Grade 11 – Book B TG

(CAPS Edition)

CONTENT:

This book was compiled and processed by E.J. Du Toit in 2008.
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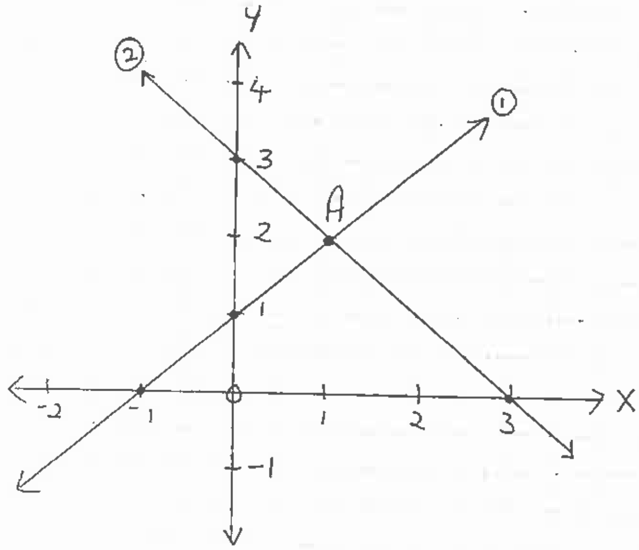
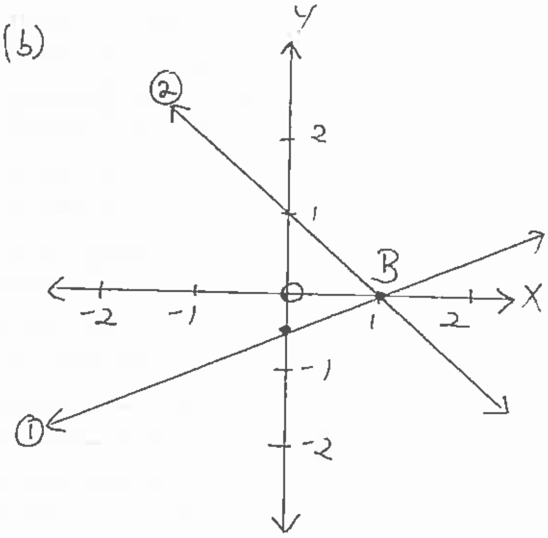
(1)(a)



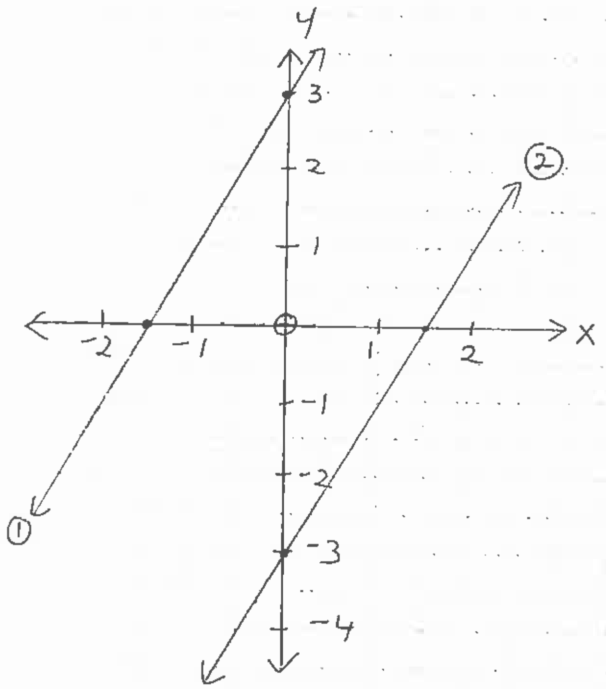
A(1;2)

(b)



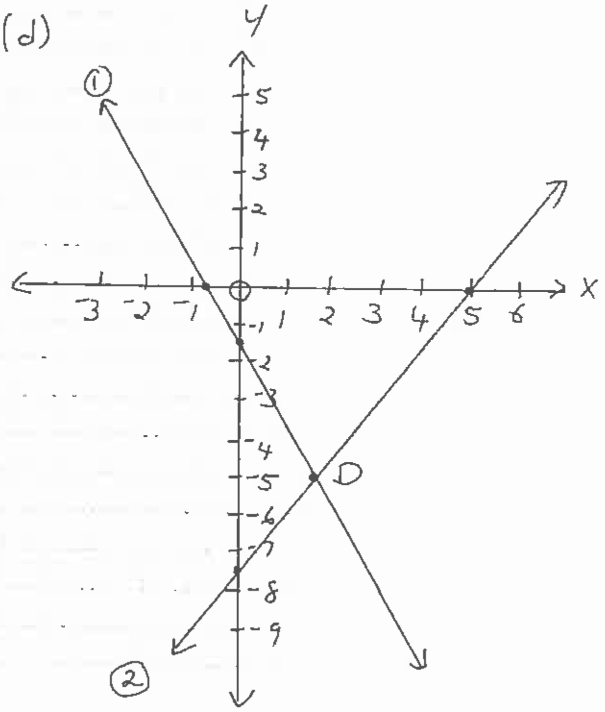
B(1;0)

(c)



No point of intersection.
Lines are parallel!

(d)



$D(\frac{3}{2};-5)$

Chapter B1

Functions

B1.1 Linear functions:

Revision!

Standard form: $y = mx + c$ with $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ as the gradient and c as the y-intercept.

Exercise 1: Date:______________

(1) Draw each set of straight lines on the same system of axes and determine the point of intersection for each set:

(a) x - y + 1 = 0 and x + y = 3

$x - y + 1 = 0$ --- ①
x-int: $(-1; 0)$
y-int: $(0; 1)$
$x + y = 3$ --- ②
x-int: $(3; 0)$
y-int: $(0; 3)$

(b) 2y + 1 = x and x + y = 1

$2y + 1 = x$ --- ①
x-int: $(1; 0)$
y-int: $(0; -\frac{1}{2})$
$x + y = 1$ --- ②
x-int: $(1; 0)$
y-int: $(0; 1)$

(c) 2x + 3 = y and 2y - 4x + 6 = 0

$2x + 3 = y$ --- ①
x-int: $(-\frac{3}{2}; 0)$
y-int: $(0; 3)$
$2y - 4x + 6 = 0$ --- ②
x-int: $(\frac{3}{2}; 0)$
y-int: $(0; -3)$

(d) 4x + 2y = -3 and 2y + 15 = 3x

$4x + 2y = -3$ --- ①
x-int: $(-\frac{3}{4}; 0)$
y-int: $(0; -\frac{3}{2})$
$2y + 15 = 3x$ --- ②
x-int: $(5; 0)$
y-int: $(0; -7\frac{1}{2})$

(2) Determine the equation of the straight line:

(a) through (1 ; 3) and (2 ; -1)
$\quad\quad x_1\ y_1 \quad\quad x_2\ y_2$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-3}{2-1} = \dfrac{-4}{1} = -4$

$y = -4x + c$
$3 = -4(1) + c$
$7 = c$
$\therefore y = -4x + 7$

(b) through (4 ; 0) and parallel to 3y + 6x - 2 = 0
$\quad\quad x\ y$

$3y = -6x + 2$
$y = -2x + \frac{2}{3}$
$\therefore m_1 = m_2 = -2$ (// lines)
$\therefore y = -2x + c$
$0 = -2(4) + c$
$8 = c$
$\therefore y = -2x + 8$

(c) through (3 ; -7) and (3 ; 4)

$x = 3$ in both points

$\therefore x = 3$

(d) through (0 ; 2) with an inclination of 135°

$m = \tan\theta$

$m = \tan 135°$

$m = -1$ through $(\overset{x}{0};\overset{c}{2})$

$\therefore y = mx + c$

$\therefore y = -x + 2$

(3) The points $\overset{A}{(3 ; 5)}$, $\overset{B}{(0 ; 4)}$ and $\overset{C}{(-1 ; m)}$ is collinear. Calculate the value of m.

$m_{AB} = \frac{4-5}{0-3} = \frac{-1}{-3} = \frac{1}{3}$

$m_{BC} = \frac{m-4}{-1-0}$

but $m_{AB} = m_{BC}$ (collinear)

$\therefore \frac{1}{3} = \frac{m-4}{-1}$

$(1)(-1) = 3(m-4)$

$-1 = 3m - 12$

$11 = 3m \quad \therefore m = \frac{11}{3}$

(4) 3x - 2y = 3 and px + 1 = 2y is perpendicular. Calculate the value of p.

$3x - 3 = 2y$ and $2y = px + 1$

$\frac{3}{2}x - \frac{3}{2} = y \qquad y = \frac{p}{2}x + \frac{1}{2}$

But lines are $\perp$:

$\therefore \frac{3}{2} \times \frac{p}{2} = -1$

$\frac{3p}{4} = -1$

$3p = -4$

$p = \frac{-4}{3}$

B1.2 Quadratic function (parabola):
B1.2.1 Sketching of the parabola:
B1.2.1.1 Standard form 1:

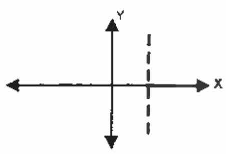
$y = ax^2 + bx + c$

Influence of a: [Form!]

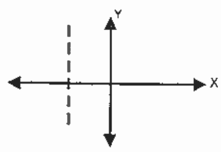
If $a > 0$:  and if $a < 0$: 

Influence of b: [Symmetry-axis!]

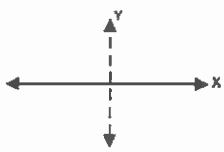
If symm-axis (x) = $\frac{-b}{2a} > 0$ then:



If symm-axis (x) = $\frac{-b}{2a} < 0$ then:



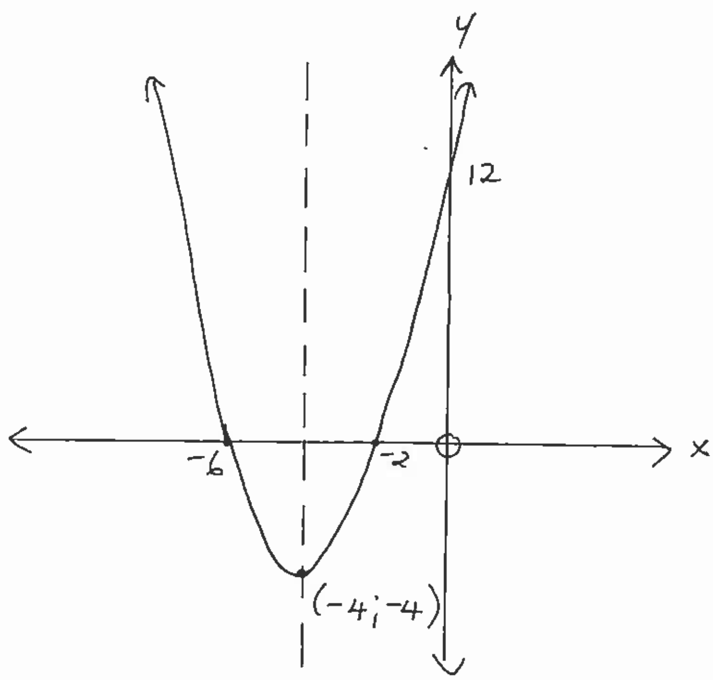
If symm-axis (x) = $\frac{-b}{2a} = 0$ then:



Influence of c: [*y*-intercept!]

c represents, similar to the straight line, the y-intercept of the parabola.

(1) (a)



E.g. 1 Sketch the following: $2y = -2x^2 + 4x + 16$

****

Step 1 *[Write the equation in the standard form]:* $y = -x^2 + 2x + 8$

Step 2 *[Interpret the form]:* $a < 0 \therefore$ 

Step 3 *[Determine the y-intercept]:* $c = 8$ *or substitute* $x = 0$ $\therefore$ *y-int:* $(0 ; 8)$

Step 4 *[Determine the x-intercept(s)] There can be two, one, or no x-intercept(s).*

Subst $y = 0 \rightarrow 0 = -x^2 + 2x + 8$

$0 = x^2 - 2x - 8$

$0 = (x - 4)(x + 2)$

$\therefore x = 4$ or $x = -2$

$\therefore$ *x-intercepts:* $(4 ; 0)$ *and* $(-2 ; 0)$

NB: If you do not find factors for the equation, make use of the formula!

Step 5 *[Determine the equation of the symmetry-axis]:* *Formula* $\rightarrow x = \frac{-b}{2a}$

From standard form: $a = -1$ *and* $b = 2 \rightarrow$ $x = \frac{-(2)}{2(-1)}$

$x = \frac{-2}{-2} = 1$

or *the symm-axis is exactly halfway between the two x-int:* $\therefore$ *symm-axis* $= \frac{4 + (-2)}{2} = \frac{2}{2} = 1$

Step 6 *[Determine the coordinates of the turning point]:*

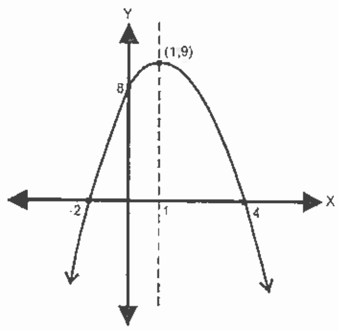
Subst $x = 1$ *(symm-axis) in the equation of step 1*

$\therefore y = -x^2 + 2x + 8$

$\therefore y = -(1)^2 + 2(1) + 8$

$\therefore y = -1 + 2 + 8 = 9$ $\therefore TP = (1 ; 9)$

Step 7 *[Draw the curve of the function]:*
Show the x-and y-intercepts and the turning point clearly.



Conclusions:

Max value of 9
Domain: $x \in \mathbb{R}$
Range: $y \leq 9$

Exercise 2: Date:________________

(1) Draw the following functions on different Cartesian planes: (Do drawings on the left!)

(a) $y = x^2 + 8x + 12$ Form: $a > 0$

y-int: $(0 ; 12)$ symm-axis: $x = \frac{-6-2}{2}$

x-int: $x = -4$

$0 = x^2 + 8x + 12$ TP: $y = x^2 + 8x + 12$

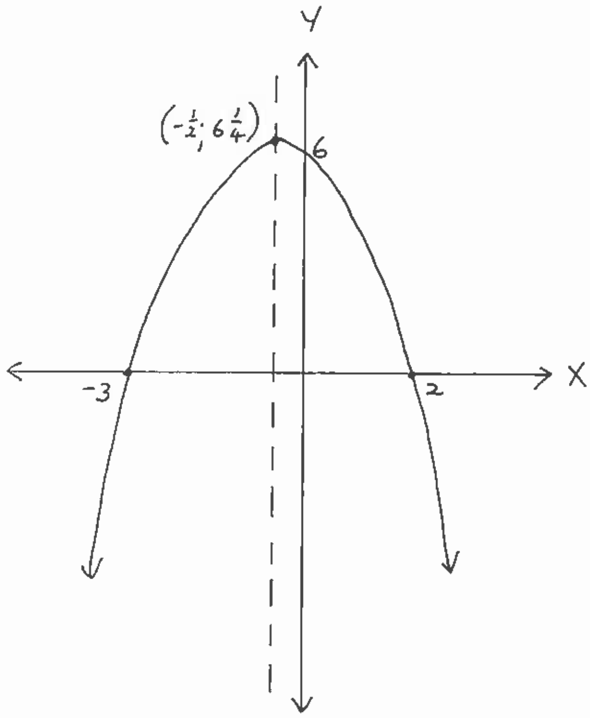
$0 = (x + 6)(x + 2)$ $= (-4)^2 + 8(-4) + 12$

$\therefore x = -6$ or $x = -2$ $y = 16 - 32 + 12$

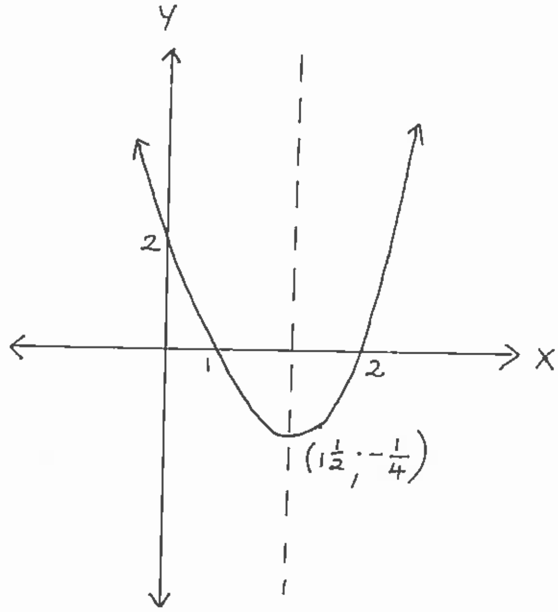
$(-6 ; 0)$ $(-2 ; 0)$ $y = -4$

$\therefore TP \rightarrow (-4 ; -4)$

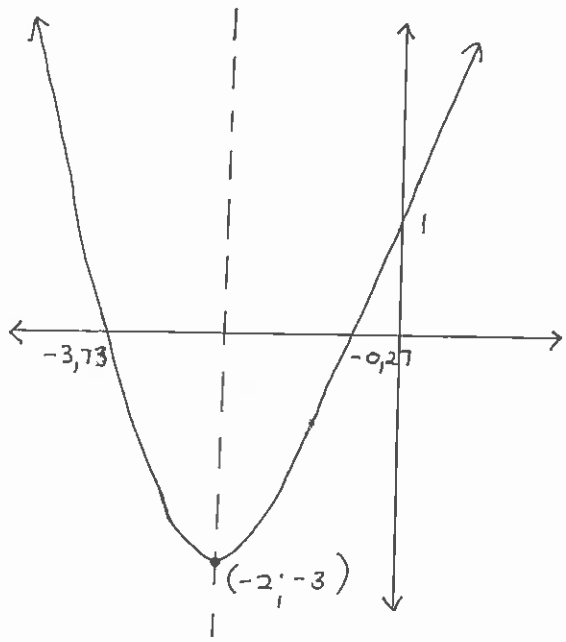
(b)


Y
$(-\frac{1}{2};6\frac{1}{4})$
6
X
-3
2

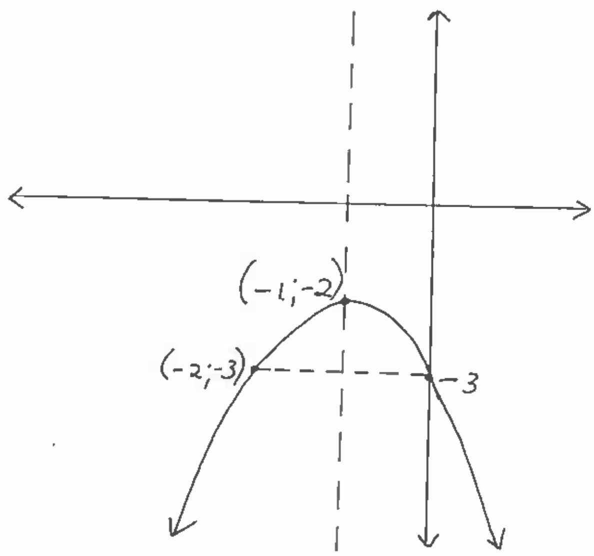
(c)


Y
2
X
1
2
$(1\frac{1}{2};-\frac{1}{4})$

(d)


1
-3,73
-0,27
(-2;-3)

(e)


(-1;-2)
(-2;-3)
-3

(b) $y = -x^2 - x + 6$

Form: $a<0$ ∩

y-int: $(0;6)$

x-int: $0 = -x^2 - x + 6$

$0 = x^2 + x - 6$

$0 = (x+3)(x-2)$

$x = -3$ or $x = 2$

$(-3;0)$ $(2;0)$

Symm-axis: $x = \frac{-b}{2a} = \frac{-(-1)}{2(-1)}$

$\therefore x = -\frac{1}{2}$

TP: $y = -(-\frac{1}{2})^2 - (-\frac{1}{2}) + 6$

$y = -(\frac{1}{4}) + \frac{1}{2} + 6$

$y = -\frac{1}{4} + \frac{2}{4} + \frac{24}{4}$

$y = \frac{25}{4}$

$\therefore$ TP $(-\frac{1}{2};\frac{25}{4})$

(c) $2y = 2x^2 - 6x + 4$

$y = x^2 - 3x + 2$

Form: $a>0$ ∪

y-int: $(0;2)$

x-int: $0 = x^2 - 3x + 2$

$0 = (x-2)(x-1)$

$x = 2$ or $x = 1$

$(2;0)$ $(1;0)$

Symm-axis: $x = \frac{2+1}{2}$

$\therefore x = \frac{3}{2} = 1\frac{1}{2}$

TP: $y = (\frac{3}{2})^2 - 3(\frac{3}{2}) + 2$

$y = \frac{9}{4} - \frac{9}{2} + 2$

$y = \frac{9}{4} - \frac{18}{4} + \frac{8}{4}$

$y = -\frac{1}{4}$

$\therefore$ TP $(\frac{3}{2};-\frac{1}{4})$

(d) $y = x^2 + 4x + 1$

Form: $a>0$ ∪

y-int: $(0;1)$

x-int: $0 = x^2 + 4x + 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)}$

$x = \frac{-4 \pm \sqrt{12}}{2}$

$x = -0{,}27$ or $x = -3{,}73$

Symm-axis: $x = \frac{-b}{2a}$

$x = \frac{-4}{2(1)}$

$x = -2$

TP: $y = (-2)^2 + 4(-2) + 1$

$= 4 - 8 + 1$

$y = -3$

$\therefore$ TP $(-2;-3)$

(e) $y = -x^2 - 2x - 3$

Form: $a<0$ ∩

y-int: $(0;-3)$

x-int: $0 = -x^2 - 2x - 3$

$0 = x^2 + 2x + 3$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(3)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{-8}}{2}$

$\therefore$ No $\mathbb{R}$ roots

$\Rightarrow$ no x-intercepts!

Symm-axis: $x = \frac{-(-2)}{2(-1)} = -1$

DP: $y = -(-1)^2 - 2(-1) - 3$

$= -(1) + 2 - 3$

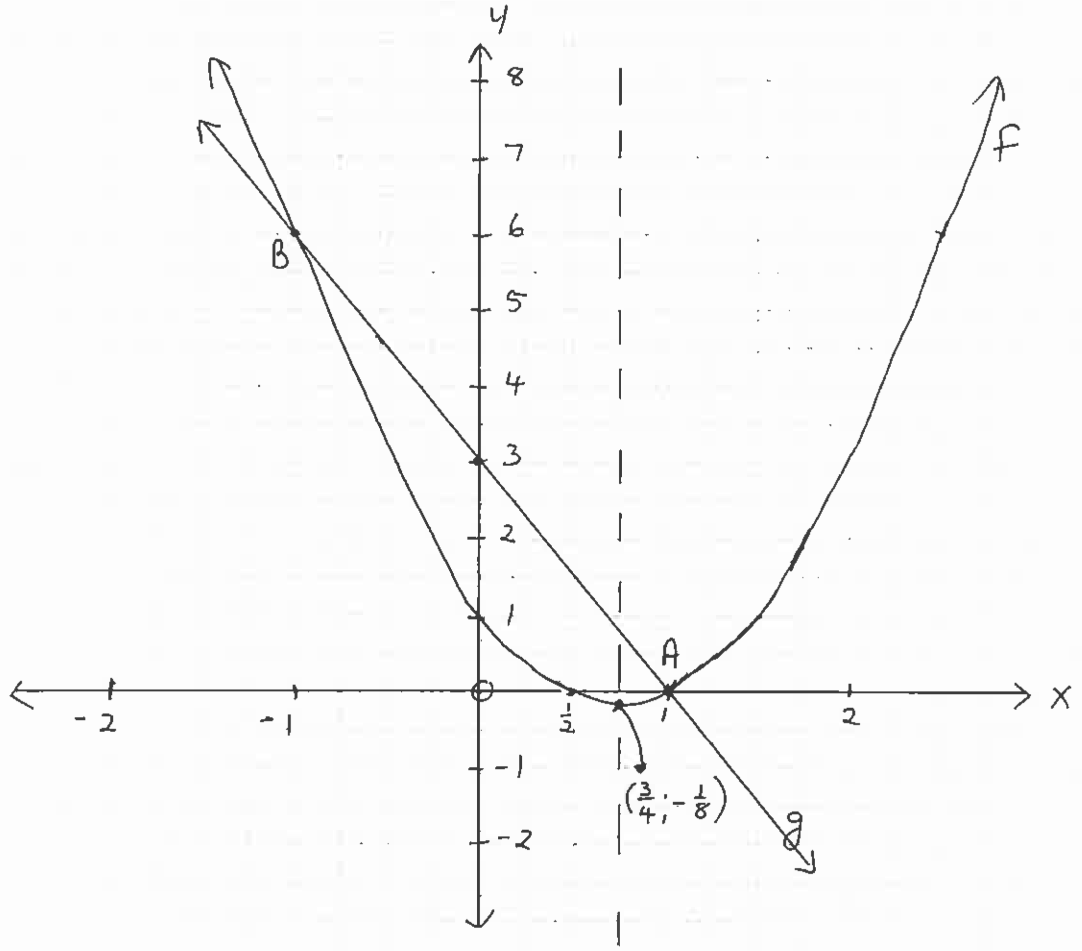
$y = -1 + 2 - 3$

$y = -2$

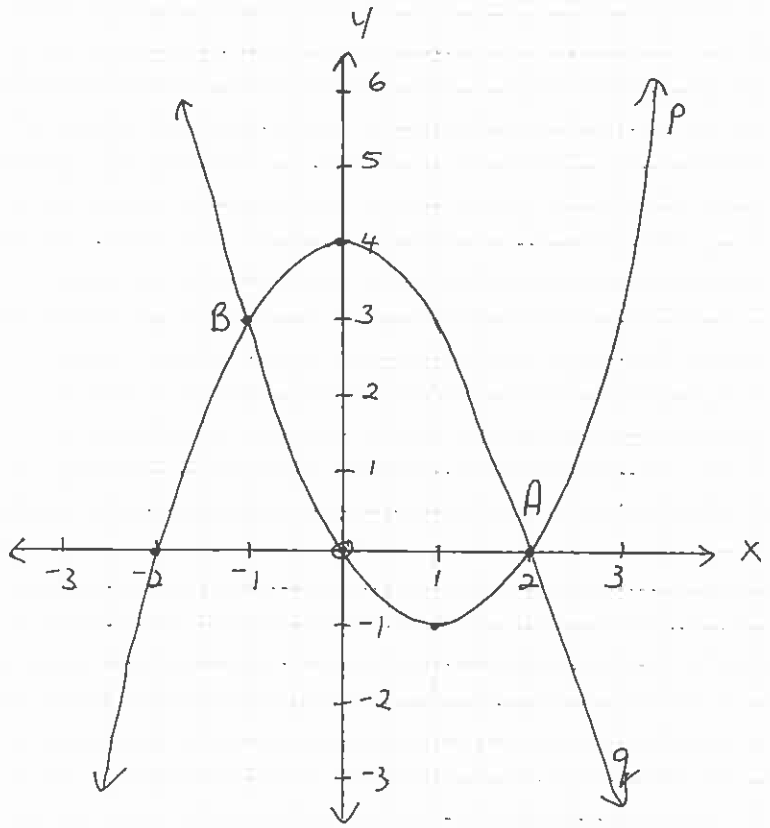
$\therefore$ TP $(-1;-2)$

Check point: $P(-2;-3)$

(2)(a)


y
8
7
6
5
4
3
2
1
-1
-2
B
A
f
g
x
-2
-1
$\frac{1}{2}$
1
2
$(\frac{3}{4};-\frac{1}{8})$

(3)(a)


y
6
5
4
3
2
1
-1
-2
-3
B
A
p
g
x
-3
-2
-1
1
2
3

(2) Consider: $f(x) = 2x^2 - 3x + 1$

(a) Sketch f. Show all calculations.

$f(x) = y = 2x^2 - 3x + 1$

Form: $a > 0$ ↺

y-int: $(0;1)$

x-int: $0 = 2x^2 - 3x + 1$

$0 = (2x-1)(x-1)$

$2x - 1 = 0$ or $x = 1$

$x = \frac{1}{2}$

Symm. axis: $x = \frac{-b}{2a}$

$x = \frac{-(-3)}{2(2)} = \frac{3}{4}$

TP: $y = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) + 1$

$= 2\left(\frac{9}{16}\right) - 3\left(\frac{3}{4}\right) + 1$

$y = \frac{9}{8} - \frac{18}{8} + \frac{8}{8}$

$\therefore y = -\frac{1}{8}$ $\therefore TP\left(\frac{3}{4}; -\frac{1}{8}\right)$

(b) Draw on the same Cartesian plane as in (a): $g: x \rightarrow -3x + 3$. Show all calculations.

$\therefore y = -3x + 3$

x-int:

$0 = -3x + 3$

$3x = 3$ $\therefore x = 1$ $\therefore (1;0)$

y-int: $(0;3)$

(c) Determine the following: (i) the domain of g.

(ii) the range of f .

(iii) The equation of the symmetry-axis of f.

(iv) The coordinates of $f \cap g$.

(i) $x \in \mathbb{R}$

(ii) $y \geq -\frac{1}{8}$

(iii) $x = \frac{3}{4}$

(iv) $A(1;0)$ and $B(-1;6)$

(3) (a) Draw on the same Cartesian plane: $p(x) = x^2 - 2x$ and $q(x) = 4 - x^2$

$p(x) = y = x^2 - 2x$ ↺

y-int: $(0;0)$

x-int: $0 = x^2 - 2x$

$0 = x(x-2)$

$x = 0$ or $x = 2$

symm. axis: $x = \frac{0+2}{2} = 1$

TP: $y = (1)^2 - 2(1)$

$y = 1 - 2 = -1$

$\therefore TP\ (1;-1)$

$q(x) = y = 4 - x^2$ ↻

y-int: $(0;4)$

x-int: $0 = 4 - x^2$

$0 = (2-x)(2+x)$

$x = 2$ or $x = -2$

symm. axis: $x = \frac{2-2}{2} = 0$

TP: $y = 4 - (0)^2$

$y = 4$

$\therefore TP\ (0;4)$

(b) Use the graph in (a) and determine the following:

(i) Domain of p. (iii) Min/Max value of q.

(ii) Range of q. (iv) x if $p(x) = q(x)$.

(i) $D_p: x \in \mathbb{R}$ (iii) Max value of 4

(ii) $R_q: y \leq 4$ (iv) At $A(2;0)$ and $B(-1;3)$

B1.2.1.2 Standard form 2:

$y = a(x - p)^2 + q$

Influence of a: [Form!]

If $a > 0$:  and if $a < 0$: 

Influence of p: [Symmetry-axis!]

The equation of the symm-axis: $x = p$

Influence of q: [Min/Max!]

q represents the y-coordinate of the turning point. $\therefore$ TP = (p ; q)

E.g. 2 Sketch the following: $y = (x - 1)^2 - 4$

****

Step 1 *[Interpret the form]:* $a > 0$ $\therefore$ 

Step 2 *[Determine the coordinates of the turning point]:* TP $= (p ; q) = (1 ; -4)$

Step 3 *[Determine the x-intercept(s)]: Subst* $y = 0$

$\therefore\ 0 = (x - 1)^2 - 4$

$4 = (x - 1)^2$

$\pm\sqrt{4} = x - 1$

$\pm 2 = x - 1$

$\therefore\ x = +2 + 1$ or $x = -2 + 1$

$x = 3$ $\quad x = -1$

or

$0 = (x - 1)^2 - 4$

$0 = x^2 - 2x + 1 - 4$

$0 = x^2 - 2x - 3$

$0 = (x - 3)(x + 1)$

$x = 3$ or $x = -1$

$\therefore$ *x-int:* $(3 ; 0)$ *and* $(-1 ; 0)$

Step 4 *[Determine the y-int]: Subst* $x = 0$

$\therefore\ y = (0 - 1)^2 - 4$

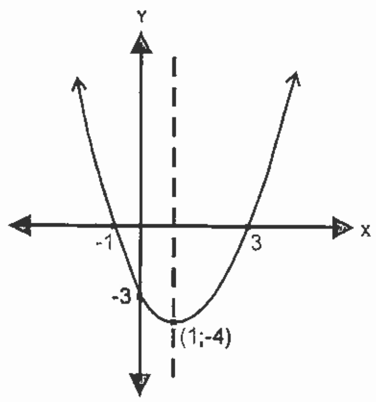
$\therefore\ y = (-1)^2 - 4$

$\therefore\ y = 1 - 4$

$\therefore\ y = -3$

$\therefore$ *y-int:* $(0 ; -3)$

Step 5 *[Draw the graph!]*



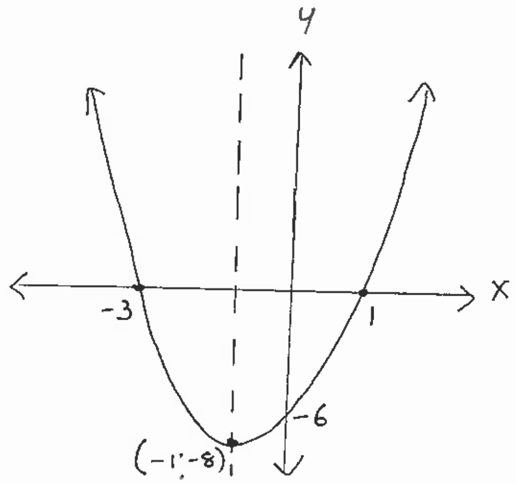
Conclusions:

Min value of -4

Domain: $x \in R$

Range: $y \geq -4$

(1)(a)



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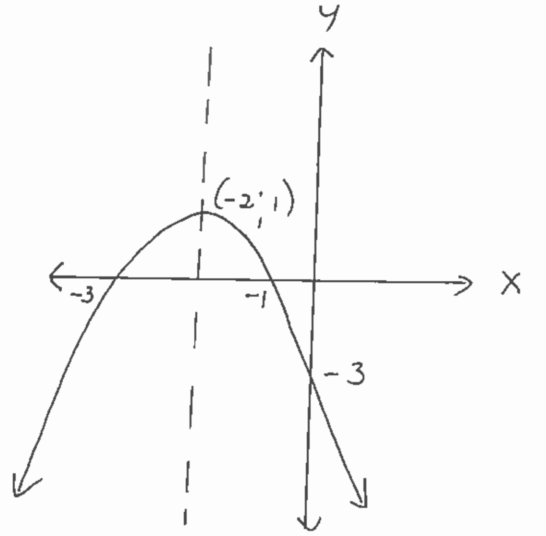

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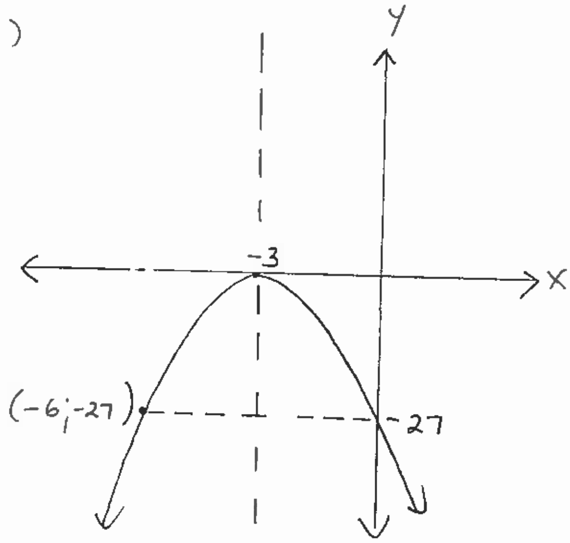

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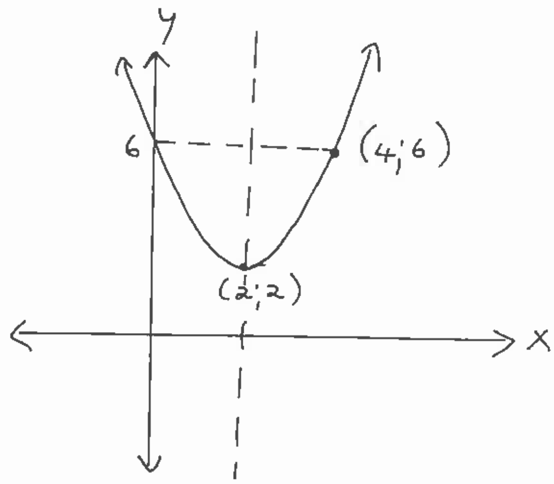
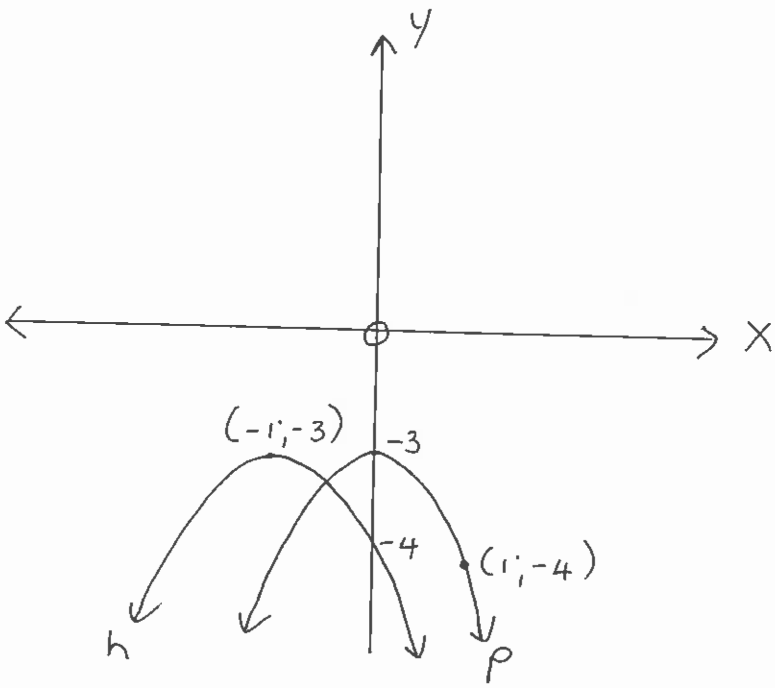


(2)



Exercise 3: Date:______________

(1) Sketch the following functions on separate Cartesian planes: (Draw left!)

(a) $y = 2(x + 1)^2 - 8$

TP: $(-1;-8)$

x-int: $0 = 2(x+1)^2 - 8$

$8 = 2(x+1)^2$

$4 = (x+1)^2$

$\pm 2 = x+1$

$\therefore x = 1$ or $x = -3$

y-int: $y = 2(0+1)^2 - 8$

$y = 2(1)^2 - 8$

$y = -6$

(b) $y = -(x + 2)^2 + 1$

TP: $(-2;1)$

x-int: $0 = -(x+2)^2 + 1$

$(x+2)^2 = 1$

$x+2 = \pm 1$

$x = -1$ or $x = -3$

y-int: $y = -(0+2)^2 + 1$

$y = -(2)^2 + 1$

$y = -4+1$

$y = -3$

(c) $y = -3(x + 3)^2$

TP: $(-3;0)$

x-int: $0 = -3(x+3)^2$

$0 = (x+3)^2$

$\therefore x = -3$ or $x = -3$

y-int: $y = -3(0+3)^2$

$= -3(3)^2$

$= -3(9)$

$y = -27$

(d) $y = (x - 2)^2 + 2$

TP: $(2;2)$

x-int: $0 = (x-2)^2 + 2$

$-2 = (x-2)^2$

$\pm\sqrt{-2} = x-2$

No $\mathbb{R}$ solution $\rightarrow$ no x-int.

y-int: $y = (0-2)^2 + 2$

$y = 4+2$

$y = 6$

(2) Consider: $h: x \rightarrow -(x + 1)^2 - 3$

(a) Draw h. Show all calculations.

TP: $(-1;-3)$

x-int: $0 = -(x+1)^2 - 3$

$(x+1)^2 = -3$

$x+1 = \pm\sqrt{-3}$

No $\mathbb{R}$-solution

$\therefore$ No x-int.

y-int: $y = -(0+1)^2 - 3$

$y = -(1)^2 - 3$

$y = -1-3$

$y = -4$

(b) Draw $p(x) = -x^2 - 3$ on the same Cartesian plane as (a).

$\therefore y = -(x+0)^2 - 3$

TP: $(0;-3)$

y-int: $(0;-3)$

x-int: $0 = -x^2 - 3$

$x^2 = -3$

$x = \pm\sqrt{-3}$

$\therefore$ No x-int.

(c) Describe the transformation of h to p as in (a) and (b). What is the influence of such transformation on the equation of h to p:

h moves 1 unit to the right to become p.
The max value of -3 stays the same for both h and p, but the symm-axis moves to the right.

(d) Determine the equation of the straight line through the turning points of the two parabolas:

$y = -3$

(e) Write down the ranges of h and p:

R_h: $y \leq -3$
R_p: $y \leq -3$

B1.2.1.3 <u>Standard form 3</u>:

$$y = a(x - x_1)(x - x_2)$$

<u>Influence of a</u>: [Form!]

If $a > 0$:  and if $a < 0$: 

<u>Influence of x_1 and x_2</u> : [x-intercepts!]

Parabola intercepts the x-axis at x_1 and x_2.

E.g. 3 Sketch the following: $y = 2(x - 3)(x + 1)$
****

<u>Step 1</u> *[Interpret the form]:* $a > 0$ $\therefore$ 

<u>Step 2</u> *[Determine the x-intercept(s)]:* $x_1 = 3$ *and* $x_2 = -1$
$\therefore$ *x-int:* $(3 ; 0)$ *and* $(-1 ; 0)$

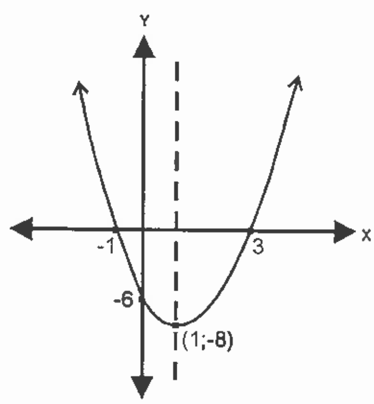
<u>Step 3</u> *[Determine the equation of the symm-axis]:* *symm-axis* $= \dfrac{x_1 + x_2}{2}$

$$x = \frac{3 + (-1)}{2} = 1$$

<u>Step 3</u> *[Determine the coordinates of the turning point]:*
Substitute $x = 1$ *(symm-axis) in equation:* $\therefore y = 2(1 - 3)(1 + 1)$
$\therefore y = 2(-2)(2) = -8$
$\therefore TP = (1 ; -8)$

<u>Step 4</u> *[Determine the y-int]: Subst* $x = 0$
$\therefore y = 2(0 - 3)(0 + 1)$
$\therefore y = 2(-3)(1)$
$\therefore y = -6$
$\therefore$ *y-int:* $(0 ; -6)$

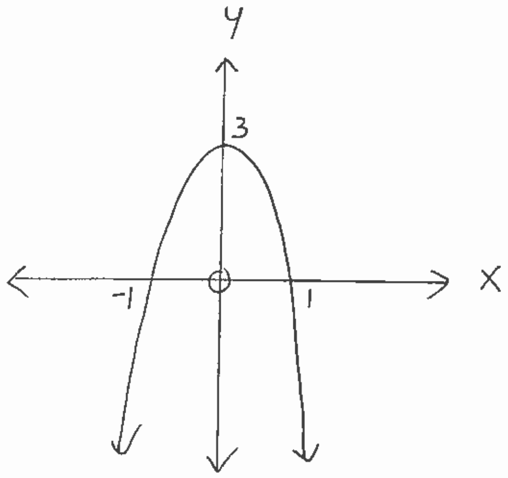
<u>Step 5</u> *[Draw the graph!]*



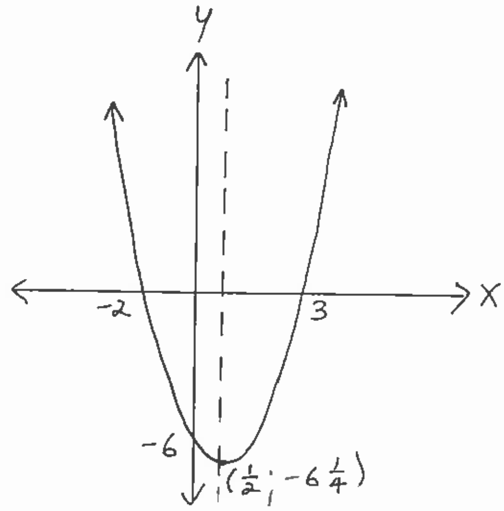
<u>Conclusions</u>:

Min value of -8
Domain: $x \in R$
Range: $y \geq -8$

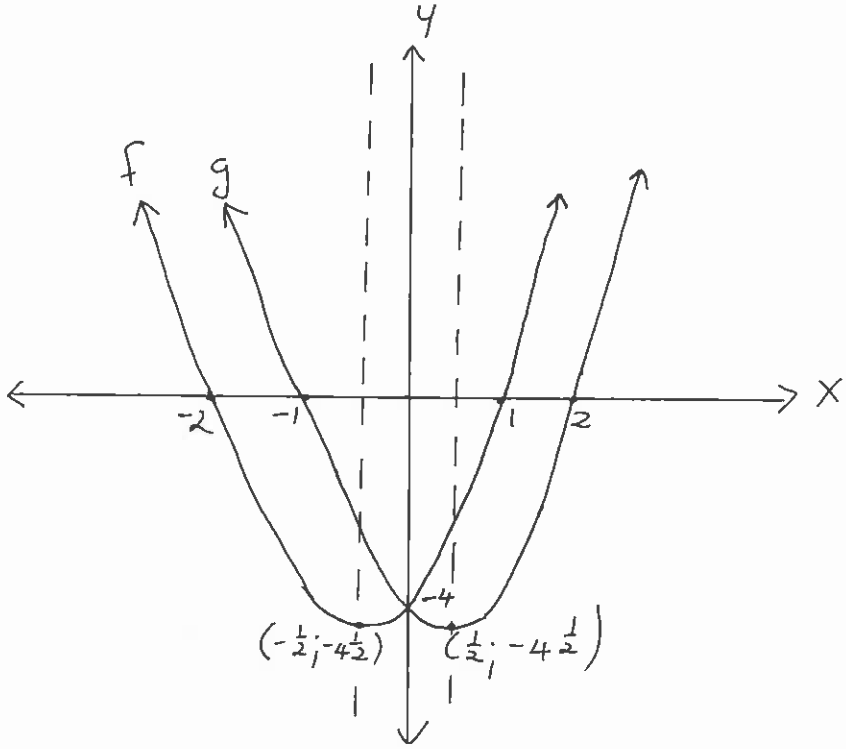
(1)(a)


y
3
-1
0
1
x

(b)


y
-2
3
x
-6
$(\frac{1}{2}; -6\frac{1}{4})$

(2)


y
f
g
x
-2
-1
1
2
-4
$(-\frac{1}{2}; -4\frac{1}{2})$
$(\frac{1}{2}; -4\frac{1}{2})$

Exercise 4: Date:______________

(1) Sketch the following on different Cartesian planes: (Draw left!)

(a) $y = -3(x + 1)(x - 1)$

x-int: $(-1;0)$ $(1;0)$

Symm. axis $x = \frac{-1+1}{2} = 0$

TP: $y = -3(0+1)(0-1)$

$y = -3(1)(-1)$

$y = 3$

$\therefore$ TP $(0;3)$

y-int: $y = -3(0+1)(0-1)$

$\therefore y = 3$

(b) $y = (x + 2)(x - 3)$

x-int: $(-2;0)$ $(3;0)$

Symm. axis: $x = \frac{-2+3}{2} = \frac{1}{2}$

TP: $y = (\frac{1}{2}+2)(\frac{1}{2}-3)$

$= (2\frac{1}{2})(-2\frac{1}{2})$

$y = -6\frac{1}{4}$

$\therefore$ TP $(\frac{1}{2}; -6\frac{1}{4})$

y-int: $y = (0+2)(0-3)$

$\therefore y = -6$

(2) Consider the following: $f(x) = 2(x - 1)(x + 2)$ and $g(x) = 2x^2 - 2x - 4$

(a) Draw f and g on the same Cartesian plane

f: x-int: $(1;0)$ and $(-2;0)$

Symm. axis: $x = \frac{1-2}{2} = -\frac{1}{2}$

TP: $y = 2(-\frac{1}{2}-1)(-\frac{1}{2}+2)$

$y = 2(-\frac{3}{2})(\frac{3}{2})$

$y = -4\frac{1}{2}$

$\therefore$ TP $(-\frac{1}{2}; -4\frac{1}{2})$

y-int: $y = 2(0-1)(0+2)$

$y = 2(-1)(2)$

$y = -4$

g: x-int:

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = 2$ or $x = -1$

Symm-axis: $x = \frac{2-1}{2} = \frac{1}{2}$

TP: $y = 2(\frac{1}{2})^2 - 2(\frac{1}{2}) - 4$

$y = -4\frac{1}{2}$

$\therefore$ TP $(\frac{1}{2}; -4\frac{1}{2})$

y-int: $y = 2(0)^2 - 2(0) - 4$

$y = -4$

(b) Write g in the form $g(x) = a(x - x_1)(x - x_2)$

$g(x) = 2x^2 - 2x - 4$

$g(x) = 2(x^2 - x - 2)$

$g(x) = 2(x-2)(x+1)$

(c) Describe the transformation of f $\rightarrow$ g. Also explain the relation between the equations of f and g and the transformation.

g is the reflection of f in the y-axis. The equations are the same except for the signs of the x-intercepts in the brackets are swapped!