

Grade 12 – Book A

(First edition – CAPS)

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Chapter A1

Sequences and series

A1.1 Arithmetic sequences and series:

A1.1.1 Arithmetic sequences:

An arithmetic sequence (AS) has a constant difference.

General term: $T_n = a + (n - 1)d$

With: $a \rightarrow$ the first term

$d \rightarrow$ constant difference: $d = T_2 - T_1 = T_3 - T_2 = T_8 - T_7 \dots\dots\dots$

$n \rightarrow$ position of the term $[n \in \mathbb{N}_0]$

Ex. 1 Consider the sequence: 5 ; 2 ; -1 ; -4 ;

(a) Calculate the general term of the sequence.

(b) Calculate T_{34}

(c) Calculate n if $T_n = -58$

(d) Solve for x if $5T_x - 3T_{x+1} = 1$

(a) $a = 5$

$$d = T_2 - T_1 = 2 - 5 = -3 \quad \text{or} \quad d = T_3 - T_2 = -1 - 2 = -3$$

$$T_n = a + (n - 1)d$$

$$\therefore T_n = 5 + (n - 1)(-3)$$

$$\therefore T_n = 5 - 3n + 3$$

$$\therefore T_n = 8 - 3n$$

(b) $T_n = 8 - 3n$

$$\therefore T_{34} = 8 - 3(34)$$

$$\therefore T_{34} = -94$$

(c) $T_n = 8 - 3n$

$$\therefore -58 = 8 - 3n$$

$$\therefore 3n = 8 + 58$$

$$\therefore 3n = 66$$

$$\therefore n = \frac{66}{3}$$

$$\therefore n = 22$$

(d) $5T_x - 3T_{x+1} = 1$

$$\therefore 5(8 - 3x) - 3[8 - 3(x + 1)] = 1$$

$$\therefore 40 - 15x - 3[8 - 3x - 3] = 1$$

$$\therefore 40 - 15x - 24 + 9x + 9 = 1$$

$$\therefore -6x = 1 - 25 = -24$$

$$\therefore x = 4$$

Exercise 1:

Date: _____

(1) Write the next three terms in each of the following sequences:

[Also indicate which of the sequences are arithmetic sequences.]

(a) 3 ; 6 ; 12 ; 24 ; _____

(b) 3 ; - 4 ; - 11 ; - 18 ; _____

(c) 113 ; 115 ; 117 ; 119 ; _____

(d) 0,17 ; 0,37 ; 0,57 ; 0,77 ; _____

(e) $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$; _____

(f) 2 ; 8 ; 18 ; 32 ; _____

(2) Write the first four terms in each of the following sequences:

[Also indicate which of the sequences are arithmetic sequences.]

(a) $T_n = -3n$ _____(b) $T_n = 2^n$ _____(c) $T_n = n + 10$ _____(d) $T_n = 4n + 1$ _____(e) $T_n = n^2$ _____(f) $T_n = \frac{n}{2}$ _____

(3) Consider the sequence: 3 ; 7 ; 11 ; 15 ;

(a) Calculate the general term of the sequence.

(b) Calculate T_{25}

(c) Calculate n if $T_n = 87$

(4) How many terms are there in the following sequence: $65 ; 59 ; 53 ; 47 ; \dots \dots \dots ; -85$?

(5) Place 6 terms between 8 and 29 for it to form an arithmetic sequence.

(6) The first three terms of an AS are: $x - 1$; $2x + 1$; $3 - x$

(a) Calculate the value of x .

(b) Write down the first five terms of the sequence.

(c) Write down the n^{th} term.

(d) Determine the 80^{th} term of the sequence.

(7) If $T_n = 5 - 2n$, calculate:

(a) the first term of the sequence.

(b) the constant difference of the sequence.

(c) $T_{24} + 3T_{56}$

(8) The general term of an AS is $T_n = 4n + 3$.

Calculate: $T_{2x} - 2T_{x-1}$

(9) The third term in an AS is equal to 18 and $T_{10} = -17$.

Determine the first three terms of the sequence.

- (10) The sum of the third term and the fourth term of an AS is 26 and the difference between the eleventh and the tenth terms of the same sequence is 4. Determine the first term and the value of term eighty-four.

A1.1.2 Arithmetic series:

A1.1.2.1 Formula:

Formula for the sum (S_n) of an AS:

$$S_n = a + [a + d] + \dots + [a + (n-2)d] + [a + (n-1)d]$$

$$+ S_n = [a + (n-1)d] + [a + (n-2)d] + \dots + [a + d] + a$$

$$\therefore 2 S_n = [2a + (n-1)d] + [2a + (n-1)d] + \dots + [2a + (n-1)d] + [2a + (n-1)d]$$

$$\therefore 2 S_n = n [2a + (n-1)d]$$

$$\therefore S_n = \frac{n}{2} [2a + (n-1)d]$$

or if $l \rightarrow$ last term

$$S_n = a + [a + d] + \dots + [l - d] + l$$

$$+ S_n = l + [l - d] + \dots + [a + d] + a$$

$$\therefore 2 S_n = [a + l] + [a + l] + \dots + [a + l] + [a + l]$$

$$\therefore 2 S_n = n [a + l]$$

$$\therefore S_n = \frac{n}{2} [a + l]$$

Ex. 2 Calculate: $5 + 2 - 1 - 4 - \dots - 94$

$$\begin{array}{ll}
 T_n = a + (n - 1)d & \text{with} \quad S_n = \frac{n}{2}[a + \ell] \\
 \therefore T_n = 5 + (n - 1)(-3) & \text{with} \quad \therefore S_{34} = \frac{34}{2}[5 + (-94)] \\
 \therefore -94 = 5 - 3n + 3 & \therefore S_{34} = -1\,513 \\
 \therefore 3n = 8 + 94 & \\
 \therefore 3n = 102 & \text{or} \quad S_n = \frac{n}{2}[2a + (n - 1)d] \\
 \therefore n = 34 & \therefore S_{34} = \frac{34}{2}[2(5) + (34 - 1)(-3)] \\
 \therefore T_{34} = -94 = \ell \text{ [last term]} & \therefore S_{34} = -1\,513
 \end{array}$$

A1.1.2.2 Sigma-notation:

$$\text{Sigma notation} \rightarrow \sum_{k=2}^8 5k - 1$$

It reads as: Calculate the sum from where $k = 2$ up to where $k = 8$ for $(5k - 1)$.

Ex. 3 Calculate n if $\sum_{k=1}^n 3k - 1 = 442$

$$\therefore \text{Calculate: } [3(1) - 1] + [3(2) - 1] + [3(3) - 1] + \dots + [3(k) - 1]$$

$$\therefore 2 + 5 + 8 + \dots + [3(n) - 1] = 442$$

$$\therefore a = 1$$

$$d = 2$$

$$n = ?$$

$$S_n = 442$$

$$\therefore S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\therefore 442 = \frac{n}{2}[2(2) + (n - 1)(3)]$$

$$\therefore 442 \times 2 = n[4 + 3n - 3]$$

$$\therefore 884 = 3n^2 + 1n$$

$$\therefore 0 = 3n^2 + 1n - 884$$

$$\therefore 0 = (3n + 52)(n - 17)$$

$$\therefore n = \frac{-52}{3} \quad \text{or} \quad n = 17$$

N/A

$$\therefore S_{17} = 442 \quad [n \in \mathbb{N}_0]$$

Ex. 4 Write the following in sigma-notation:

$$29 + 25 + 21 + 17 + \dots \quad (\text{to 18 terms})$$

$$T_n = a + (n - 1)d$$

$$\therefore T_n = 29 + (n - 1)(-4)$$

$$\therefore T_n = 29 - 4n + 4$$

$$\therefore T_n = 33 - 4n$$

$$\therefore \text{Sigma notation} \rightarrow \sum_{n=1}^{18} 33 - 4n$$

Exercise 2:

Date: _____

(1) Calculate:

(a) $5 + 8 + 11 + 14 + \dots$ to 16 terms

(b) $9 + 12 + 15 + 18 + \dots + 264$

(c) $36 + 31 + 26 + 21 + \dots$ to 34 terms

(d) $\sum_{k=1}^7 (4k - 1)$

(e) $-7 - 2 + 3 + 8 + \dots + 123$

(f) $\sum_{n=3}^{12} (3 - n)$

(g) n if $\sum_{k=1}^n (3k - 2) = 92$

(h) $-66 - 64 - 62 - 60 - \dots - 22$

(i) $\sum_{i=1}^{300} \left(\frac{i}{2}\right)$

(j) n if $0,3 + 1,1 + 1,9 + 2,7 + \dots$ (to n terms) = $24,8$

(2) The n^{th} term of an AS is $2n + 3$. Determine:

(a) the first three terms of the sequence.

(b) the 18th term of the sequence.

(c) how many terms in the sequence have a sum of 4 352.

(3) The following is given: $\sum_{t=2}^{11} (3 - 3t)$

(a) Write down the first three terms.

(b) Determine the sum of the series.

(b) Determine n for which $S_n < 403$.

(6) Given: $-1 ; 2 ; 5 ; 8 ; \dots \dots \dots$

(a) Determine the twentieth term of the sequence.

(b) Determine the sum of the first twenty terms.

(c) Which term in the sequence is equal to 56?

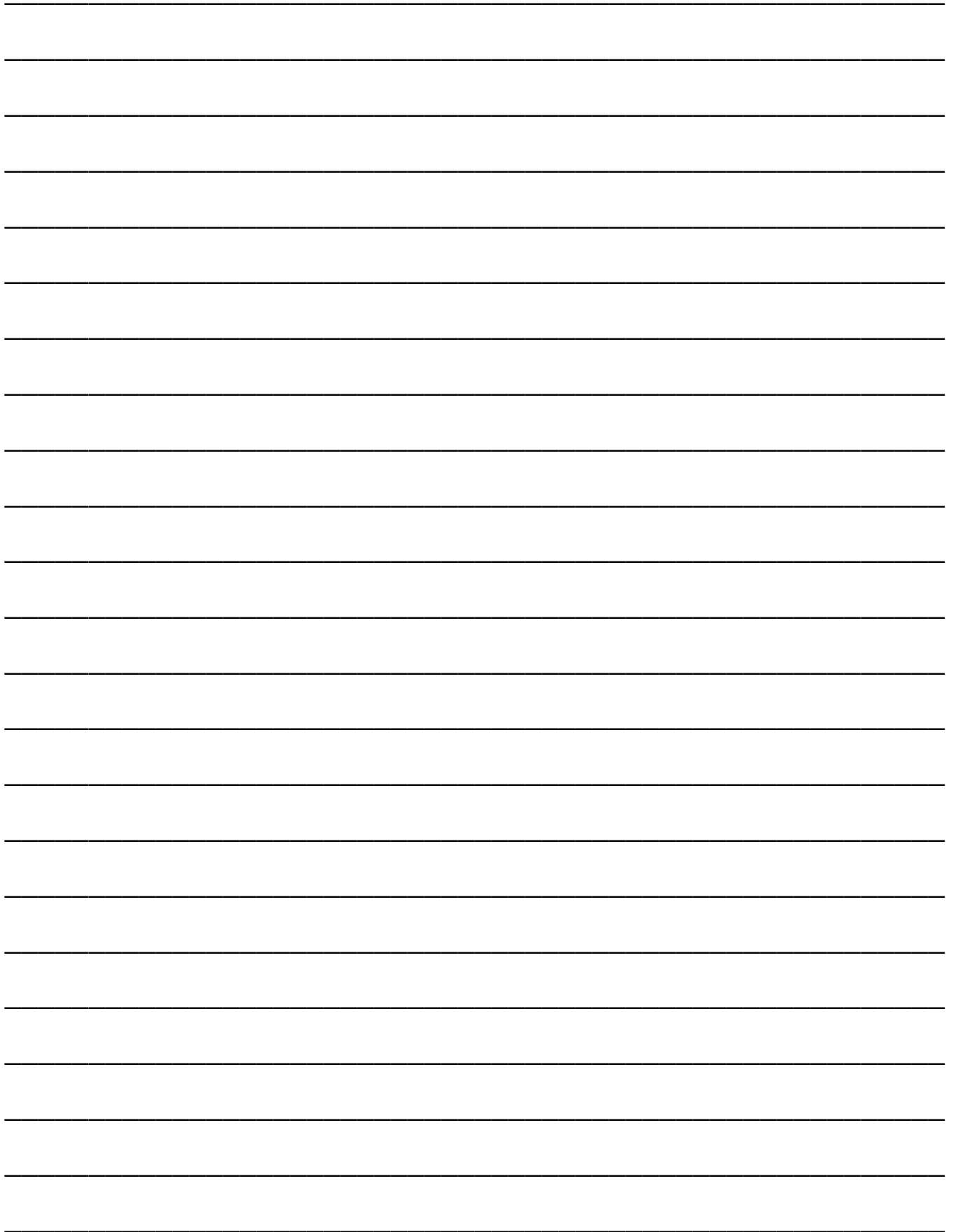
(d) How many terms in the sequence must be added to give a sum of 259?

(7) The first three terms of an AS are: $7x - 1$; $2x + 3$ and $3 - 5x$

(a) Determine the value of x .

(b) Write down the first four terms of the sequence.

(c) Calculate: T_{18} .



(c) Determine the thirteenth term.

(9) Given arithmetic sequence: $3k + 2$; $5k + 1$; $7k$; $9k - 1$;

(a) Determine the constant difference in terms of k .

(b) Determine the sum of the first twelve terms in terms of k .

A1.1.3 Applications:

Ex. 5 Rachel is training for a marathon. She runs 20 km on the first day's preparation. She decides to run 3 km further as the previous day on each day of her preparation.

- (a) If she follows this exercise program, calculate during which day of exercise she will run 56 km.
 (b) Calculate Rachel's total distance she ran for the first 10 preparatory exercises.

(a) AS: 20 ; 23 ; 26 ; 29 ; ; 56

$$\therefore a = 20$$

$$d = 3$$

$$n = ?$$

$$T_n = 56$$

$$T_n = a + (n - 1)d$$

$$\therefore 56 = 20 + (n - 1)(3)$$

$$\therefore 56 = 20 + 3n - 3$$

$$\therefore -3n = 17 - 56$$

$$\therefore -3n = -39$$

$$\therefore n = 13$$

\therefore Rachel will run 56 km on the **thirteenth** day.

(b) $S_{10} = ?$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\therefore S_{10} = \frac{10}{2}[2(20) + (10 - 1)(3)]$$

$$\therefore S_{10} = 335$$

\therefore Rachel's total distance for the first 10 days is 335 km.

Exercise 3:

Date: _____

- (1) Cory cuts sixteen pieces of string for a project. The shortest piece of string is 28 cm in length and the longest piece is 88 cm in length. If the lengths of the sixteen pieces of string are written down, it forms an AS.

- (a) Calculate the length of the 13th string in the sequence.

- (b) Calculate the total length of all 16 pieces of string.
