \dfrac{p^2 - 2p + 3p - 3}{(p-1)(p+1)(p-2)}$

$= \dfrac{p^2 + p - 3}{(p-1)(p+1)(p-2)}$

(5) $\dfrac{2(x-1)}{x^2 - 4} - \dfrac{3}{6 - x - x^2}$

$= \dfrac{2x - 2}{x^2 - 4} + \dfrac{3}{x^2 + x - 6}$

$= \dfrac{(2x-2)}{(x-2)(x+2)} + \dfrac{3}{(x+3)(x-2)}$

$= \dfrac{(2x-2)(x+3) + 3(x+2)}{(x-2)(x+2)(x+3)}$

$= \dfrac{2x^2 + 6x - 2x - 6 + 3x + 6}{(x-2)(x+2)(x+3)}$

$= \dfrac{2x^2 + 7x}{(x-2)(x+2)(x+3)}$

(6) $\dfrac{y^3 - 4y}{6y^2} \div \dfrac{y^2 - 2y - 8}{y^3 - 4y}$

$= \dfrac{y(y^2-4)}{6y^2} \div \dfrac{\cancel{(y-4)}(y+2)}{y\cancel{(y-4)}}$

$= \dfrac{\cancel{y}(y-2)\cancel{(y+2)}}{6y^{\cancel{2}}} \times \dfrac{\cancel{y}}{\cancel{(y+2)}}$

$= \dfrac{y-2}{6}$

(7) $\dfrac{p^2 + 9}{p + 3} \times \dfrac{(p - 3)^2}{p} \div \dfrac{p^4 - 81}{1}$

$= \dfrac{(p^2+9)}{(p+3)} \times \dfrac{(p-3)(p-3)}{p} \times \dfrac{1}{p^4 - 81}$

$= \dfrac{\cancel{(p^2+9)}}{(p+3)} \times \dfrac{(p-3)(p-3)}{p} \times \dfrac{1}{(p^2-9)\cancel{(p^2+9)}}$

$= \dfrac{1}{(p+3)} \times \dfrac{(p-3)\cancel{(p-3)}}{p} \times \dfrac{1}{\cancel{(p-3)}(p+3)}$

$= \dfrac{(p-3)}{p(p+3)^2}$

(8) $\dfrac{2}{m^2 + 3m + 2} + \dfrac{m}{m^2 - 4} + 3$

$= \dfrac{2}{(m+2)(m+1)} + \dfrac{m}{(m-2)(m+2)} + \dfrac{3}{1}$

$= \dfrac{2(m-2) + m(m+1) + 3(m+2)(m+1)(m-2)}{(m+2)(m+1)(m-2)}$

$= \dfrac{2m - 4 + m^2 + m + 3(m^2 - 4)(m+1)}{(m+2)(m+1)(m-2)}$

$= \dfrac{m^2 + 3m - 4 + 3(m^3 + m^2 - 4m - 4)}{(m+2)(m+1)(m-2)}$

$= \dfrac{m^2 + 3m - 4 + 3m^3 + 3m^2 - 12m - 12}{(m+2)(m+1)(m-2)} = \dfrac{3m^3 + 4m^2 - 9m - 16}{(m+2)(m+1)(m-2)}$

A2.2 Vergelykings met breuke:

Oefening 2:

Los die volgende vergelykings op:

(1) $x = \dfrac{5}{x-4}$ KGV $= x-4$ $\therefore x \neq 4$

$x(x-4) = 5$

$x^2 - 4x = 5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$\underline{x = 5}$ of $\underline{x = -1}$

(2) $\dfrac{y-2}{y-1} = \dfrac{2y-1}{y+7}$ KGV: $(y-1)(y+7)$ $\therefore y \neq 1$ $y \neq -7$

$(y-2)(y+7) = (2y-1)(y-1)$

$y^2 + 5y - 14 = 2y^2 - 3y + 1$

$0 = y^2 - 8y + 15$

$0 = (y-5)(y-3)$

$\underline{y = 5}$ of $\underline{y = 3}$

(3) $\dfrac{m}{m+1} = \dfrac{m-2}{m+3}$ KGV: $(m+1)(m+3)$ $\therefore m \neq -1$ $m \neq -3$

$m(m+3) = (m-2)(m+1)$

$m^2 + 3m = m^2 - m - 2$

$3m + m = -2$

$4m = -2$

$m = \frac{-2}{4}$

$\underline{m = \frac{-1}{2}}$

(4) $1 - \dfrac{1}{p-1} = \dfrac{p-3}{p-1}$ KGV: $(p-1)$ $\therefore p \neq 1$

$1(p-1) - 1 = p - 3$

$p - 1 - 1 = p - 3$

$p - 2 = p - 3$

$-2 = -3$?

$\therefore$ Geen $\mathbb{R}$-oplossing

(5) $\dfrac{4}{x^2-4} - \dfrac{10}{x^2-x-6} = \dfrac{1}{x+2}$ KGV: $(x-2)(x+2)(x-3)$ $\therefore x \neq 2$ $x \neq -2$ $x = 3$

$\dfrac{4}{(x-2)(x+2)} - \dfrac{10}{(x-3)(x+2)} = \dfrac{1}{(x+2)}$

$4(x-3) - 10(x-2) = (x-2)(x-3)$

$4x - 12 - 10x + 20 = x^2 - 5x + 6$

$-6x + 8 = x^2 - 5x + 6$

$0 = x^2 + 6x - 5x + 6 - 8$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x \neq -2$ of $\underline{x = 1}$

NUT

(6) $\dfrac{10}{y^2-2y-8} + \dfrac{5}{y+2} = -1$ KGV $= (y-4)(y+2)$ $\therefore y \neq 4$ $y \neq -2$

$\dfrac{10}{(y-4)(y+2)} + \dfrac{5}{(y+2)} = \dfrac{-1}{1}$

$10 + 5(y-4) = -1(y-4)(y+2)$

$10 + 5y - 20 = -1(y^2 - 2y - 8)$

$5y - 10 = -y^2 + 2y + 8$

$y^2 - 2y - 8 + 5y - 10 = 0$

$y^2 + 3y - 18 = 0$

$(y+6)(y-3) = 0$

$\underline{y = -6}$ of $\underline{y = 3}$

(7) $\dfrac{y}{y-1} = 2 + \dfrac{2}{1-y} + \dfrac{2}{y+1}$ KGV $= (y-1)(y+1)$ $\therefore y \neq 1$ $y \neq -1$

$\dfrac{y}{y-1} = \dfrac{2}{1} - \dfrac{2}{y-1} + \dfrac{2}{y+1}$

$y(y+1) = 2(y-1)(y+1) - 2(y+1) + 2(y-1)$

$y^2 + y = 2(y^2-1) - 2y - 2 + 2y - 2$

$y^2 + y = 2y^2 - 2 - 4$

$0 = y^2 - y - 6$

$0 = (y-3)(y+2)$

$\underline{y = 3}$ of $\underline{y = -2}$

(8) $\dfrac{6}{m^2 - 9} - \dfrac{1}{3 - m} = \dfrac{2m}{m + 3}$

KGV = $(m-3)(m+3)$
$\therefore m \neq 3 \quad m \neq -3$

$$\frac{6}{(m-3)(m+3)} - \frac{1}{(3-m)} = \frac{2m}{(m+3)}$$

$$\frac{6}{(m-3)(m+3)} + \frac{1}{(m-3)} = \frac{2m}{(m+3)}$$

$$6 + 1(m+3) = 2m(m-3)$$

$$6 + m + 3 = 2m^2 - 6m$$

$$m + 9 = 2m^2 - 6m$$

$$0 = 2m^2 - 7m - 9$$

$$0 = (2m - 9)(m + 1)$$

$2m - 9 = 0$ of $m = -1$

$m = \frac{9}{2}$

(9) $\dfrac{3 - x}{x^2 + 6x + 5} = \dfrac{3}{x^2 + x} - \dfrac{2}{5 + x}$

KGU = $(x+5)(x+1)(x)$
$\therefore x \neq -5 \quad x \neq -1 \quad x \neq 0$

$$\frac{3-x}{(x+5)(x+1)} = \frac{3}{x(x+1)} - \frac{2}{(x+5)}$$

$$(3-x)(x) = 3(x+5) - 2(x)(x+1)$$

$$x(3-x) = 3(x+5) - 2x(x+1)$$

$$3x - x^2 = 3x + 15 - 2x^2 - 2x$$

$$3x - x^2 = -2x^2 + x + 15$$

$$3x - x^2 + 2x^2 - x - 15 = 0$$

$$x^2 + 2x - 15 = 0$$

$$(x+5)(x-3) = 0$$

$x \neq -5$ of $x = 3$

NUT

(10) $\dfrac{2x}{3x - 6} - 1 = \dfrac{2(x + 1)}{x^2 - 4} - \dfrac{1}{x + 2}$

KGV: $3(x-2)(x+2)$
$\therefore x \neq 2 \quad x \neq -2$

$$\frac{2x}{3(x-2)} - \frac{1}{1} = \frac{2(x+1)}{(x-2)(x+2)} - \frac{1}{(x+2)}$$

$$2x(x+2) - 1(3)(x-2)(x+2) = 3 \times 2(x+1) - 1 \times 3(x-2)$$

$$2x^2 + 4x - 3(x^2 - 4) = 6(x+1) - 3(x-2)$$

$$2x^2 + 4x - 3x^2 + 12 = 6x + 6 - 3x + 6$$

$$-x^2 + 4x + 12 = 3x + 12$$

$$0 = 3x + 12 + x^2 - 4x - 12$$

$$0 = x^2 - x$$

$$0 = x(x-1)$$

$x = 0$ of $x = 1$

A2.3 Oplos van vergelykings mbv kwadrering of worteltrekking:

Oefening 3:

Los op vir x: (waar nodig, laat jou antwoord in eenvoudigste wortelvorm.)

(1) $\sqrt{2x + 3} = 4$

$$(\sqrt{2x+3})^2 = (4)^2$$

$$2x + 3 = 16$$

$$2x = 16 - 3$$

$$2x = 13$$

$$x = \frac{13}{2}$$

Toets:

$LK = \sqrt{2x+3}$

$= \sqrt{2 \times \frac{13}{2} + 3}$

$= \sqrt{13+3}$

$= \sqrt{16}$

$= 4 = RK$

(2) $(x - 1)^2 - 4 = 0$

$$(x-1)^2 = 4$$

$$x - 1 = \pm\sqrt{4} = \pm 2$$

$x - 1 = +2$ of $x - 1 = -2$

$x = 3 \qquad x = -1$

(3) $\sqrt{x + 12} - x = 0$

$(\sqrt{x+12})^2 = (x)^2$

$x + 12 = x^2$

$0 = x^2 - x - 12$

$0 = (x-4)(x+3)$

$x = 4$ of $x = -3$

Toets:

$LK = \sqrt{x+12} - x$	$LK = \sqrt{x+12} - x$
$= \sqrt{4+12} - 4$	$= \sqrt{-3+12} - (-3)$
$= \sqrt{16} - 4$	$= \sqrt{9} + 3$
$= 4 - 4 = 0$	$= 3 + 3$
$= RK \quad \therefore x = 4$	$= 6 \neq RK$
	$\therefore x \neq -3$

(5) $x + 1 = \sqrt{2x + 5}$

$(x+1)^2 = (\sqrt{2x+5})^2$

$x^2 + 2x + 1 = 2x + 5$

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$x = 2$ of $x \neq -2$

Toets!! $\left.\begin{array}{l} LK = -1 \\ RK = 1 \end{array}\right\} LK \neq RK$

(7) $\sqrt{5 - 3x} = \sqrt{x + 1}$

$(\sqrt{5-3x})^2 = (\sqrt{x+1})^2$

$5 - 3x = x + 1$

$-3x - x = 1 - 5$

$-4x = -4$

$x = \frac{-4}{-4}$

$x = 1$

Toets!! $LK = \sqrt{2}$ $RK = \sqrt{2}$

$\therefore LK = RK \quad \therefore x = 1$

(4) $\sqrt{x + 1} = x - 1$

$(\sqrt{x+1})^2 = (x-1)^2$

$x + 1 = x^2 - 2x + 1$

$0 = x^2 - 3x$

$0 = x(x-3)$

$x = 0$ of $x = 3$

Toets:

$LK = \sqrt{x+1}$	$LK = \sqrt{x+1}$
$= \sqrt{0+1}$	$= \sqrt{3+1}$
$= \sqrt{1} = 1$	$= \sqrt{4} = 2$
$RK = x - 1$	$RK = x - 1$
$= 0 - 1 = -1 \neq LK$	$= 3 - 1 = 2 = LK$
$\therefore x \neq 0$	$\therefore x = 3$

(6) $x - \sqrt{x} = 2$

$x - 2 = \sqrt{x}$

$(x-2)^2 = (\sqrt{x})^2$

$x^2 - 4x + 4 = x$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4$ of $x \neq 1$

Toets!! $\left.\begin{array}{l} LK = 0 \\ RK = 2 \end{array}\right\} LK \neq RK$

(8) $4(x + 3)^2 - 16 = 9$

$4(x+3)^2 = 9 + 16$

$4(x+3)^2 = 25$

$(x+3)^2 = \frac{25}{4}$

$x + 3 = \pm\sqrt{\frac{25}{4}}$

$x + 3 = \pm\frac{5}{2}$

$x + 3 = \frac{5}{2}$ of $x + 3 = -\frac{5}{2}$

$x = \frac{5}{2} - 3$ $\qquad x = -\frac{5}{2} - 3$

$x = \frac{5}{2} - \frac{6}{2}$ $\qquad x = -\frac{5}{2} - \frac{6}{2}$

$x = -\frac{1}{2}$ $\qquad x = \frac{-11}{2} = -5\frac{1}{2}$

(9) $x - \sqrt{3 - 2x} = 0$

$x = \sqrt{3-2x}$

$(x)^2 = (\sqrt{3-2x})^2$

$x^2 = 3 - 2x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x \neq -3$ of $x = 1$

$\left.\begin{array}{l} LK = -6 \\ RK = 0 \end{array}\right\} LK \neq RK$ Toets!!

(10) $2\sqrt{1 - x} + 1 = x$

$2\sqrt{1-x} = x - 1$

$(2\sqrt{1-x})^2 = (x-1)^2$

$4(1-x) = x^2 - 2x + 1$

$4 - 4x = x^2 - 2x + 1$

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

$x \neq -3$ of $x = 1$

$\left.\begin{array}{l} LK = 5 \\ RK = -3 \end{array}\right\} LK \neq RK$ Toets!!

A2.4 K-metode / Substitusie:

Oefening 4:

Los die volgende vergelykings op:

(1) $(y^2 - 3y)^2 - 2(y^2 - 3y) = 8$

stel $(y^2 - 3y) = k$

$\therefore k^2 - 2k - 8 = 0$

$(k-4)(k+2) = 0$

$k - 4 = 0$ of $k + 2 = 0$

$y^2 - 3y - 4 = 0$ $\qquad y^2 - 3y + 2 = 0$

$(y-4)(y+1) = 0$ $\qquad (y-2)(y-1) = 0$

$y = 4$ of $y = -1$ $\qquad y = 2$ of $y = 1$

(2) $(x^2 - 5x)^2 = 36$

Stel $(x^2 - 5x) = k$

$\therefore k^2 = 36$

$k = \pm 6$

$\therefore x^2 - 5x = 6$ of $x^2 - 5x = -6$

$x^2 - 5x - 6 = 0$ $\qquad x^2 - 5x + 6 = 0$

$(x-6)(x+1) = 0$ $\qquad (x-2)(x-3) = 0$

$x = 6$ of $x = -1$ $\qquad x = 2$ of $x = 3$

(3) $\dfrac{1}{x^2 - x - 1} = x^2 - x - 1$

Stel $x^2 - x - 1 = k$

$\dfrac{1}{k} = k$

$1 = k^2$

$\pm 1 = k$

$k = 1$ of $k = -1$

$x^2 - x - 1 = 1$ | $x^2 - x - 1 = -1$

$x^2 - x - 2 = 0$ | $x^2 - x = 0$

$(x-2)(x+1) = 0$ | $x(x-1) = 0$

$x = 2$ of $x = -1$ | $x = 0$ of $x = 1$

(4) $4(m^2 - m) - 7 = \dfrac{2}{m^2 - m}$

Stel $(m^2 - m) = k$

$\therefore 4k - 7 = \dfrac{2}{k}$

$4k^2 - 7k = 2$

$4k^2 - 7k - 2 = 0$

$(k - 2)(4k + 1) = 0$

$k - 2 = 0$ of $4k + 1 = 0$

$m^2 - m - 2 = 0$ | $4(m^2 - m) + 1 = 0$

$(m-2)(m+1) = 0$ | $4m^2 - 4m + 1 = 0$

$m = 2$ of $m = -1$ | $(2m-1)(2m-1) = 0$

$2m = 1$

$m = \frac{1}{2}$

(5) $\sqrt{x - 3} = 2 - \dfrac{1}{\sqrt{x - 3}}$

stel $\sqrt{x-3} = k$

$k = 2 - \frac{1}{k}$

$k^2 = 2k - 1$

$k^2 - 2k + 1 = 0$

$(k-1)(k-1) = 0$

$k = 1$

$\therefore \sqrt{x-3} = 1$

$x - 3 = (1)^2$

$x - 3 = 1$

$x = 4$

Toets: $LK = \sqrt{4-3}$

$= \sqrt{1} = 1 = RK$

(6) $x^2 - 5x + 3 - \dfrac{9}{x^2 - 5x + 3} = 0$

Stel $x^2 - 5x + 3 = 0$

$k - \frac{9}{k} = 0$

$k^2 - 9 = 0$

$(k-3)(k+3) = 0$

$k - 3 = 0$ of $k + 3 = 0$

$x^2 - 5x + 3 - 3 = 0$ | $x^2 - 5x + 3 + 3 = 0$

$x^2 - 5x = 0$ | $x^2 - 5x + 6 = 0$

$x(x-5) = 0$ | $(x-2)(x-3) = 0$

$x = 0$ of $x = 5$ | $x = 2$ of $x = 3$

(7) $(y^2 - 2y)^2 - 2y^2 + 4y - 3 = 0$

$(y^2 - 2y)^2 - 2(y^2 - 2y) - 3 = 0$

Stel $(y^2 - 2y) = k$

$\therefore k^2 - 2k - 3 = 0$

$(k-3)(k+1) = 0$

$k - 3 = 0$ of $k + 1 = 0$

$y^2 - 2y - 3 = 0$ | $y^2 - 2y + 1 = 0$

$(y-3)(y+1) = 0$ | $(y-1)(y-1) = 0$

$y = 3$ of $y = -1$ | $y = 1$

(8) $x^2 + x + 2 = \dfrac{-8}{x^2 + x - 4}$

Stel $(x^2 + x) = k$

$\therefore k + 2 = \dfrac{-8}{k - 4}$

$(k+2)(k-4) = -8$

$k^2 - 2k - 8 + 8 = 0$

$k^2 - 2k = 0$

$k(k-2) = 0$

$k = 0$ of $k - 2 = 0$

$x^2 + x = 0$ | $x^2 + x - 2 = 0$

$x(x+1) = 0$ | $(x+2)(x-1) = 0$

$x = 0$ of $x = -1$ | $x = -2$ of $x = 1$