Graad 12 – Boek A
(Eerste KABV uitgawe)

ONDERWYSERS HANDLEIDING

INHOUD:

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Hoofstuk A1
Rye en reekse

A1.1 Rekenkundige rye en reekse:
A1.1.1 Rekenkundige rye:

'n Rekenkundige ry (RR) het 'n konstante verskil.

Algemene term: $T_n = a + (n - 1)d$

Met: $a \rightarrow$ die eerste term

$d \rightarrow$ konstante verskil: $d = T_2 - T_1 = T_3 - T_2 = T_8 - T_7$ ....

$n \rightarrow$ posisie van die term $[n \in \mathbb{N}_0]$

Vb. 1 Beskou die ry: $5 ; 2 ; -1 ; -4 ; $ ... .

(a) Bereken die algemene term van die ry.

(b) Bereken T_{34}

(c) Bereken n as $T_n = -58$

(d) Los op vir x as $5T_x - 3T_{x+1} = 1$

(a) $a = 5$

$d = T_2 - T_1 = 2 - 5 = -3$ of $d = T_3 - T_2 = -1 - 2 = -3$

$T_n = a + (n - 1)d$

$\therefore \quad T_n = 5 + (n - 1)(-3)$

$\therefore \quad T_n = 5 - 3n + 3$

$\therefore \quad T_n = 8 - 3n$

(b) $T_n = 8 - 3n$

$\therefore T_{34} = 8 - 3(34)$

$\therefore T_{34} = -94$

(c) $T_n = 8 - 3n$

$\therefore -58 = 8 - 3n$

$\therefore 3n = 8 + 58$

$\therefore 3n = 66$

$\therefore \quad n = \dfrac{66}{3}$

$\therefore \quad n = 22$

(d) $5T_x - 3T_{x+1} = 1$

$\therefore \quad 5(8 - 3x) - 3[8 - 3(x + 1)] = 1$

$\therefore \quad 40 - 15x - 3[8 - 3x - 3] = 1$

$\therefore \quad 40 - 15x - 24 + 9x + 9 = 1$

$\therefore \quad -6x = 1 - 25 = -24$

$\therefore \quad x = 4$

<u>Oefening 1</u>:   Datum:_____

(1)   Skryf die volgende drie terme in elk van die volgende rye neer:

        [Dui ook aan watter van die rye is rekenkundige rye. **RR**]

        $\times 2 \quad \times 2 \quad \times 2$

(a) $3 \; ; \; 6 \; ; \; 12 \; ; \; 24 \; ; \;$ **$48 \; ; \; 96 \; ; \; 192$**

        $-7 \qquad -7 \qquad -7$

(b) $3 \; ; \; -4 \; ; \; -11 \; ; \; -18 \; ; \;$ **$-25 \; ; \; -32 \; ; \; -39$**   **RR**

        $+2 \qquad +2 \qquad +2$

(c) $113 \; ; \; 115 \; ; \; 117 \; ; \; 119 \; ; \;$ **$121 \; ; \; 123 \; ; \; 125$**   **RR**

        $+0,2 \qquad +0,2 \qquad +0,2$

(d) $0,17 \; ; \; 0,37 \; ; \; 0,57 \; ; \; 0,77 \; ; \;$ **$0,97 \; ; \; 1,17 \; ; \; 1,37$**   **RR**

(e) $\dfrac{1}{2} \; ; \; \dfrac{2}{3} \; ; \; \dfrac{3}{4} \; ; \; \dfrac{4}{5} \; ; \;$ **$\dfrac{5}{6} \; ; \; \dfrac{6}{7} \; ; \; \dfrac{7}{8}$**   **[Teller vermeerder met 1 en ook die noemer.]**

        $+6 \quad +10 \quad +14 \quad +18 \quad +22 \quad +26$

(f) $2 \; ; \; 8 \; ; \; 18 \; ; \; 32 \; ; \;$ **$50 \; ; \; 72 \; ; \; 98$**   **[Vermeerder telkens met 4 meer.]**

(2)   Skryf die eerste vier terme in elk van die volgende rye neer:

        [Dui ook aan watter van die rye is rekenkundige rye. **RR**]

(a) $T_n = -3n \rightarrow$ **$-3 \; ; \; -6 \; ; \; -9 \; ; \; -12$**   **RR**

(b) $T_n = 2^n \rightarrow$ **$2 \; ; \; 4 \; ; \; 8 \; ; \; 16$**

(c) $T_n = n + 10 \rightarrow$ **$11 \; ; \; 12 \; ; \; 13 \; ; \; 14$**   **RR**

(d) $T_n = 4n + 1 \rightarrow$ **$5 \; ; \; 9 \; ; \; 13 \; ; \; 17$**   **RR**

(e) $T_n = n^2 \rightarrow$ **$1 \; ; \; 4 \; ; \; 9 \; ; \; 16$**

(f) $T_n = \dfrac{n}{2} \rightarrow$ **$\dfrac{1}{2} \; ; \; 1 \; ; \; 1\dfrac{1}{2} \; ; \; 2$**   **RR**

(3)   Beskou die ry: $3 \; ; \; 7 \; ; \; 11 \; ; \; 15 \; ; \; \dots\dots$

(a) Bereken die algemene term van die ry.

   $a = 3$ **$T_n = a + (n-1)d$**

   $d = 7 - 3 = 4$ **$= 3 + (n-1)(4)$**

   $T_n = ?$ **$= 3 + 4n - 4$**

                          $\therefore \; T_n = 4n - 1$

(b)   Bereken T_{25}

$$T_n = 4n - 1$$

$$T_{25} = 4(25) - 1$$

∴ $T_{25} = 99$

(c)   Bereken n as $T_n = 87$

$$T_n = 4n - 1$$

∴ $$87 = 4n - 1$$

∴ $$87 + 1 = 4n$$

∴ $$4n = 88$$

∴ $$n = 22$$

(4)   Hoeveel terme is daar in die volgende ry: $65 \,;\, 59 \,;\, 53 \,;\, 47 \,;\, ……….;\, -85$?

$a = 65$ $T_n = a + (n - 1)d$

$d = 59 - 65 = -6$ $-85 = 65 + (n - 1)(-6)$

$T_n = -85$ ∴ $-85 = 65 - 6n + 6$

$n = ?$ ∴ $6n = 71 + 85$

                         ∴ $6n = 156$

                              ∴ $n = 26$

(5)   Plaas 6 terme tussen 8 en 29 sodat dit 'n rekenkundige ry vorm.

∴ $8 \,;\, \_\_ \,;\, \_\_ \,;\, \_\_ \,;\, \_\_ \,;\, \_\_ \,;\, \_\_ \,;\, 29$

$a = 8$ $T_n = a + (n - 1)d$

$d = ?$ $29 = 8 + (8 - 1)d$

$T_8 = 29$ $29 = 8 + 7d$

$n = 8$ ∴ $21 = 7d$

                          ∴ $d = 3$

∴ $8 \,;\, 11 \,;\, 14 \,;\, 17 \,;\, 20 \,;\, 23 \,;\, 26 \,;\, 29$

$$\overset{T_1}{}\quad\overset{T_2}{}\quad\overset{T_3}{}$$

(6)   Die eerste drie terme van 'n RR is: $x - 1$; $2x + 1$; $3 - x$

(a)   Bereken die waarde van x.

Vir RR: $T_2 - T_1 = T_3 - T_2$

$\therefore (2x + 1) - (x - 1) = (3 - x) - (2x + 1)$

$\therefore \quad 2x + 1 - x + 1 = 3 - x - 2x - 1$

$\therefore \quad\quad\quad x + 2 = 2 - 3x$

$\therefore \quad\quad\quad x + 3x = 2 - 2$

$\therefore \quad\quad\quad\quad 4x = 0$

$\therefore \quad\quad\quad\quad\quad x = 0$

(b)   Skryf die eerste vyf terme van die ry neer.

$T_1 = x - 1 = 0 - 1 = -1$

$T_2 = 2x + 1 = 2(0) + 1 = 0 + 1 = 1$

$T_3 = 3 - x = 3 - (0) = 3 \quad\quad\quad\quad \rightarrow \quad d = 2$

$\therefore T_4 = 5$

en $T_5 = 7 \quad\quad\quad \therefore$ **Ry:** -1 ; 1 ; 3 ; 5 ; 7

(c)   Skryf die n^{de} term neer.

$a = -1 \quad\quad\quad\quad\quad\quad\quad T_n = a + (n - 1)d$

$d = 2 \quad\quad\quad\quad\quad\quad\quad\quad = -1 + (n - 1)(2)$

$T_n = ? \quad\quad\quad\quad\quad\quad\quad = -1 + 2n - 2$

$\therefore T_n = 2n - 3$

(d)   Bepaal die 80ste term van die ry.

$T_n = 2n - 3$

$\therefore T_{80} = 2(80) - 3$

$\therefore T_{80} = 157$

(7)   As $T_n = 5 - 2n$, bereken:

(a)   die eerste term van die ry.

$$T_n = 5 - 2n$$

$$\therefore T_1 = 5 - 2(1)$$

$$\therefore T_1 = 3$$

(b)   die konstante verskil van die ry.

$$T_2 = 5 - 2(2) = 5 - 4 = 1$$

$$\therefore d = T_2 - T_1 = 1 - 3$$

$$\therefore d = -2$$

(c) $T_{24} + 3\,T_{56}$

$$= [5 - 2(24)] + 3[5 - 2(56)]$$

$$= [-43] + 3[-107]$$

$$= -364$$

(8)   Die algemene term van 'n RR is $T_n = 4n + 3$.

Bereken: $T_{2x} - 2\,T_{x-1}$

$$T_{2x} - 2\,T_{x-1}$$

$$= [4(2x) + 3] - 2[4(x - 1) + 3]$$

$$= [8x + 3] - 2[4x - 4 + 3]$$

$$= [8x + 3] - 2[4x - 1]$$

$$= 8x + 3 - 8x + 2$$

$$= 5$$

(9)   In 'n RR is die derde term gelyk aan 18 en $T_{10} = -17$.

Bepaal die eerste drie terme van die ry.

$$T_3 = a + (3 - 1)d = a + 2d = 18 \qquad \text{en}$$

$$T_{10} = a + (10 - 1)d = a + 9d = -17$$

$$\therefore \quad a + 9d = -17$$
$$- (a \mp 2d \equiv 18)$$

$$a + 2(-5) = 18 \quad \rightarrow \quad a + -10 = 18$$

$$7d = -35$$

$$\therefore a = 28$$

$$\therefore \qquad d = -5$$

$$\therefore \text{Ry:}\ \ 28\ ;\ 23\ ;\ 18\ ;\ 13\ ;\ \dots\dots$$

(10)   Die som van die derde term en die vierde term van 'n RR is 26 en die
verskil tussen dieselfde ry se elfde en tiende term is 4. Bepaal die eerste term en
dan ook die waarde van term vier-en-tagtig.

$$T_3 + T_4 = 26 \qquad \text{en} \qquad T_{11} - T_{10} = 4 \ \to \ d$$

$$(a + 2d) + (a + 3d) = 26$$

$$2a + 5d = 26$$

$$\therefore \quad 2a + 5(4) = 26$$

$$\therefore \qquad 2a = 26 - 20 = 6$$

$$\therefore \qquad a = 3 \ \to \ T_1$$

$$\therefore \qquad T_{84} = a + 83d$$

$$\therefore \qquad T_{84} = 3 + 83(4)$$

$$\therefore \qquad T_{84} = 335$$

A1.1.2 <u>Rekenkundige reekse:</u>
A1.1.2.1 <u>Formule:</u>

Formule vir die som (S_n) van 'n RR:

$$S_n = a \qquad + \ [a + d] \qquad + \ .... + [a + (n-2)d] + [a + (n-1)d]$$
$$+ \ S_n = [a + (n-1)d] + [a + (n-2)d] + ... + \qquad [a + d] \qquad + \qquad a$$

$$\therefore \ 2\,S_n = [2a + (n-1)d] + [2a + (n-1)d] + ... + [2a + (n-1)d] + [2a + (n-1)d]$$
$$\therefore \ 2\,S_n = n\,[2a + (n-1)d]$$
$$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

of as $\ell \to$ laaste term

$$S_n = a + [a + d] + ... + [\ell - d] + \ell$$
$$+ \ S_n = \ell + [\ell - d] + ... + [a + d] + a$$

$$\therefore \ 2\,S_n = [a + \ell] + [a + \ell] + ... + [a + \ell] + [a + \ell]$$
$$\therefore \ 2\,S_n = n\,[a + \ell]$$
$$\therefore \quad S_n = \frac{n}{2}[a + \ell]$$

Vb. 2 Bereken: $5 + 2 - 1 - 4 - \ldots\ldots - 94$

$T_n = a + (n - 1)d$ met $S_n = \frac{n}{2}[a + \ell]$

$\therefore \quad T_n = 5 + (n - 1)(-3)$ met $\therefore \quad S_{34} = \frac{34}{2}[5 + (-94)]$

$\therefore -94 = 5 - 3n + 3$ $\therefore \quad S_{34} = -1\,513$

$\therefore \quad 3n = 8 + 94$

$\therefore \quad 3n = 102$ of $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad n = 34$ $\therefore \quad S_{34} = \frac{34}{2}[2(5) + (34 - 1)(-3)]$

$\therefore \quad T_{34} = -94 = \ell$ [laaste term] $\therefore \quad S_{34} = -1\,513$

A1.1.2.2 <u>Sigma-notasie:</u>

Sigma notasie $\rightarrow \displaystyle\sum_{k=2}^{8} 5k - 1$

Wat lees as: Bereken die som vanaf waar $k = 2$ tot by $k = 8$ vir $(5k - 1)$.

Vb. 3 Bereken n **as** $\displaystyle\sum_{k=1}^{n} 3k - 1 = 442$

$\therefore$ **Bereken:** $[3(1) - 1] + [3(2) - 1] + [3(3) - 1] + \ldots\ldots\ldots + [3(k) - 1]$

$\therefore 2 + 5 + 8 + \ldots\ldots + [3(n) - 1] = 442$

$\therefore \quad a = 1$ $\therefore \quad S_n = \frac{n}{2}[2a + (n - 1)d]$

$\quad\quad d = 2$ $\therefore \quad 442 = \frac{n}{2}[2(2) + (n - 1)(3)]$

$\quad\quad n = ?$ $\therefore \quad 442 \times 2 = n[4 + 3n - 3]$

$\quad\quad S_n = 442$ $\therefore \quad\quad 884 = 3n^2 + 1n$

                         $\therefore \quad\quad\quad 0 = 3n^2 + 1n - 884$

                         $\therefore \quad\quad\quad 0 = (3n + 52)(n - 17)$

                         $\therefore \quad\quad\quad n = \frac{-52}{3}$ of $n = 17$

                                    Nvt

                         $\therefore \quad S_{17} = 442$ $[n \in \mathbb{N}_0]$

Vb. 4 Skryf die volgende in sigma-notasie:

$29 \; + \; 25 \; + \; 21 \; + \; 17 \; + \; .....$ (tot **18** terme)

$T_n \; = \; a \; + \; (n \; - \; 1)d$

$\therefore \quad T_n \; = \; 29 \; + \; (n \; - \; 1)(-4)$

$\therefore \quad T_n \; = \; 29 \; - \; 4n \; + \; 4$

$\therefore \quad T_n \; = \; 33 \; - \; 4n$

$\therefore$ **Sigma notasie** $\rightarrow \displaystyle\sum_{n\,=\,1}^{18} 33 \; - \; 4n$

Oefening 2: Datum:_____

(1) Bereken:

(a) $5 \; + \; 8 \; + \; 11 \; + \; 14 + ...$ tot 16 terme

$a = 5$ $\qquad S_n \; = \; \frac{n}{2}[2a \; + \; (n \; - \; 1)d]$

$d = 8 \; - \; 5 \; = \; 3$ $\qquad S_{16} \; = \; \frac{16}{2}[2 \times 5 \; + \; (16 \; - \; 1)(3)]$

$n \; = \; 16$ $\qquad\qquad = \; 8[10 \; + \; 15 \times 3]$

$S_n \; = \; ?$ $\qquad\qquad \boxed{S_{16} \; = \; 440}$

(b) $9 \; + \; 12 \; + \; 15 \; + \; 18 + ... \; + \; 264$

$a \; = \; 9$ $\qquad T_n \; = \; a \; + \; (n \; - \; 1)d$

$d \; = \; 3$ $\qquad 264 \; = \; 9 \; + \; (n - 1)(3)$

$n \; = \; ?$ $\qquad 264 \; = \; 9 \; + \; 3n \; - \; 3$

$T_n \; = \; 264$ $\qquad 258 \; = \; 3n$

$S_n \; = \; ?$ $\qquad \therefore \quad n \; = \; 86$

$\therefore \qquad S_n \; = \; \frac{n}{2}[2a \; + \; (n \; - \; 1)d]$

$\qquad S_{86} \; = \; \frac{86}{2}[2 \times 9 \; + \; (86 \; - \; 1)(3)]$

$\qquad\qquad = \; 43[18 \; + \; 85 \times 3]$

$\boxed{S_{86} \; = \; 11\,739}$ of $S_{86} \; = \; \frac{86}{2}[9 \; + \; 264] \; = \; \boxed{11\,739}$

(c) $36 + 31 + 26 + 21 + \ldots\ldots$ tot 34 terme

$a = 36$ $\qquad$ $S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 31 - 36 = -5$ $\qquad$ $S_{34} = \frac{34}{2}[2 \times 36 + (34 - 1)(-5)]$

$n = 34$ $\qquad\qquad$ $= 17[72 + 33 \times -5]$

$S_n = ?$ $\qquad\qquad$ $\boxed{S_{34} = -1\,581}$

(d) $\displaystyle\sum_{k=1}^{7} (4k - 1)$

Ry: $4(1) - 1 \ ; \ 4(2) - 1 \ ; \ 4(3) - 1 \ ; \ \ldots\ldots \ ; \ 4(7) - 1$

$\qquad = 3 \ ; \ 7 \ ; \ 11 \ ; \ \ldots\ldots \ ; \ 27$

$a = 3$ $\qquad$ $S_n = \frac{n}{2}[2a + (n - 1)d]$ $\qquad$ of $S_n = \frac{n}{2}[a + \ell]$

$d = 4$ $\qquad$ $S_7 = \frac{7}{2}[2 \times 3 + (7 - 1)(4)]$ $\qquad$ $S_7 = \frac{7}{2}[3 + 27]$

$n = 7$ $\qquad\qquad$ $= 3,5[6 + 6 \times 4]$ $\qquad$ $\boxed{S_7 = 105}$

$\ell = 27$ $\qquad$ $\boxed{S_7 = 105}$

$S_n = ?$

(e) $-7 - 2 + 3 + 8 + \ldots\ldots + 123$

$a = -7$ $\qquad$ $T_n = a + (n - 1)d$ $\qquad$ $\therefore$ $S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 5$ $\qquad$ $123 = -7 + (n - 1)(5)$ $\qquad$ $S_{27} = \frac{27}{2}[2 \times -7 + (27 - 1)(5)]$

$n = ?$ $\qquad$ $123 = -7 + 5n - 5$ $\qquad\qquad$ $= 13,5[-14 + 26 \times 5]$

$T_n = 123$ $\qquad$ $135 = 5n$ $\qquad\qquad$ $\boxed{S_{27} = 1\,566}$

$S_n = ?$ $\qquad$ $\therefore$ $n = 27$ $\qquad\qquad$ of $\boxed{S_{27} = \frac{27}{2}[-7 + 123] = 1\,566}$

(f) $\displaystyle\sum_{n=3}^{12} (3 - n)$

Ry: $3 - (3)$; $3 - (4)$; $3 - (5)$; ...... ; $3 - (12)$

$= 0$; -1 ; -2 ; ... ... ; -9

$a = 0$ $\qquad$ $S_n = \frac{n}{2}[2a + (n - 1)d]$ $\qquad$ of $\qquad$ $S_n = \frac{n}{2}[a + \ell]$

$d = -1$ $\qquad$ $S_{10} = \frac{10}{2}[2 \times 0 + (10 - 1)(-1)]$ $\qquad$ $S_{10} = \frac{10}{2}[0 + (-9)]$

$n = 10$ $\qquad\qquad$ $= 5[0 + 9 \times -1]$ $\qquad\qquad$ $S_{10} = -45$

$\ell = -9$ $\qquad\qquad$ $S_{10} = -45$

$S_n = ?$

(g) n as $\displaystyle\sum_{k=1}^{n} (3k - 2) = 92$

Ry: $3(1) - 2$; $3(2) - 2$; $3(3) - 2$; ...... ; $3(n) - 2$

$= 1$; 4 ; 7 ; ... ...

$a = 1$ $\qquad\qquad$ $S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 3$ $\qquad\qquad$ $92 = \frac{n}{2}[2 \times 1 + (n - 1)(3)]$

$n = ?$ $\qquad\qquad$ $184 = n[2 + 3n - 3]$

$S_n = 92$ $\qquad\qquad$ $184 = 2n + 3n^2 - 3n$

$\qquad\qquad\qquad$ $0 = 3n^2 - n - 184$

$\qquad\qquad\qquad$ $0 = (3n + 23)(n - 8)$

$\therefore$ $n = -\frac{23}{3}$ Nvt $(n \in \mathbb{N}_0)$ $\qquad$ of $\qquad$ $n = 8$

(h) $-66 - 64 - 62 - 60 - \ldots\ldots.. - 22$

$a = -66$

$T_n = a + (n - 1)d$

$\therefore$ $S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 2$

$-22 = -66 + (n - 1)(2)$

$S_{23} = \frac{23}{2}[2 \times -66 + (23 - 1)(2)]$

$n = ?$

$-22 = -66 + 2n - 2$

$= 11,5[-132 + 22 \times 2]$

$T_n = -22$

$46 = 2n$

$S_{23} = -1\,012$

$S_n = ?$

$\therefore$ $n = 23$

of $\quad S_{23} = \frac{23}{2}[-66 + (-22)] = -1\,012$

(i) $\displaystyle\sum_{i = 1}^{300}\left(\frac{i}{2}\right)$

Ry: $\frac{(1)}{2}$; $\frac{(2)}{2}$; $\frac{(3)}{2}$ $\ldots\ldots$; $\frac{(300)}{2}$

$= \frac{1}{2}$; 1 ; $1\frac{1}{2}$; $\ldots\ldots$; 150

$a = \frac{1}{2}$

$S_{300} = \frac{300}{2}\left[2 \times \frac{1}{2} + (300 - 1)\left(\frac{1}{2}\right)\right]$

of $S_n = \frac{n}{2}[a + \ell]$

$d = \frac{1}{2}$

$S_{300} = \frac{300}{2}\left[1 + (299)\left(\frac{1}{2}\right)\right]$

$S_{300} = \frac{300}{2}\left[\frac{1}{2} + 150\right]$

$n = 300$

$= 150\left[1 + (299)\left(\frac{1}{2}\right)\right]$

$S_{300} = 22\,575$

$\ell = 150$

$S_{300} = 22\,575$

$S_n = ?$

(j) n as $0,3 + 1,1 + 1,9 + 2,7 + \ldots\ldots$ (tot n terme) $= 24,8$

$a = 0,3$

$d = 0,8$

$n = ?$

$S_n = 24,8$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$24,8 = \frac{n}{2}[2 \times 0,3 + (n - 1)(0,8)]$

$49,6 = n[0,6 + 0,8n - 0,8]$

$49,6 = 0,6n + 0,8n^2 - 0,8n$

$0 = 0,8n^2 - 0,2n - 49,6 \qquad [\times 10]$

$0 = 8n^2 - 2n - 496 \leftarrow$

$0 = 4n^2 - n - 248$

$0 = (4n + 31)(n - 8)$

$\therefore n = \frac{-31}{4}$ Nvt ($n \in \mathbb{N}_0$) of $n = 8$

(2) Die n^{de} term van 'n RR is $2n + 3$. Bepaal:

(a) die eerste drie terme van die ry.

$T_n = 2n + 3$

$T_1 = 2(1) + 3 = 2 + 3 = 5$

$T_2 = 2(2) + 3 = 4 + 3 = 7$

$T_3 = 2(3) + 3 = 6 + 3 = 9$

$\therefore 5 ; 7 ; 9$

(b) die 18de term van die ry.

$T_n = 2n + 3$

$T_{18} = 2(18) + 3$

$T_{18} = 36 + 3$

$\therefore T_{18} = 39$

(c) hoeveel terme in die ry het 'n som van 4 352.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$4\,352 = \frac{n}{2}[2(5) + (n - 1)(2)]$$

$$8\,704 = n[10 + 2n - 2]$$

$$8\,704 = 8n + 2n^2$$

$$0 = 8n + 2n^2 - 8\,704$$

$$0 = 2n^2 + 8n - 8\,704$$

$$0 = n^2 + 4n - 4\,352$$

$$0 = (n + 68)(n - 64)$$

$$n + 68 = 0 \quad \text{of} \quad n - 64 = 0$$

$$\therefore \quad n = -68 \text{ Nvt } (n \in \mathbb{N}_0) \qquad \text{of} \qquad n = 64$$

(3)   Die volgende is gegee: $\displaystyle\sum_{t=2}^{11} (3 - 3t)$

(a) Skryf die eerste drie terme neer.

$$T_1 = 3 - 3(2) = 3 - 6 = -3 \quad \rightarrow \quad T_1 = -3$$

$$T_2 = 3 - 3(3) = 3 - 9 = -6 \quad \rightarrow \quad T_2 = -6$$

$$T_3 = 3 - 3(4) = 3 - 12 = -9 \quad \rightarrow \quad T_3 = -9$$

(b) Bepaal die som van die reeks.

$$a = -3 \qquad\qquad S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$d = -3 \qquad\qquad S_{10} = \frac{10}{2}[2 \times -3 + (10 - 1)(-3)]$$

$$n = 10 \qquad\qquad\qquad = 5[-6 + 9 \times -3]$$

$$S_n = ? \qquad\qquad\qquad = 5[-6 - 27]$$

$$\qquad\qquad\qquad\qquad S_{10} = -165$$

(4) As $S_n = n^2 + n$, bepaal T_7.

$$S_7 = 7^2 + 7 = 49 + 7 = 56$$

$$S_6 = 6^2 + 6 = 36 + 6 = 42$$

Maar $T_7 = S_7 - S_6 = 56 - 42$

$$\therefore T_7 = 14$$

(5) In 'n RR is $S_6 = 123$ en $T_5 = 25$.

(a) Bepaal die eerste term en die konstante verskil van die ry.

$$S_6 = 123 \qquad\qquad \text{en} \qquad\qquad T_5 = 25$$

Maar $S_6 = \frac{6}{2}[2a + (6-1)d]$ $\qquad\qquad T_5 = a + (5-1)d$

$\therefore 123 = 3[2a + 5d]$ $\qquad\qquad\qquad 25 = a + 4d$

$\therefore 123 = 6a + 15d \longleftarrow \qquad 25 - 4d = a$

$\therefore 123 = 6(25 - 4d) + 15d$

$\therefore 123 = 150 - 24d + 15d$

$\therefore 123 - 150 = -9d$

$\therefore \qquad\qquad -27 = -9d$

$\therefore \qquad\qquad d = \frac{-27}{-9} = 3$

$\therefore \qquad\qquad a = 25 - 4(3)$

$\therefore \qquad\qquad a = 25 - 12$

$\therefore \qquad\qquad a = 13$

(b) Bepaal n waarvoor $S_n < 403$.

$$S_n = \frac{n}{2}[2a + (n - 1)d] < 403$$

$$\therefore \quad \frac{n}{2}[2(13) + (n - 1)(3)] < 403$$
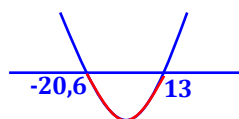
$$\therefore \quad n[26 + 3n - 3] < 806$$

$$\therefore \quad 26n + 3n^2 - 3n < 806$$

$$\therefore \quad 3n^2 + 23n - 806 < 0$$

$$\therefore \quad (3n + 62)(n - 13) < 0$$

$$\therefore \quad 0 \leq n < 13$$

$$(n \in \mathbb{N}_0)$$

(6) Gegee: $-1 \; ; \; 2 \; ; \; 5 \; ; \; 8 \; ;$ …… …

(a) Bepaal die twintigste term van die ry.

$a = -1$ $T_n = a + (n - 1)d$

$d = 3$ $\therefore \quad T_{20} = -1 + (20 - 1)(3)$

$n = 20$ $\therefore \quad T_{20} = -1 + (19)(3) = -1 + 57$

$T_n = ?$ $\therefore \quad T_{20} = 56$

(b) Bepaal die som van die eerste twintig terme.

$a = -1$ $S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 3$ $\therefore \quad S_{20} = \frac{20}{2}[2(-1) + (20 - 1)(3)]$

$n = 20$ $\therefore \quad S_{20} = 10[-2 + (19)(3)] = 10[-2 + 57]$

$S_n = ?$ $\therefore \quad S_{20} = 550$

(c) Watter term in die ry is gelyk aan 56?

$a = -1$ $T_n = a + (n - 1)d$

$d = 3$ $\therefore \quad T_n = -1 + (n - 1)(3)$

$n = ?$ $\therefore \quad 56 = -1 + 3n - 3$

$T_n = 56$ $\therefore \quad 56 + 1 + 3 = 3n$

       $\therefore \quad\quad\quad\quad 3n = 60$

       $\therefore \quad\quad\quad\quad n = 20$

(d) Hoeveel terme moet bymekaar getel word om 'n som van 259 te kry?

$a = -1$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 3$

$\therefore \ 259 = \frac{n}{2}[2(-1) + (n - 1)(3)]$

$n = ?$

$259 = \frac{n}{2}[-2 + 3n - 3]$

$S_n = 374$

$\therefore \ 518 = 3n^2 - 5n$

$$0 = 3n^2 - 5n - 518$$

$$\therefore \quad 0 = (3n + 37)(n - 14)$$

$$\therefore \quad n = \frac{-37}{3} \text{ Nvt } (n \in \mathbb{N}_0) \qquad \text{of} \qquad n = 14$$

(7)   Die eerste drie terme van 'n RR is: $7x - 1$; $2x + 3$ en $3 - 5x$

(a) Bepaal die waarde van x.

$\text{RR} \quad \rightarrow \quad d \quad \rightarrow \quad T_2 - T_1 = T_3 - T_2$

$\therefore (2x + 3) - (7x - 1) = (3 - 5x) - (2x + 3)$

$\therefore \quad 2x + 3 - 7x + 1 = 3 - 5x - 2x - 3$

$-5x + 4 = -7x$

$-5x + 7x = -4$

$2x = -4$

$\therefore \qquad x = -2$

(b) Skryf die eerste vier terme van die ry neer.

$T_1 = 7x - 1 = 7(-2) - 1 = -14 - 1 = -15$

$T_2 = 2x + 3 = 2(-2) + 3 = -4 + 3 = -1$

$T_3 = 3 - 5x = 3 - 5(-2) = 3 + 10 = 13$

$\therefore$ Die ry: -15 ; -1 ; 13 ; 27 ; ... ...

(c) Bereken T_{18}.

$a = -15$

$T_n = a + (n - 1)d$

$d = 14$

$\therefore \ T_{18} = -15 + (18 - 1)(14)$

$n = 18$

$\therefore \ T_{18} = -15 + (17)(14)$

$= -15 + 238$

$T_n = ?$

$\therefore \ T_{18} = 223$

(d) Hoeveel terme moet by die eerste 18 terme getel word om 'n som van 2 360 te kry?

$a = -15$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 14$

$\therefore \quad 2\,360 = \frac{n}{2}[2(-15) + (n - 1)(14)]$

$n = ?$

$2\,360 = \frac{n}{2}[-30 + 14n - 14]$

$S_n = 2\,360$

$\therefore \quad 4720 = 14n^2 - 44n$

$0 = 14n^2 - 44n - 4\,720$

$0 = 7n^2 - 22n - 2\,360$

$\therefore \quad 0 = (7n + 118)(n - 20)$

$\therefore \quad n = \frac{-118}{7}$ Nvt $(n \in \mathbb{N}_0)$ $\quad$ of $\quad n = 20$

∴ **Twee terme moet by die eerste 18 terme getel word om 'n som van 2 360 te gee.**

(8) In 'n RR is die eerste term 23 en die laaste term is -369.
Die som van die reeks is $-9\,861$.

(a) Bepaal die aantal terme in die reeks.

$a = 23$

$S_n = \frac{n}{2}[a + \ell]$

$\ell = -369$

$\therefore \quad -9\,861 = \frac{n}{2}[23 + (-369)]$

$n = ?$

$-9\,861 = \frac{n}{2}[23 - 369]$

$S_n = -9\,861$

$\therefore \quad -9\,861 = \frac{n}{2} \times -346$

$\therefore \quad -9\,861 = -173n$

$\therefore \quad n = \frac{-9\,861}{-173}$

$\therefore \quad n = 57$

(b) Bepaal die konstante verskil van die terme in die reeks.

$a = 23$

$T_n = a + (n - 1)d$

$d = ?$

$\therefore \quad -369 = 23 + (57 - 1)d$

$n = 57$

$\therefore \quad -369 = 23 + 56d$

$T_n = -369$

$\therefore \quad -369 = 23 + 56d$

$\therefore \quad -56d = 23 + 369$

$\therefore \quad -56d = 392$

$\therefore \quad d = -7$

(c) Bepaal die dertiende term.

$a = 23$ $\qquad T_n = a + (n - 1)d$

$\quad d = -7$ $\qquad \therefore \quad T_{13} = 23 + (13 - 1)(-7)$

$\quad n = 13$ $\qquad \therefore \quad T_{13} = 23 - 84$

$\quad T_{13} = ?$ $\qquad \therefore \quad T_{13} = -61$

(9) Gegee rekenkundige ry: $3k + 2 \; ; \; 5k + 1 \; ; \; 7k \; ; \; 9k - 1 \; ; \; \ldots\ldots\ldots$

(a) Bepaal die konstante verskil in terme van k.

$\qquad d = T_2 - T_1 \qquad\qquad\qquad \text{of} \qquad\qquad d = T_3 - T_2$

$\therefore \quad d = (5k + 1) - (3k + 2) \qquad\qquad\qquad d = (7k) - (5k + 1)$

$\qquad = 5k + 1 - 3k - 2 \qquad\qquad\qquad\qquad = 7k - 5k - 1$

$\therefore \quad d = 2k - 1$

(b) Bepaal die som van die eerste twaalf terme in terme van k.

$a = 3k + 2 \qquad\qquad\qquad S_n = \dfrac{n}{2}[2a + (n - 1)d]$

$d = 2k - 1 \qquad\qquad\qquad \therefore S_{12} = \dfrac{12}{2}[2(3k + 2) + (12 - 1)(2k - 1)]$

$n = 12 \qquad\qquad\qquad\qquad \therefore S_{12} = \dfrac{12}{2}[2(3k + 2) - (11)(2k - 1)]$

$S_n = ? \qquad\qquad\qquad\qquad \therefore S_{12} = 6[6k + 4 + 22k - 11]$

$\qquad\qquad\qquad\qquad\qquad\quad \therefore S_{12} = 6[28k - 7]$

$\qquad\qquad\qquad\qquad\qquad\quad \therefore S_{12} = 168k - 42$

A1.1.3 Toepassings:

Vb. 5 Rachel oefen vir 'n marathon. Sy hardloop 20 km op die eerste dag se voorbereiding. Sy besluit om elke dag as sy oefen 3 km verder te hardloop as die vorige keer.

(a) Indien sy hierdie oefenprogram volg, bereken tydens watter dag van oefening sy 56 km sal hardloop.

(b) Bereken Rachel se totale afstand wat sy gehardloop het vir die eerste 10 voorbereidende oefeninge.

(a) RR: $20 \; ; \; 23 \; ; \; 26 \; ; \; 29 \; ; \; \ldots\ldots \; ; \; 56$

$\therefore \quad a = 20 \qquad\qquad\qquad\qquad T_n = a + (n - 1)d$

$\quad d = 3 \qquad\qquad\qquad\qquad \therefore \quad 56 = 20 + (n - 1)(3)$

$\quad n = ? \qquad\qquad\qquad\qquad \therefore \quad 56 = 20 + 3n - 3$

$\quad T_n = 56 \qquad\qquad\qquad\qquad \therefore \quad -3n = 17 - 56$

$\qquad\qquad\qquad\qquad\qquad\qquad \therefore \quad -3n = -39$

$\qquad\qquad\qquad\qquad\qquad\qquad \therefore \qquad n = 13$

$\therefore$ Rachel sal op die dertiende dag 56 km hardloop.

(b) S_{10} = ? $\qquad S_n = \frac{n}{2}[2a + (n-1)d]$

$$\therefore S_{10} = \frac{10}{2}[2(20) + (10-1)(3)]$$

$$\therefore S_{10} = 335$$

$\therefore$ **Rachel se totale afstand vir die eerste 10 dae is 335 km.**

Oefening 3: $\qquad\qquad\qquad$ Datum:_____

(1) Cory sny sestien stukkies tou vir 'n projek. Die kortste van die toue is 28 cm lank en die langste van die toue is 88 cm lank. Indien die lengte van die sestien toue van kort tot lank neergeskryf word, vorm dit 'n RR.

(a) Bereken die lengte van die 13de tou in die ry.

Ry: 28 ; … … . … ; 88

$a = 28$ $\qquad\qquad T_n = a + (n-1)d$

$d = ?$ $\qquad\qquad \therefore \qquad T_{16} = 28 + (16-1)d$

$n = 16$ $\qquad\qquad \therefore \qquad 88 = 28 + 15d$

$T_{16} = 88$ $\qquad \therefore \qquad -15d = 28 - 88 = -60$

$\qquad\qquad\qquad \therefore \qquad d = \frac{-60}{-15} = 4$

$a = 28$

$d = 4$ $\qquad\qquad \therefore \qquad T_{13} = 28 + (13-1)(4)$

$n = 13$ $\qquad\qquad \therefore \qquad T_{13} = 28 + 48$

$T_{13} = ?$ $\qquad\qquad \therefore \qquad T_{13} = 76$

$\therefore$ **Die 13de tou is 76 cm lank.**

(b) Bereken die totale lengte van al 16 toue.

$a = 28$ $\qquad\qquad S_n = \frac{n}{2}[2a + (n-1)d]$

$d = 4$ $\qquad\qquad \therefore \qquad S_{16} = \frac{16}{2}[2(28) + (16-1)(4)]$

$n = 16$ $\qquad\qquad\qquad\qquad = 8[56 + 15 \times 4]$

$S_n = ?$ $\qquad\qquad \therefore \qquad S_{16} = 928$

$\therefore$ $\qquad$ **Die totale lengte van die 16 toue is 928 cm.**