Grade 11 – Book D TG

(CAPS Edition)

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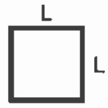
Chapter D1

Area and volume

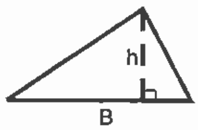
D1.1 Revision:

* Rectangle: Perimeter = $2L + 2B$
Area = $L \times B$

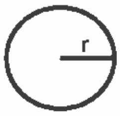
* Square: Perimeter = $4L$
Area = L^2



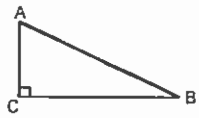
* Triangle: Perimeter = side + side + side
Area = $\frac{1}{2} B \times \perp H$



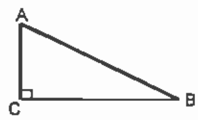
* Circle: Circumference = $2\pi r$
Area = πr^2



* Theorem of Pythagoras: $AB^2 = AC^2 + BC^2$



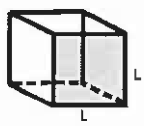
* Trigonometrical functions: E.g. $\sin \hat{B} = \frac{t}{s} = \frac{AC}{AB}$
or $\cos \hat{B} = \frac{a}{s} = \frac{BC}{AB}$
or $\tan \hat{B} = \frac{t}{a} = \frac{AC}{BC}$



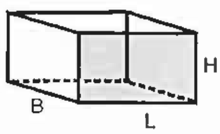
D1.2 Surface area:

* Right prism: Surface area = Perimeter of base × $\perp H$ + 2 × area of base

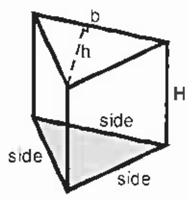
E.g. Square base: Surf. area = $(4L)H + 2L^2$ but $L = H$
(Cube) ∴ Surf. area = $(4L)L + 2L^2$
∴ Surf. area = $(4L)L + 2L^2 = 4L^2 + 2L^2$
∴ **Surf. area = $6L^2$**



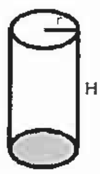
E.g. Rectangular base: Surf. area = $(2L + 2B)H + 2LB$
∴ **Surf. area = $2LH + 2BH + 2LB$**



E.g. Triangular base: **Surf. area = (side + side + side)H + 2 × $\frac{1}{2}$bh**
∴ **Surf. area = (side + side + side)H + bh**



E.g. Circular base: **Surf. area = $(2\pi r)H + 2(\pi r^2)$**
(Cylinder) **= $2\pi r\,(H + r)$**

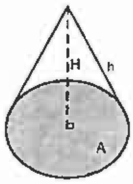


* Right cone: **Surf. area = $\pi r(h + r)$** **With: A the area of the base– use with volume!**

r the radius of the base (circle)

h the slant height of the cone

H the perpendicular height – use with volume!

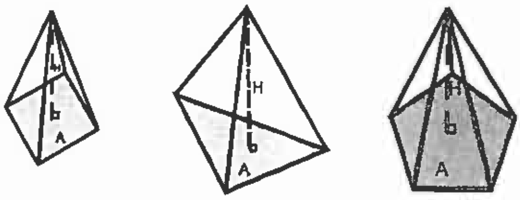


* Pyramid: **Surf. area = A + $\frac{1}{2}$ph** **With: A the area of the base**

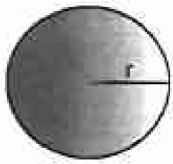
p the perimeter of the base

h the slant height of the pyramid

H the perpendicular height – use with volume!



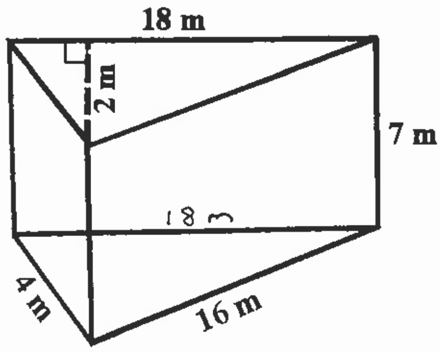
* Sphere: **Surf. area = $4\pi r^2$ with r the radius.**



Exercise 1: Date:_______________

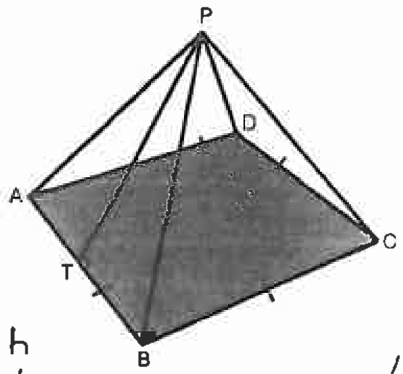
Calculate the total surface area of the following: [If necessary, round off to the nearest integer.]

(1)



Area = (sy+sy+sy)H + 2 × ½bh
= (4+16+18)(7) + (18×2)
= (38)(7) + 36
= 266 + 36
= 302 m²

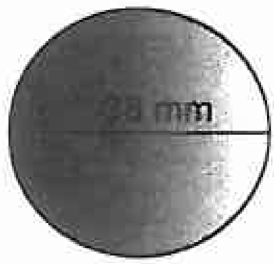
(2)



h ↓ PT = 34 mm en BC = 3,1 cm = 31 mm ↑ L

Area = A + ½ph
= L² + ½(4L)h
= (31)² + ½(4×31)(34)
= 961 + 2108
= 3 069 mm²

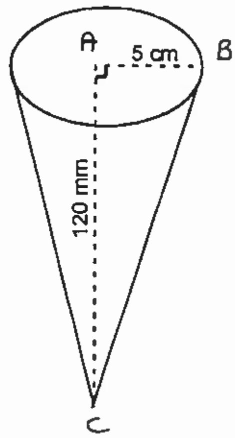
(3)



$d = 38 \rightarrow r = 19mm$

$Area = 4\pi r^2$
$= 4\pi(19)^2$
$= 4\ 536,459...$
$\approx 4\ 536\ mm^2$

(4)



$120\ mm = 12\ cm$

$Area = \pi r(h + r)$
$= \pi(5)(13^* + 5)$
$= \pi(5)(18)$
$= 282,74...$
$\approx 283\ cm^2$

$* \ AB^2 + AC^2 = BC^2$ (Pyth)
$5^2 + 12^2 = BC^2$
$169 = BC$
$\therefore BC = 13cm$

D1.3 <u>Volumes</u>:

* Right prism: $V = Base \times Height$
  E.g. Square base: $V = L^3$
  (Cube)
  E.g. Rectangular base: $V = L \times B \times H$
  E.g. Triangular base: $V = \frac{1}{2}bh \times H$
  E.g. Circular base: $V = \pi r^2 H$
  (Cylinder)

* Right cone: $V = \frac{1}{3}AH$

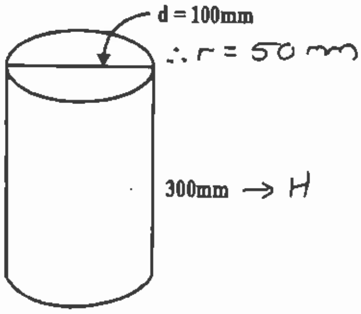
* Pyramid: $V = \frac{1}{3}AH$

* Sphere: $V = \frac{4}{3}\pi r^3$

Exercise 2: Date:______________

Calculate the volume of each of the following: [Where necessary, rounded off to 2 decimals.]

(1)

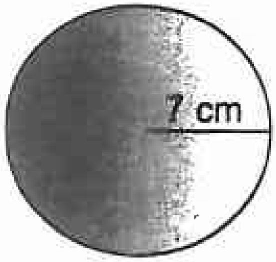


$V = \pi r^2 H$

$= \pi(50)^2(300)$

$= 2\ 356\ 194{,}49\ mm^3$

(2)



$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(7)^3$

$= 1436{,}75504$

$\approx 1436{,}76\ cm^3$

(3) A solid metal block is melted and remolded into spherical balls. The solid block is of cubical form with each side equal to 1,2 m. = 120cm Each spherical ball should have a diameter of 10 cm. Calculate the number of spherical balls that can be molded from the given cubical metal block.

Cube: $V = L^3$

$= (120)^3$

$= 1\ 728\ 000 cm^3$

Spere: $V = \frac{4}{3}\pi r^3$

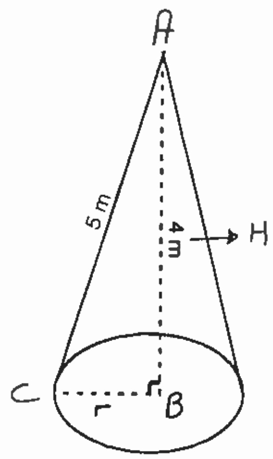
$V = \frac{4}{3}\pi(10)^3$

$V = 4188{,}79...$

Number $= \frac{1\ 728\ 000}{4\ 188{,}79}$

$= 412{,}529...$

(4)



$BC^2 = AC^2 - AB^2$

$BC^2 = 5^2 - 4^2 = 9$

$BC = 3\ m$

$V = \frac{1}{3} AH$

$= \frac{1}{3}(\pi r^2)H$

$= \frac{1}{3}(\pi \times 3^2)(4)$

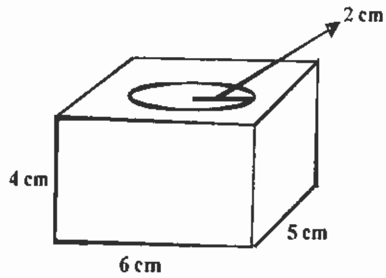
$V = 37{,}699...$

$V \approx 37{,}70\ m^3$

D1.4 Combinations:

Exercise 3: Date:______________

(1) The sketch is an illustration of a wooden Block, that forms part of a set of toys, with a cylindrical hole in the middle. The block should be painted with one layer of lead-free paint
Calculate: (Correct to 1 dec.)
(a) The volume of wood needed for this block expressed in mm^3.
(b) The total surface area of the block that should be painted.



(a) Prism Volume = $L \times B \times H$
= $6 \times 5 \times 4$
= $120\ cm^3$

Cylinder Volume = $\pi r^2 H$
= $\pi (2)^2 (4)$
= $50,265...\ cm^3$

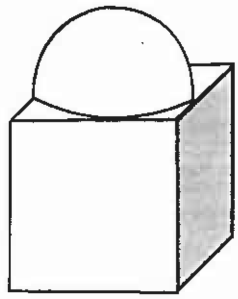
∴ Block Volume = $120\ cm^3 - 50,265...$
= $69,73...$
≈ $69,7\ cm^3$

(b) Area Prisma = $2LH + 2BH + 2LB$
= $2(6)(4) + 2(5)(4) + 2(6 \times 5)$
= $148\ cm^2$

Area Cylinder = $2\pi r(H+r)$
= $2\pi(2)(4+2)$
= $75,398...\ cm^2$

Total surface area = $148 + 75,398... - (2 \times \pi r^2)$ (2 circles)
= $223,398... - 25,132...$
= $198,265...$
≈ $198,3\ cm^2$

(2) A semi sphere is mounted on a cube, with sides 8 cm each.
[Correct to the nearest integer.]
(a) Determine the maximum diameter that the sphere can possibly have.
(b) Calculate the total surface area of the solid body.
(c) Calculate the volume of the solid body.



↓
L

(a) Max. diameter = 8 cm
↳ $r = 4$ cm

(b) Area cube = $6L^2$
$= 6(8)^2$
$= 384 \text{ cm}^2$

① Cube − area circle $= 384 - \pi(4)^2$
$= 333,7345\text{---}$

② Area $\frac{1}{2}$ sphere $= \frac{1}{2}(4\pi r^2)$
$= 2\pi(4)^2$
$= 100,5309\text{---}$

∴ Total surface area $= 333,7345\text{---} + 100,5309\text{---}$
$= 434,265\text{---}$
$\approx 434 \text{ cm}^2$

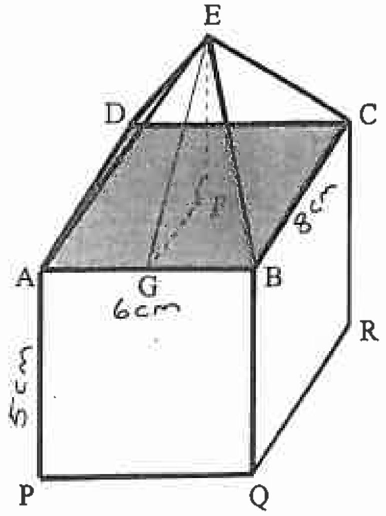
(c) Cube volume $= L^3$
$= 8^3$
$= 512 \text{ cm}^3$

$\frac{1}{2}$ Sphere volume $= \frac{1}{2} \times (\frac{4}{3}\pi r^3)$
$= \frac{1}{2} \times (\frac{4}{3}\pi \times 4^3)$
$= 134,041\text{---} \text{ cm}^3$

∴ Total volume $= 512 + 134,041\text{--}$
$= 646,041\text{---}$
$\approx 646 \text{ cm}^3$

(3) As seen in the diagram, ABCD is a common base of both the pyramid and the prism.
ABCD is a rectangle with AB = 6 cm and BC = 8 cm. The height of the prism is 5 cm.
EF = 4 cm is the perpendicular height of the pyramid.

Calculate: [Correct to 1 dec.]
(a) the length of FG.
(b) the length of the slant height of the pyramid.
(c) the total surface area of the solid body.



(a) $FG = \frac{1}{2} BC$ [EF as perpendicular]
$\therefore FG = 4$ cm

(b) In $\triangle EFG$:
$EG^2 = EF^2 + FG^2$ [Pyth.]
$= 4^2 + 4^2$
$EG^2 = 16 + 16$
$EG = 32$
$EG \approx 5{,}7$ cm

contact area with pyramide

(c) Area prisma $= 2LH + 2BH + 2LB - [1LB]$
$= 2(8)(5) + 2(6)(5) + 1(8)(6)$
$= 188 \text{ cm}^2$

Area piramide $= A + \frac{1}{2}ph - A$ → contact area with prism
$= \frac{1}{2}ph$
$= \frac{1}{2}(2\times6 + 2\times8)(4)$
$= \frac{1}{2}(28)(4)$
$= 56 \text{ cm}^2$

Total surface area $= 188 \text{ cm}^2 + 56 \text{ cm}^2$
$= 244 \text{ cm}^2$

Chapter D2

Euclidian Geometry

D2.1 Revision:

(1) Angles:

Type of angle:	Example:	Angle size:
Acute angle		Bigger than 0° but smaller than 90°.
Right angle		Equal to 90°.
Obtuse angle		Bigger than 90° but smaller than 180°.
Flat angle		Equal to 180°.
Re-entrant angle		Bigger than 180° but smaller than 360°.
Revolution		Equal to 360°.

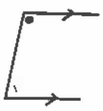
(2) Parallel lines:

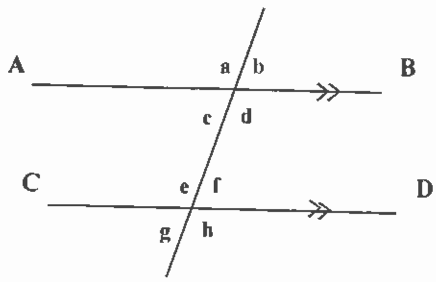
* If two lines are parallel to each other, the following will be true:

(a) Corresponding angles:
E.g. $a = e$; $b = f$;
$c = g$ and $d = h$.

(b) Alternate angles:
E.g. $e = d$; $c = f$;
$a = h$ and $b = g$.

(c) Co-interior angles:
E.g. $c + e = 180°$ and
$d + f = 180°$.





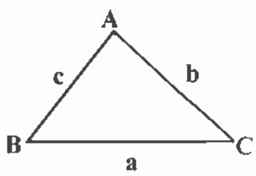
* To prove that lines are parallel one of the following have to be true:

(a) A pair of corresponding angles has to be equal or
(b) A pair of alternate angles has to be equal or
(c) Together a pair of co-interior angles has to equal 180°.

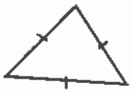
(3) Triangles:

* Naming of sides and angles:

A, B and C represent the angles.
a, b and c represent the sides.



* Types of triangles:

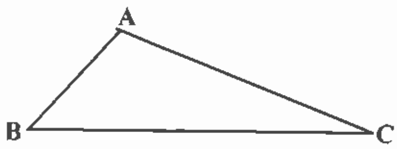
Type of triangle:	**Example:**	**Description:**
Right angled triangle		One angle is equal to 90°.
Acute angled triangle		All the angles are acute angles.
Obtuse angled triangle		One of the angles is an obtuse angle.
Isosceles triangle		Two of the sides are of equal length.
Equilateral triangle		All three sides are of equal length.
Scalene triangle		All three sides have different lengths.

* Characteristics of triangles:

(a) In a triangle the longest side is always opposite the largest angle.
(b) In an isosceles triangle the angles opposite the equal sides are always of equal size.
(c) In an equilateral triangle all the angles are equal to 60°.

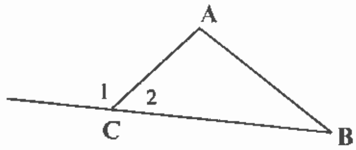
(d) The sum of the interior angles of all triangles is 180°.

$\therefore\ \hat{A} + \hat{B} + \hat{C} = 180°$



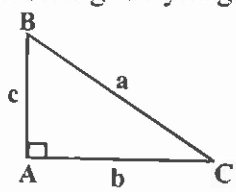
(e) The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

$\therefore\ \hat{C}_1 = \hat{A} + \hat{B}$



* Theorem of Pythagoras:

According to Pythagoras: "The square on the hypotenuse side of a right angled triangle is equal to the sum of the squares on the other two sides."



$\therefore\ a^2 = b^2 + c^2$

* Similar triangles:

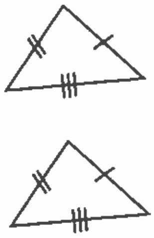
Triangles are similar to each other if:
(a) all the pairs of corresponding angles are equal and
(b) all the pairs of corresponding sides have the same proportion.

If two triangles are similar, then:
(a) all the corresponding angles are equal and
(b) all the corresponding sides are in the same proportion.

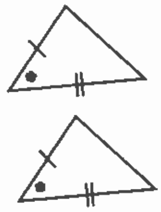
* Congruent triangles:

Two triangles are congruent to each other if one of the following conditions is true:

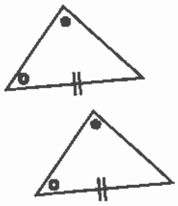
(a) All three pairs of sides are equal.



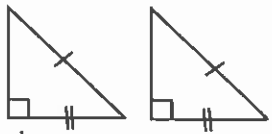
(b) Two pairs of corresponding sides and the included angle have to be equal.



(c) Two pairs of angles and a corresponding side are equal.

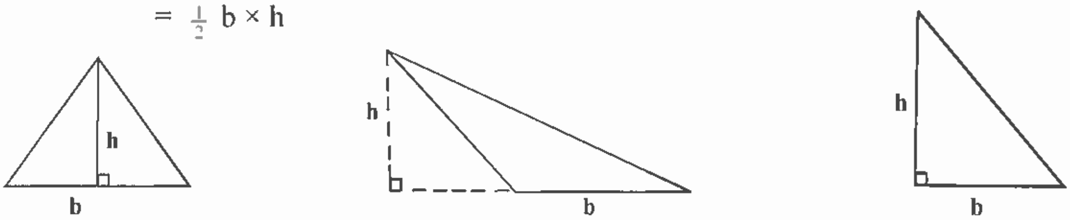


(d) In a right angle triangle the hypotenuse and a corresponding right angle are equal.



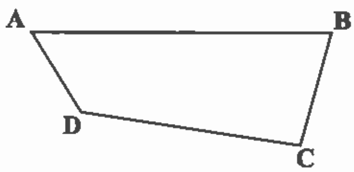
* The area of a triangle:

$$\text{Area } \Delta = \tfrac{1}{2} \text{ base} \times \text{perpendicular height}$$
$$= \tfrac{1}{2}\, b \times h$$

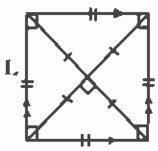
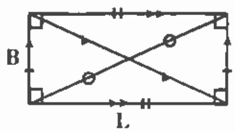


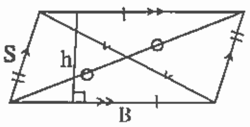
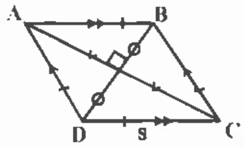
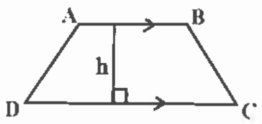
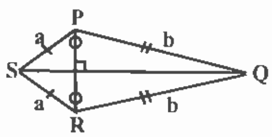
(4) Kinds of quadrangles:

* The sum of the interior angles of a quadrangle is equal to 360°.
  $\hat{A} + \hat{B} + \hat{C} + \hat{D} = 360°$



* Types of quadrangles:

Type of quadrangle:	Characteristics:	Perimeter:	Area:
Square 	*All the sides are of equal length. *The corresponding sides are parallel. *All the angles are 90°. *The diagonals bisects each other perpendicular and they bisect the angles.	4L	L^2
Rectangle 	*Opposite sides are of equal length and are parallel. * All the angles are 90°. * Diagonals bisect each other.	2L + 2B	L × B

Parallelogram 	*Opposite sides are of equal length and are parallel. *The opposite angles are of equal size. * Diagonals bisect each other.	2B + 2S	B × ⊥h
Rhombus 	*All the sides are of equal length. *The opposite sides are parallel. *The opposite angles are of equal size. *The diagonals bisect each other perpendicular and they bisect the angles.	4s	$\frac{1}{2}$ AC × BD
Trapezium 	*Only one pair of opposite sides are parallel.	AB+BC+ CD+DA	$\frac{1}{2}$h × (AB + CD)
Kite 	*The pairs of adjacent sides are of equal length. *One pair of opposite angles are of equal size. *The diagonals are perpendicular and the longest diagonal bisects the shorter diagonal.	2a + 2b	$\frac{1}{2}$ × SQ × PR

* A quadrilateral is a parallelogram if:
  - both pairs of opposite sides are parallel. (Per definition!)
  - both pairs of opposite sides are equal in length.
  - both pairs of opposite angles are equal.
  - one pair of opposite sides is parallel and equal in length.
  - the diagonals bisect one another.

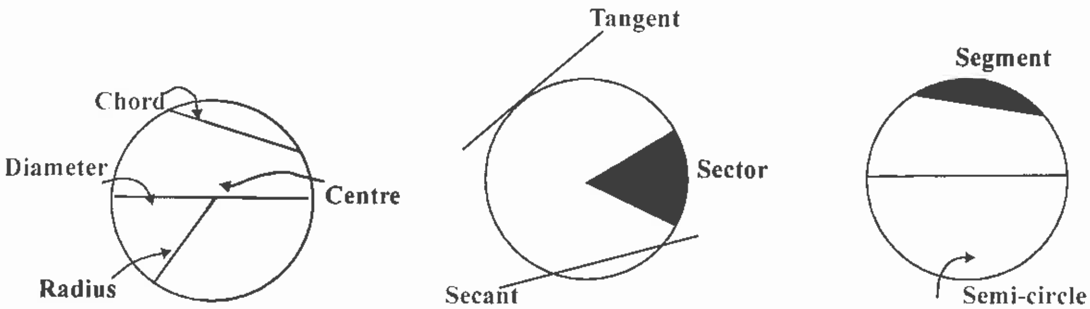
* A rhombus is a parallelogram of which:
  - one pair of adjacent sides is equal in length.
  - the diagonals are perpendicular to one another.

* A rectangle is a parallelogram of which:
  - one of the angles is 90°.
  - the diagonals are equal in length.

* A square is a:
  - rectangle of which al the sides are of equal length.
  - rhombus for which all angles are 90°.

(5) Circles:

* Terminology:



* Area and circumference:

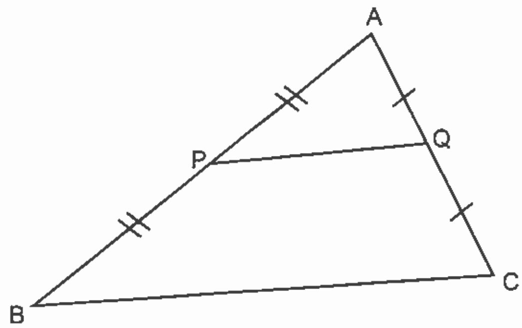
Circumference = $2 \times \pi \times r$ and Area = $\pi \times r^2$

Remember: $\pi = \frac{22}{7}$ and Diameter (d) = 2 × radius (r)

(6) Midpoint theorem:

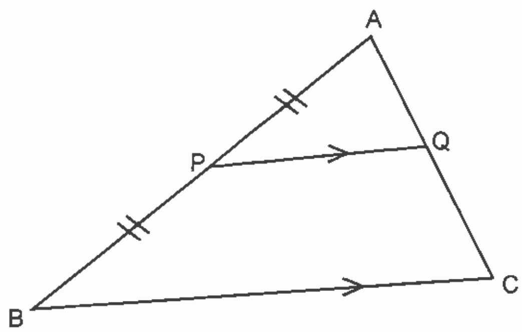
The line segment joining the mid-points of two sides of a triangle, is parallel to the third side and equal half the length of the third side.

$\therefore$ If AP = PB and AQ = QC,
then PQ // BC and
PQ = $\frac{1}{2}$BC.



Converse: The line segment through the midpoint of one side of a triangle, parallel to another side, bisect the third side and also equal to half the length of the third side.

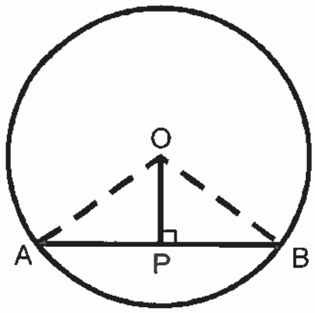
$\therefore$ If AP = PB and PQ // BC
then AQ = QC and
PQ = $\frac{1}{2}$BC.



D2.2 Centre of a circle:

Theorem 1:

"The line drawn from the centre of a circle, perpendicular to a chord, bisects the chord." [Line from cntr ⊙ to mdpt chord]



Prove:

Given: A circle with centre O with OP ⊥ AB.

To be proven: AP = PB

Construction: Join O with A and O with B.

Prove: In △ AOP and △ BOP:

* AO = BO [radii of circle O]
* $A\hat{P}O = B\hat{P}O$ [OP ⊥ AB]
* OP = OP [common]

∴ △ AOP ≡ △ BOP [hypotenuse and side right angled △]

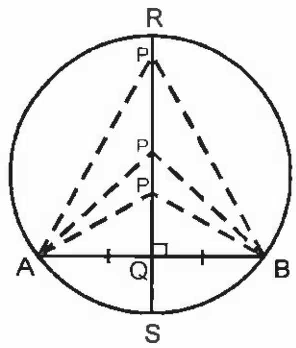
∴ AP = PB [≡]

Converse of theorem 1:

"The line joining the centre of a circle and the midpoint of a chord, is perpendicular to the chord.

Theorem 2:

"The perpendicular bisector of a chord passes through the centre of a circle." [Perpendicular bisector on chord]



Prove:

Given: A circle with AQ = QB and RS ⊥ AB.

To be proven: The centre of the circle passes through RS.

Construction: Choose P as any point on line RS.

Join P with A and with B.

Prove: In △ AQP and △ BQP:

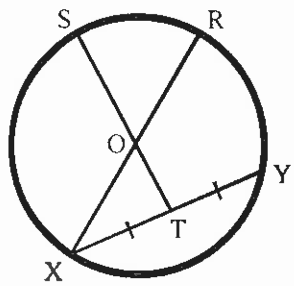
* AQ = BQ [given]
* $A\hat{Q}P = B\hat{Q}P$ [OD ⊥ AB]
* QP = QP [common]

∴ △ AOP ≡ △ BOP [side, angle, side]

∴ AP = PB

But the midpoint of a circle lies the same distance from any two (or more) points (as e.g. A and B) on the circumference of the circle. ∴ The centre of the circle should pass through RS.

E.g.1 O is the centre of the circle with XT = TY. XR = 20 cm and XY = 16 cm. Calculate the length of ST.



****

XO = OR = 10 cm [Radius is halve of the diameter]

XT = TY = 8 cm [Given]

OT ⊥ XY [Line from cntr ⊙ to midpt chord]

∴ In △OXT:

$OX^2 = OT^2 + XT^2$ *[Pythagoras]*

∴ $10^2 = OT^2 + 8^2$

∴ $OT^2 = 36$

∴ $OT = 6$ *cm*

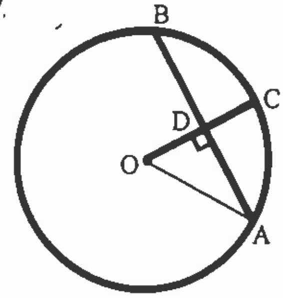
∴ *ST = Radius (OS) + OT*

= 10 cm + 6 cm

∴ ***ST = 16 cm***

Exercise 1: Date: ______________

(1) If OD = 5 cm and AB = 24 cm, calculate the length of the diameter of the circle with midpoint O.



AD = DB = 12 [line ⊥ on radius]

In △ ADO:

$OA^2 = OD^2 + AD^2$

$= 5^2 + 12^2$

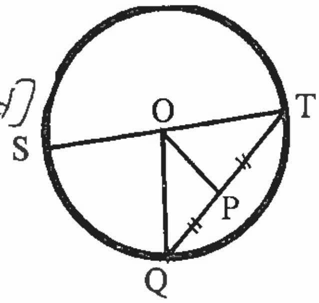
$= 25 + 144$

$OA^2 = 169$

OA = 13 cm → radius

∴ Diameter = 2 × 13 = 26 cm

(2) Calculate the length of QT if OS = 10 mm and OP = 6 mm, with O the centre of the circle and QP = PT.



OP ⊥ QT [Line midp ⊙ → midpt. chord]

OS = 10 = OQ [radii]

In △ OPQ:

$OQ^2 = OP^2 + PQ^2$ [Pyth.]

$10^2 = (6)^2 + PQ^2$

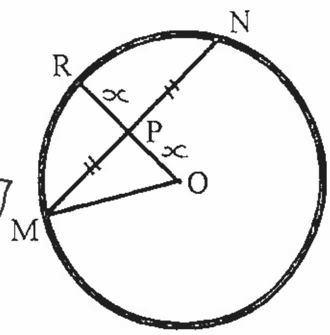
$100 - 36 = PQ^2$

$64 = PQ^2$

∴ PQ = 8

∴ QT = 2 × 8 = 16 cm

(3) O is the centre of the circle with MP = PN. Calculate the length of OR, correct to 1 dec, if MN = 18 cm and RP = PO = *x*.



MP = PN = 9 cm

OR = 2x = OM [radii]

OR ⊥ MN [Line midpt ⊙ → midpt. chord]

In △ OPM:

$OM^2 = OP^2 + PM^2$

$(2x)^2 = (x)^2 + (9)^2$

$4x^2 = x^2 + 81$

$3x^2 = 81$

$x^2 = 27$

$x = 5,196\ldots \Rightarrow x \approx 5,2$ ∴ OR ≈ 10,4

(b) $NY = NP + PY = 25 + 20 = 45$ cm

In Δ ANY :

$AN^2 = NY^2 + AY^2$

$= (45)^2 + (15)^2$

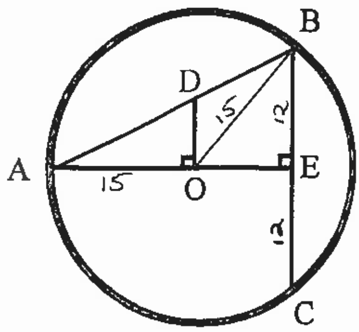
$AN^2 = 2025 + 225$

$AN^2 = 2\,250$

$AN = 47{,}43\ldots$

$AN \approx 47$

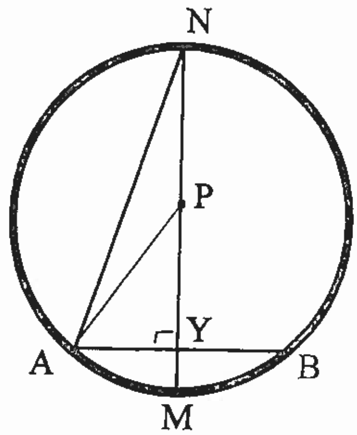
(4) Circle O has a radius of 15 cm and BC = 24 cm.
Calculate: (a) the length OE (b) area $\triangle ABE$



(a) $BE = EC = 12cm$ [line ⊥ radius]
In $\triangle OBE$:
$OB^2 = BE^2 + OE^2$
$15^2 = 12^2 + OE^2$
$OE^2 = 81$
$\therefore OE = 9cm$

(b) Area $\triangle ABE = \frac{1}{2} AE \times BE$
$= \frac{1}{2}(15+9) \times (12)$
$= \frac{1}{2}(24)(12)$
$= 144 \text{ cm}^2$

(5) P is the centre of the circle with MN = 50 cm.
NM ⊥ AB and YP = 4YM.
Calculate the length of: (Correct to the nearest integer.)
(a) AY (b) AN

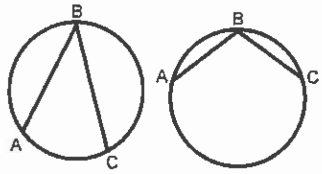


(a) $MN = 50 \rightarrow NP = PM = 25 cm$
$AY = YB$ [Chord ⊥ radius]
Let $YM = x$
$\therefore YP = 4x$
But $PM = 25 = PY + YM$
$\therefore 25 = 4x + x$
$25 = 5x$
$\therefore x = 5$ $\therefore YP = 5 \times 4 = 20 cm$
In $\triangle APY$:
$AP^2 = PY^2 + AY^2$
$(25)^2 = (20)^2 + AY^2$
$625 - 400 = AY^2$
$225 = AY^2$
$15 = AY$

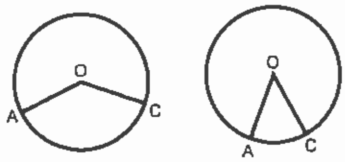
← (b)

Terminology:

* $A\hat{B}C$ is an angle on the arc of the circle; also called the inscribed angle.



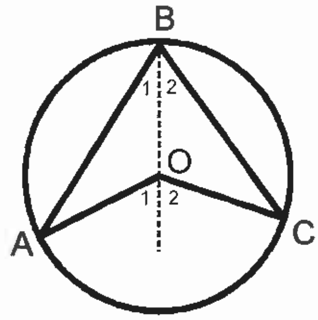
* $A\hat{O}C$ is an angle at the centre of the circle with O as the centre of the circle; also called the central angle.



Theorem 3:

"The central angle subtended by an arc of a circle is double the size of an inscribed angle subtended by the same arc at the circle and lies on the same side of the centre of the centre." [cent ∠⊙ = 2 × inscr∠]

Prove:



Given: Central angle $A\hat{O}C$ and inscribed angle $A\hat{B}C$ subtended by the same arc AC.

To be proven: $A\hat{O}C = 2 \times A\hat{B}C$

Construction: Join B with O and extend.

Prove: In △ AOB:

$\hat{O}_1 = \hat{A} + \hat{B}_1$ [ext ∠ of △]

As in △ COB:

$\hat{O}_2 = \hat{C} + \hat{B}_2$ [ext ∠ of △]

$\therefore \hat{O}_1 + \hat{O}_2 = \hat{A} + \hat{B}_1 + \hat{C} + \hat{B}_2$

but $\hat{A} = \hat{B}_1$ and $\hat{C} = \hat{B}_2$ [∠' opposite equal sides with radii AO = OB = OC]

$\therefore \hat{O}_1 + \hat{O}_2 = \hat{B}_1 + \hat{B}_1 + \hat{B}_2 + \hat{B}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{B}_1 + 2\hat{B}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{B}_1 + \hat{B}_2)$

$\therefore A\hat{O}C = 2 \times A\hat{B}C$

The following sketches can also be used to prove the theorem above: