Grade 9 – Textbook

(Revised CAPS edition)

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Chapter A1

Real numbers

A1.1 The number system:

Complete: * Natural numbers: $\mathbb{N}$ = {_____}

* Whole numbers: $\mathbb{N}_0$ = {_____}

* Integers: $\mathbb{Z}$ = {_____}

* Rational numbers: $\mathbb{Q}$ = {_____}

A1.2 Irrational numbers:

These numbers are non-terminating and non-recurring decimals and is therefore all real numbers that cannot be exactly expressed as a ratio of two integers.

E.g. 1 Irrational numbers:

- $\sqrt{2}$ or $\sqrt{7}$ or $\sqrt{\frac{3}{4}}$ **etc. because 2 ; 7 and 3 are not perfect squares!**
- $\sqrt[3]{12}$ or $\sqrt[3]{100}$ **etc. because 12 and 100 are not perfect cubes!**

Whereas the following numbers are rational numbers:

- $\sqrt{4}$ or $\sqrt{0,01}$ or $\sqrt{\frac{25}{9}}$ **etc. because 4 ; 0,01 ; 25 and 9 are perfect squares!**
- $\sqrt[3]{27}$ or $\sqrt[3]{125}$ **etc. because 27 and 125 are perfect cubes!**

The real numbers, $\mathbb{R}$ consist of the rational numbers, $\mathbb{Q}$ and the irrational numbers, $\mathbb{Q}'$. Remember that all terminating and recurring decimals are rational numbers.

Exercise 1:

Which of the following numbers are rational numbers $(\mathbb{Q})$ and which are irrational numbers $(\mathbb{Q}')$?

(1) 14 (2) $\frac{1}{5}$ (3) $\sqrt{81}$ (4) 0,12

(5) $\sqrt{18}$ (6) $12,\dot{2}\dot{3}$ (7) $-\sqrt{\frac{12}{3}}$ (8) 0,2945 ...

(9) $\sqrt[3]{64}$ (10) π (11) $\sqrt[5]{32}$ (12) $\frac{11}{7}$

☺ **Which of the following numbers are irrational numbers between 2 and 10?**

$\sqrt{10}$	$-\sqrt{20}$	$\sqrt{32}$	π	$\sqrt{25}$	$\sqrt[3]{9}$	$3,\dot{2}$	$\sqrt{99}$	$2,15$	$\sqrt{\frac{18}{2}}$

A1.3 Conversion of common fractions to decimals:

E.g. 2 Express the following as decimal fractions, without using a calculator:

(a) $\frac{3}{8} = \frac{3{,}000\ldots}{8} = \frac{3,\ {}^{3}0\ {}^{6}0\ {}^{4}0}{8} = 0{,}375$

(b) $1\frac{2}{3} = 1\frac{2{,}000\ldots}{3} = 1\frac{2,\ {}^{2}0\ {}^{2}0\ {}^{2}0\ldots}{3} = 1{,}666\ldots = 1{,}\dot{6}$

Exercise 2:

Express the following as decimal fractions, without using a calculator:

(1) $\frac{23}{7}$ (2) $1\frac{1}{3}$ (3) $\frac{-2}{12}$ (4) $\frac{3}{5}$

(5) $\frac{1}{8}$ (6) $\frac{7}{9}$ (7) $\frac{17}{25}$ (8) $\frac{5}{100}$

(9) $\frac{4}{11}$ (10) $-3\frac{6}{7}$ (11) $-5\frac{5}{6}$ (12) $\frac{11}{4}$

☺ **You scored $^{14}/_{30}$ for a maths test. Calculate your percentage, correct to the nearest integer.**

A1.4 Conversion of decimal fractions to common fractions:

E.g. 3 Express the following as common fractions in its simplest form:

(a) $4{,}5 = 4\frac{5}{10} = 4\frac{5}{10}\left(\frac{\div 5}{\div 5}\right) = 4\frac{1}{2}$

(b) $-0{,}12 = -\frac{12}{100} = -\frac{12}{100}\left(\frac{\div 4}{\div 4}\right) = -\frac{3}{25}$

Exercise 3:

Express the following as a common fraction in its simplest form:

(1) 2,4 (2) 0,25 (3) 33,6 (4) −0,5

(5) −1,2 (6) 0,125 (7) 5,02 (8) 7,3

(9) 100,75 (10) 0,0005 (11) −2,1 (12) 1,45

☺ **If you and your friends ate $0{,}84$ of your birthday cake, what part of the cake was left over? (Express your answer as a common fraction in its simplest form!)**

A1.5 Conversion of recurring decimals to common fractions:

E.g. 4 Convert the following to a common fraction in its simplest form:

(a) $0,\dot{1} = \frac{1}{9}$; $0,\dot{3} = \frac{3}{9} = \frac{1}{3}$; $0,\dot{5} = \frac{5}{9}$; $0,\dot{8} = \frac{8}{9}$

(b) $3,\dot{2}\dot{4} = 3\frac{24}{99} = 3\frac{8}{33}$; $0,\dot{4}\dot{2}\dot{1} = \frac{421}{999}$; $15,\dot{1}\dot{6}\dot{5}\dot{3} = 15\frac{1653}{9999} = 15\frac{551}{3333}$

(c) $0,0\dot{3} = 0,\dot{3} \div 10 = \frac{3}{9} \div \frac{10}{1} = \frac{3}{9} \times \frac{1}{10} = \frac{3}{90} = \frac{1}{30}$

(d) $0,00\dot{4}\dot{6} = 0,\dot{4}\dot{6} \div 100 = \frac{46}{99} \div \frac{100}{1} = \frac{46}{99} \times \frac{1}{100} = \frac{46}{9900} = \frac{23}{4950}$

(e) $0,5\dot{7} = 0,5 + 0,0\dot{7} = 0,5 + 0,\dot{7} \div 10 = \frac{5}{10} + \frac{7}{9} \times \frac{1}{10} = \frac{5 \times 9}{10 \times 9} + \frac{7}{90} = \frac{45+7}{90} = \frac{52}{90} = \frac{26}{45}$

Exercise 4:

Convert the following to a common fraction in its simplest form:

(1)	$4,\dot{6}$	(2)	$0,\dot{1}\dot{5}$	(3)	$0,\dot{9}$	(4)	$0,\dot{7}$
(5)	$17,\dot{4}$	(6)	$2,\dot{2}$	(7)	$0,\dot{8}\dot{5}$	(8)	$2,\dot{2}\dot{6}\dot{3}$
(9)	$12,3\dot{7}$	(10)	$0,00\dot{3}$	(11)	$1,\dot{1}\dot{3}\dot{5}$ or $1,\overline{135}$	(12)	$4,\dot{7}\dot{2}$
(13)	$0,5\dot{8}$	(14)	$0,2\dot{3}\dot{7}$	(15)	$2,2\dot{6}$	(16)	$3,25\dot{8}$
(17)	$188,\dot{2}$	(18)	$1,\overline{214}$	(19)	$0,1\dot{6}\dot{7}\dot{3}$	(20)	$86,535\dot{2}$

☺ **Add the following without using a calculator and by first converting the numbers to common fractions: $0,251$ and $0,\dot{2}\dot{5}$?**

A1.6 Representation of sets of numbers:

Sets of numbers can be represented or written in the following ways:

A1.6.1 Set builder notation:

E.g. 5 Write the following sets of numbers in set builder notation:

(a) All natural numbers greater than 6: $\{x \,/\, x > 6 \,;\, x \in \mathbb{N}\}$

(b) All real numbers between -2 and 5: $\{m : -2 < m < 5 \;;\; m \in \mathbb{R}\}$

Exercise 5:

Write the following sets of numbers in set builder notation:

(1) The real numbers between 1 and 6.
(2) The whole numbers smaller than 10.
(3) The real numbers from -2 up to and including 3.
(4) The natural numbers greater than 4.
(5) $\{-8;\ -7;\ -6;\ -5;\ -4;\ -3;\ -2;\ -1\}$
(6) $\{17;\ 18;\ 19;\ \ldots\ldots\ldots..\}$
(7) The real numbers greater than -20 but smaller than or equal to 1.
(8) $\{\ldots\ldots.;\ -6;\ -5;\ -4;\ -3\}$
(9) All even integers between 0 and 20.
(10) $\{-1;\ 0;\ 1\}$

A1.6.2 Interval notation:

Only sets that form part of real numbers can be represented using interval notation!

E.g. 6 Write the following in interval notation:

(a) The real numbers between -2 and 4, including 4: $x \in (-2\ ;\ 4]$ Open, closed interval!

(b) $\{m\ /\ m > 7;\ m \in \mathbb{R}\}$: $m \in (7\ ;\ \infty)$ Open interval!

Exercise 6:

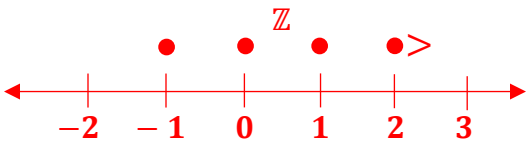
Write the following in interval notation:

(1) The real numbers between -2 and 15.
(2) The real numbers smaller than 10.
(3) The real numbers from -2 up to and including 3.
(4) $\{x\ /\ -7 < x \leq 0;\ x \in \mathbb{R}\}$
(5) $\{y\ /\ y \leq 17;\ y \in \mathbb{R}\}$
(6) $\{p\ /\ -1 < p < 1;\ p \in \mathbb{R}\}$
(7) The real numbers greater than -8 but smaller than or equal to 11.
(8) $\{x\ /\ 3 < x \leq 8;\ x \in \mathbb{N}\}$
(9) The set of real numbers.
(10) $\{t\ /\ t > 0;\ t \in \mathbb{R}\}$

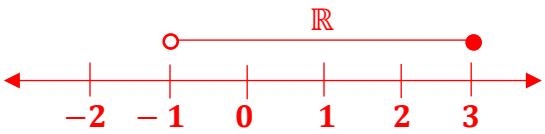
A1.6.3 Number lines:

E.g. 7 Represent the following on a number line:

(a) $\{-1\,;\,0\,;\,1\,;\,2\,;\,\ldots\ldots\ldots\}$



(b) $\{x:\ -1 < x \leq 3\ ;\ x \in \mathbb{R}\}$



Exercise 7:

Represent the following on a number line:

(1) $\{t:\ t > 4;\ t \in \mathbb{R}\}$

(2) $\{x\ /\ 1 < x \leq 7;\ x \in \mathbb{N}\}$

(3) $\{x:\ -3 \leq x \leq 6;\ x \in \mathbb{N}_0\}$

(4) $\{x\ /\ 2\frac{1}{2} \leq x \leq 8;\ x \in \mathbb{Z}\}$

(5) $m \in (-2\,;\,2]$

(6) All whole numbers between 4 and 8.

(7) $\{y\ /\ y < 7;\ y \in \mathbb{N}\}$

(8) $\{\ldots\ldots\,;\,-2;\,-1;\,0;\,1\}$

(9) $\{p\ /\ p < -3 \text{ or } p > 1;\ p \in \mathbb{Z}\}$

(10) $k \in [-3;\,\infty)$

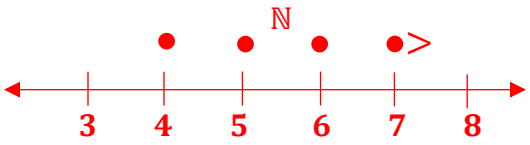
A1.6.4 Solving of linear inequalities:

E.g. 8 Solve for x in each of the following and represent the solution on a number line:

(a) $x - 2 \geq 2$ **if** $x \in \mathbb{N}$

$x \geq 2 + 2$

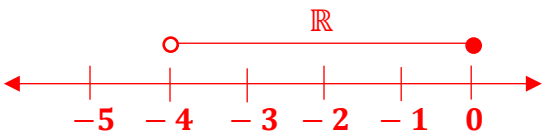
$x \geq 4$



(b) $-3 < x + 1 \leq 1$ **if** $x \in \mathbb{R}$

$-3 - 1 < x \leq 1 - 1$

$-4 < x \leq 0$



Exercise 8:

(1) Solve for x in each of the following and represent the solution on a number line:

(a) $x + 1 < 3;\quad x \in \mathbb{N}_0$

(b) $2x \geq -8;\quad x \in \mathbb{R}$

(c) $x - 4 \leq 0;\quad x \in \mathbb{Z}$

(d) $2x + 3 > 7;\quad x \in \mathbb{N}$

(e) $-6 < x - 1 \leq 6;\quad x \in \mathbb{R}$

(f) $x + 7 \geq -1;\quad x \in \mathbb{Z}$

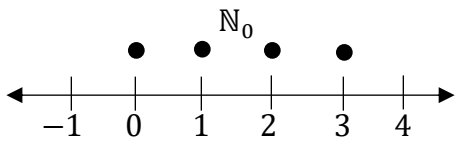
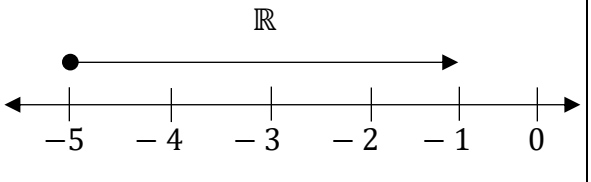
(2) Represent the following on a number line:

(a) $\{x : 2x < -2;\ x \in \mathbb{R}\}$
(b) $\{x : -2 \leq x + 1 \leq 4;\ x \in \mathbb{Z}\}$
(c) $\{y : y - 3 < -1;\ y \in \mathbb{N}\}$
(d) $\{x : x \leq -1;\ x \in \mathbb{R}\}$
(e) $\{x / x < 3;\ x \in \mathbb{Z}\}$
(f) $\{p / 2p \geq -5;\ p \in \mathbb{R}\}$
(g) $\{m : -1 \leq 2m - 1 < 7;\ m \in \mathbb{R}\}$
(h) $\{x : 2x - 3 < 7;\ x \in \mathbb{N}_0\}$

A1.6.5 Combinations:

Exercise 9:

Redraw the table below and complete the missing representations in the table:

	Set builder notation:	Interval notation:	Number line:
(1)	$\{x / -1 < x \leq 2;\ x \in \mathbb{R}\}$		
(2)		$x \in [-2\,;5]$	
(3)		$y \in (-\infty\,;3]$	
(4)			
(5)	$\{y /\ y \geq 3;\ y \in \mathbb{N}\}$		
(6)		$m \in (0\,;4]$	
(7)			
(8)	$\{m : m \leq 6;\ m \in \mathbb{R}\}$		
(9)	$\{x / -1 < x < 2;\ x \in \mathbb{Z}\}$		
(10)		$x \in (-1;\ \infty)$	

☺ **A farmer has enough grazing for 13 head of cattle. Write down the possible permutations he can allow on his farm. Represent your answer on a number line.**

A1.7 REVISION EXERCISE:

(1) Complete the following table by making a ✓ in the appropriate column(s) to which each number belongs:

		$\mathbb{N}$	$\mathbb{N}_0$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{Q}'$	$\mathbb{R}$			$\mathbb{N}$	$\mathbb{N}_0$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{Q}'$	$\mathbb{R}$
(1)	$\frac{-2}{3}$							(5)	$\frac{\sqrt{4}}{\sqrt{9}}$						
(2)	0							(6)	$\sqrt[3]{8}$						
(3)	$0,\dot{3}$							(7)	$-1,7$						
(4)	$\sqrt{27}$							(8)	111						

(2) Express each of the following as a decimal fraction:

(a) $\frac{3}{7}$ (b) $\frac{4}{25}$ (c) $-1\frac{5}{9}$

(d) $\frac{-7}{8}$ (e) $\frac{1}{6}$ (f) $\frac{23}{12}$

(3) Express each of the following as a common fraction in its simplest form:

(a) $0,45$ (b) $-3,25$ (c) $17,2$

(4) Express each of the following as common fraction in its simplest form:

(a) $2,0\dot{1}$ (b) $0,44$

(c) $0,08\dot{3}$ (d) $25,4\dot{2}\dot{5}$

(5) Solve for x. Give the solution (i) in set builder notation
(ii) in interval notation and
(iii) as a representation on a number line.

(a) $-4 \leq 2x;\ x \in \mathbb{Z}$ (b) $x + 1 < 7;\ x \in \mathbb{N}$

(c) $-1 < x + 1 < 2;\ x \in \mathbb{R}$ (d) $x - 2 \geq -5;\ x \in \mathbb{R}$

(6) Write the following in interval notation and represent them on a number line:

(a) $\{x: 0 > x \geq -4;\ x \in \mathbb{R}\}$ (b) $\{p/\ p < 1,5;\ p \in \mathbb{R}\}$

(c) $\{m/\ m \geq -2\frac{1}{3}\ ;\ m \in \mathbb{Z}\}$ (d) $\{x: x < 1 \text{ or } x > 5;\ x \in \mathbb{R}\}$

(7) Write the following in set builder notation and represent them on a number line:

(a) $y \in (-\infty\,; 2]$ (b) $t \in \left[-1\frac{1}{4}\ ;\ 6\right]$

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Chapter A2

Exponents

A2.1 Exponential laws:

Basic exponential laws:

(1) $x^m \times x^n = x^{m+n}$

(2) $x^m \div x^n = x^{m-n}$

(3) $(x^m)^n = x^{mn}$

(4) $(xy)^m = x^m y^m$

(5) $x^0 = 1$

(6) $x^{-m} = \frac{1}{x^m}$

(7) $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

E.g. 1 Simplify and write your answer as a positive exponent:

(a) $(-3xy^{-3}k^0)^3 = (-3)^3 \times (x^1)^3 \times (y^{-3})^3 \times (k^0)^3$

$= -27x^3y^{-9}k^0 = \frac{-27x^3(1)}{y^9} = \frac{-27x^3}{y^9}$

(b) $\frac{a^{3\frac{1}{2}} \times a \times a^{\frac{1}{2}}}{b^3 \times b^{-6}} = \frac{a^{3\frac{1}{2}+1+\frac{1}{2}}}{b^{3-6}} = \frac{a^5}{b^{-3}} = a^5b^3$

(c) $3^{x+2y} \cdot 9^{-y} = 3^{x+2y} \cdot (3^2)^{-y} = 3^{x+2y} \cdot 3^{-2y} = 3^{x+2y-2y} = 3^x$

(d) $\sqrt{9m^6 + 16m^6} = \sqrt{25m^6} = \sqrt{25} \times \sqrt[2]{m^6} = 5m^{\frac{6}{2}} = 5m^3$

E.g. 2 Simplify and write your answer as a positive exponent:

(a) $\frac{5^0 \times m^{12} \times n^4 \times n^{-2}}{m^5 \times m \times n^6}$

$= \frac{1 \times m^{12} \times n^{4-2}}{m^{5+1} \times n^6}$

$= \frac{m^{12} \times n^2}{m^6 \times n^6}$

$= m^{12-6} \times n^{2-6}$

$= m^6 \times n^{-4}$

$= \frac{m^6}{n^4}$

(b) $\frac{(5^3)^2 \times (5 \times 7)^3}{(7^2 \times 5^{-4})^{-1}}$

$= \frac{5^6 \times 5^3 \times 7^3}{7^{-2} \times 5^4}$

$= \frac{5^9 \times 7^3}{7^{-2} \times 5^4}$

$= 5^{9-4} \times 7^{3-(-2)}$

$= 5^5 \times 7^{3+2}$

$= 5^5 \times 7^5$

Exercise 1:

(1) Simplify without a calculator (write answers as positive exponents!):

(a) $a^4 \,.\, b^{-7} \,.\, b \,.\, a^3$

(b) $(3xy - 2x^5y)^{2\times3-6}$

(c) $\dfrac{a^4b^3 \times ab}{a^5b^4}$

(d) $\dfrac{3^{-3} \times m^5 \times m^{-2}}{n^{-8} \times n^5}$

(e) $\sqrt{2\dfrac{1}{4}p^{-4}}$

(f) $(4mn^3p^{-3})^5$

(g) $(2p^3q)^2 \times 2^7p^{-4}q^{-2}$

(h) $4^k \times 2^{2k}$

(i) $4^0(x^2)^3 \div x^2 \times x^3$

(j) $(-12x^{-6}y)(-4x^3y^3z)$

(k) $(3^2a^{-2}b)^{-1}$

(l) $2^{x+y} \,.\, 2^{x-y}$

(m) $\dfrac{5^{n+1} \,.\, 5^{4n}}{5^{2-3n}}$

(n) $\dfrac{c^4 \times c^5}{c \times c^8}$

(o) $2\sqrt{8} \times \sqrt{2x^2}$

(p) $2^{mn} \times 3^{mn}$

(q) $(m^2n)(mn^2)^0$

(r) $2x^3 + 5x^2$

(s) $\dfrac{(-3pq)(5pq)}{-30p^2q^6}$

(t) $\sqrt{\dfrac{24y^9}{6y^3}}$

(u) $m^2n^4 \times mn^3$

(v) $(3y)^2(-3y^2)$

(w) $\dfrac{(2a^7c^6)^2}{(4ac^3)^{-2}}$

(x) $\dfrac{x^{-1} \,.\, x^{-2} \,.\, x^{-3}}{x^5}$

(2) Simplify without a calculator (write answers as positive exponents!):

(a) $\dfrac{(2x^2y^{-1})^{-1}\,(-xy)^2}{4(x^4y^3)^2\,(xy^6)^0}$

(b) $\sqrt{\dfrac{12n^3}{3n^5}} \times \sqrt{\dfrac{25}{n^4}}$

(c) $\dfrac{-4(a^4b^2)^{\frac{1}{2}}}{\sqrt{a^6}}$

(d) $\left(\dfrac{144r^4s^3t}{-24r^2t^5}\right)^2$

(e) $\sqrt{16y^8} + \sqrt[3]{8y^{12}}$

(f) $12p^3q \div (2p^2q^{-1})^2 \times 3q^{-3}(pq^5)^0$

(3) Prove that: $2^{x+5} \times 4^{x-1} = 8^{x+1}$

(4) Are the following statements true or false?

(a) $3^2 \times 2^3 = 6^6$

(b) $x^3 \times y^3 = (xy)^3$

(c) $\sqrt[4]{64x^{-4}} = \dfrac{2}{x}$

(d) $(x - y)^2 = x^2 - y^2$

(e) $\dfrac{1}{9p^2} = 9p^{-2}$

☺ **If the length of a rectangle is $2x^2y^3$ and the width is $3xy^4$, determine**
(a) the perimeter of the rectangle and
(b) double the area of the rectangle in terms of x and y.

A2.2. Scientific notation:

E.g. 3 (a) Write in scientific notation:

(i) $4\,320 = 4{,}32 \times 10^3$

(ii) $0{,}03166 = 3{,}166 \times 10^{-2}$

(b) Write as a decimal number:

(i) $1{,}1 \times 10^6 = 1\,100\,000$

(ii) $5{,}08 \times 10^{-1} = 0{,}508$

E.g. 4 Simplify, without using a calculator and write the answer in scientific notation:

(i) $(3,5 \times 10^{-5}) \div (7,0 \times 10^{2})$

$= (3,5 \div 7,0) \times (10^{-5} \div 10^{2})$

$= \left(\frac{3,5}{7,0}\right) \times (10^{-5\,-\,2})$

$= \frac{\cancel{3,5}^{\,1}}{\cancel{7,0}^{\,2}} \times (10^{-7})$

$= \frac{1}{2} \times 10^{-7}$

$= 0,5 \times 10^{-7}$

$= (5,0 \times 10^{-1}) \times 10^{-7}$

$= 5,0 \times 10^{-8}$

(ii) $(4,6 \times 10^{3}) + (3,2 \times 10^{2})$

$= (4,6 \times 10^{1} \times 10^{2}) + (3,2 \times 10^{2})$

$= 10^{2}(4,6 \times 10^{1} + 3,2)$

$= 10^{2}(46 + 3,2)$

$= 10^{2}(49,2)$

$= 49,2 \times 10^{2}$

$= 4,92 \times 10^{1} \times 10^{2}$

$= 4,92 \times 10^{3}$

or $(4,6 \times 10^{3}) + (3,2 \times 10^{2})$

$= 4\,600 + 320$

$= 4\,920$

$= 4,92 \times 10^{3}$

Exercise 2:

(1) Write in scientific notation:

(a) 0,000346 (b) 15,19 (c) 7 000 (d) 67 000 000 000

(e) 0,743 (f) 3,0003 (g) 0,000000001 (h) 10 001

(2) Write as a decimal number:

(a) $3,1 \times 10^{-2}$ (b) $6,6006 \times 10^{6}$ (c) $4,21 \times 10^{3}$ (d) $7,0 \times 10^{-1}$

(e) $9,4736 \times 10^{-7}$ (f) $7,5 \times 10^{-4}$ (g) $2,2 \times 10^{4}$ (h) 6×10^{8}

(3) Simplify, without using a calculator and write the answer in scientific notation:

(a) $(4,0 \times 10^{3}) \times (1,1 \times 10^{-2})$ (b) $(7,6 \times 10^{3}) + (5,3 \times 10^{4})$

(c) $(5,4 \times 10^{-6})(9,0 \times 10^{4})$ (d) $(6,3 \times 10^{12}) \div (2,1 \times 10^{3})$

(e) $(8,59 \times 10^{-2}) - (5,1 \times 10^{-2})$ (f) $(1,2 \times 10^{-8}) \times (1,1 \times 10^{-4})$

(g) $(7,652 \times 10^{3}) + (6,48 \times 10^{-1})$ (h) $(7,2 \times 10^{-2}) \div (1,2 \times 10^{-4})$

☺ **If the die planet Jupiter is 800 million km from earth, and it was possible for you to travel from earth to Jupiter and back, how far would you have travelled? Express your answer in scientific notation.**

A2.3 Equations:

E.g. 5 Solve for x:

(a) $2^x = 8$

$2^x = 2^3$

EB ⇔ EE

$\therefore\ x = 3$

(b) $5^x = 1$

$5^x = 5^0$

EB ⇔ EE [Equal Base ⇔ Equal Exponent]

$\therefore\ x = 0$

(c) $8^x = \frac{1}{16}$

$(2^3)^x = \frac{1}{2^4}$

$2^{3x} = 2^{-4}$

EB ⇔ EE

$\therefore\ 3x = -4$

$\therefore\ x = \frac{-4}{3}$

(d) $x^4 = 81$

$x^4 = 3^4$

EE ⇔ EB

$\therefore\ x = \pm 3$ → $(+3)^4 = 81$ and $(-3)^4 = 81$

Exercise 3:

Solve for x:

(1) $3^x = 9$

(2) $100^x = 10$

(3) $7^x = 1$

(4) $25^x = 125$

(5) $x^3 = 27$

(6) $3^x = \frac{1}{3}$

(7) $2 \times 2^x = 4^x$

(8) $6^{x-3} = 1$

(9) $4^x = 0{,}5$

(10) $64 = 8^{3x-1}$

(11) $2^{x+2} = 32$

(12) $5^x \times 25^{x+1} = 5^3$

(13) $12^{x+1} = 144^x$

(14) $2x^5 = -64$

A2.4 REVISION EXERCISE:

(1) Simplify (without a calculator!):

(a) $t^2 \times t^8 \times t \div t^{-7}$ (b) $(a^{m-n})^2 \,.\, (a^{2m})$ (c) $(-3m^4n^4)^4$

(d) $\dfrac{-pq \times 2p^2q^3}{32p^{-5}q^{-1}}$ (e) $\sqrt{\dfrac{75x^7y}{27xy^5}}$ (f) $\left[\dfrac{-3m^{-3}n^{-4}}{2m^{-1}n^{-5}}\right]^{-3}$

(g) $\dfrac{32m^6n}{24mn^3} \times \dfrac{12m^3}{8n^{-3}}$ (h) $3x^8(4y^2)^0$ (i) $8x^4y^2 - 3(x^2y)^2$

(j) $\dfrac{x^4y^6z^7 \div xy^3z^9}{xyz}$ (k) $\dfrac{(5 \times a^2 \times b^{-3})^2 \times (-2a^0 \times b)^4}{8b^{-6} \times (3cd)^{1-1}}$

(l) $\dfrac{\sqrt{144p^{144}}}{\sqrt[3]{27p^{27}}}$ (m) $\dfrac{-2x^0}{(-3x)^0} + \dfrac{(x^{-1})^2}{3x^{-2}}$

(n) $m^{a-b} \div m^{a+b}$ (o) $\sqrt[3]{3^{15}} + \left(4 - \sqrt{16}\right)^3 - \left(\sqrt{8}\right)^2$

(2) Solve for x:

(a) $8^{x-3} = 0{,}25$ (b) $3^{2x-3} = 1$

(c) $x^{\frac{2}{3}} = 25$ (d) $9^{2x} = 27^{x-1}$

(e) $3^2 \times 3^x = 3^{2-x}$ (f) $\dfrac{1}{8} = 4^x$

(g) $5^{x+2} = \sqrt{5}$ (h) $3x^2 = 12$

(3) Simplify and write the answer in scientific notation:

(a) $2(7{,}8 \times 10^5)$

(b) $450 \times 3\,000 \div 125$

(c) $(8{,}4 \times 10) \div (1{,}2 \times 10^{-1})$

(d) $(9{,}3 \times 10^2) + (2{,}5 \times 10^{-1})$

(e) $(5{,}4 \times 10^3) \times (2 \times 10^2)$

(f) $\dfrac{(8 \times 10^2)(2 \times 10^{-3})(3 \times 10^3)}{(6 \times 10^{-2})(5 \times 10^6)}$

(g) $\dfrac{(3 \times 10^{-2}) \times (6 \times 10^5)}{(2 \times 10) \div (4 \times 10^{-3})}$

(h) $\dfrac{(4{,}8 \times 10^5)(2{,}0 \times 10^2)}{(8 \times 10^{11})}$

(4) The speed of light is 3×10^8 m/s. If the sun is $1{,}5 \times 10^{11}$ kilometres from earth, how many seconds will it take a ray of sunlight to reach the earth? Write the answer in scientific notation.

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Chapter A3

Number patterns

A3.1 Completing the number patterns:

E.g. 1 Write the next four numbers in each sequence:

(a) 44 ; 39 ; 34 ; 29 ; ….. (b) 3 ; 4 ; 6 ; 9 ; 13 ; ……

****

(a) 44 ; 39 ; 34 ; 29 ; 24 ; 19 ; 14 ; 9 (b) 3 ; 4 ; 6 ; 9 ; 13 ; 18 ; 24 ; 31 ; 39

Exercise 1:

Write the next six numbers in each sequence:

(1) 8 ; 11 ; 14 ; …..

(2) 1 ; −2 ; 4 ; −8 ; …..

(3) 3 ; 3 ; 5 ; 7 ; 7 ; 9 ; 11 ; 11 ; …..

(4) 9 ; 16 ; 25 ; 36 ; …..

(5) 3 ; 5 ; 9 ; 15 ; 23 ; …..

(6) −10 ; −20 ; −40 ; −80 ; …..

(7) 4 011 ; 4 007 ; 4 003 ; …..

(8) 1 ; 1 ; 2 ; 3 ; 5 ; 8 ; …..

(9) 3 ; 6 ; 8 ; 16 ; 18 ; 36 ; 38 ; …..

(10) −37 ; −41 ; −45 ; …..

(11) 1 ; 2 ; 4 ; 5 ; 7 ; 8 ; 10 ; …..

(12) −1 ; −8 ; −27 ; −64 ; …..

E.g. 2 Write the next four numbers in each sequence and write the general rule in words:

(a) 8 ; 12 ; 16 ; 20 ; …… (b) 2 ; −4 ; 8 ; −16 ; ……

****

(a) 8 ; 12 ; 16 ; 20 ; 24 ; 28 ; 32; 36

Add 4.

(b) 2; −4 ; 8 ; −16 ; 32 ; −64 ; 128 ; −256

Multiply by −2.

Exercise 2:

(1) Write the next four numbers in each sequence and write the pattern in words:

(a) $259\,;\ 252\,;\ 245\,;\ 238\,;\ \ldots\ldots$

(b) $10\,;\ 10\,;\ 8\,;\ 8\,;\ 6\,;\ 6\,;\ \ldots\ldots$

(c) $5\,;\ 10\,;\ 20\,;\ 40\,;\ \ldots\ldots$

(d) $2\,;\ 5\,;\ 10\,;\ 17\,;\ 26\,;\ \ldots\ldots$

(e) $21\,875\,;\ 4\,375\,;\ 875\,;\ 175\,;\ \ldots\ldots$

(f) $-33\,;\ -34\,;\ -36\,;\ -39\,;\ \ldots\ldots$

(g) $8\,;\ -8\,;\ 8\,;\ -8\,;\ \ldots\ldots$

(h) $5\,;\ 8\,;\ 13\,;\ 21\,;\ 34\,;\ \ldots\ldots$

(i) $17\,;\ 26\,;\ 35\,;\ 44\,;\ \ldots\ldots$

(j) $4\,;\ 7\,;\ 13\,;\ 22\,;\ 34\,;\ \ldots\ldots$

(2) Fill in the missing numbers for each sequence:

(a) $\_\_\_\_\,;\ \_\_\_\_\,;\ \_\_\_\_\ ;\ 17\,;\ 20\,;\ 23\,;\ 26\,;\ \_\_\_\_$

(b) $-2\,;\ \_\_\_\_\,;\ -8\,;\ \_\_\_\_\,;\ -32\,;\ \_\_\_\_$

(c) $56\,;\ \_\_\_\_\,;\ 48\,;\ \_\_\_\_\,;\ 40\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ 24$

(d) $2\,;\ -1\,;\ \_\_\_\,;\ \_\_\_\_\,;\ -10\,;\ \_\_\_\_\,;\ -16\,;\ \_\_\_\_\,;\ \_\_\_\_$

(e) $\_\_\_\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ 9\,375\,;\ 46\,875\,;\ 234\,375\,;\ \_\_\_\_$

(f) $\_\_\_\_\,;\ \_\_\_\_\,;\ 5\,;\ 7\,;\ 4\,;\ 6\,;\ 3\,;\ \_\_\_\_\,;\ \_\_\_\_$

(g) $\_\_\_\_\,;\ 16\,;\ 54\,;\ 128\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ 1\,024$

(h) $\_\_\_\_\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ 16\,;\ 25\,;\ 32\,;\ 33\,;\ 64\,;\ 41\,;\ \_\_\_\_$

(i) $4\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ 60\,;\ \_\_\_\_\,;\ 74$

(j) $0\,;\ 3\,;\ 8\,;\ \_\_\_\_\,;\ \_\_\_\_\,;\ 35\,;\ 48\,;\ \_\_\_\_\,;\ \_\_\_\_$

A3.2 Tables:

E.g. 3 Complete the table by applying the given rule.

n	1	2	3	5	7	8	11	20
$n^2 - 5$	-4	-1	4	20	44	59	116	395

E.g. 4 Write the relation between row 1 and row 2 and then complete the table.

n	1	2	3	6	7	8	9	20
T_n	-3	-6	-9	-18	-21	-24	-27	-60

Relation/rule: $T_n = -3n$ $\therefore n$ is multiplied by -3 each time to obtain the value of T_n!

E.g. 5 Complete the following table and answer the questions:

(a)

Position in sequence	1	2	3	4		7	9	12
Term	3	7	11		19			

(b) Describe the pattern in words.

(c) If the position of the term in the sequence is represented by n, write the general term in terms of n.

(d) Determine the 64^{th} term.

****

T_0

(a)

Position in sequence	1	2	3	4	5	7	9	12
Term	3	7	11	15	19	27	35	47
	$\leftarrow -4$	$\leftarrow -4$						

(b) Add 4.

(c) General term (T_n) = $4n - 1$. The 4 is the constant difference between the terms.
The constant (-1) is determined by substitution $\rightarrow T_1 = 4(1) = 4 - 1 = 3$
Or the constant (-1) can be obtained by calculating T_0: $T_0 = 3 - 4 = -1$

(d) For 64^{th} term, n should be substituted by 64:
$\therefore$ Term 64: $T_{64} = 4(64) - 1 = 256 - 1 = 255$

Exercise 3:

(1) Redraw the tables and complete:

(a)

n	1	2	3	4		9	11
$n - 4$	-3	-2			3		

(b)

m	1	2	4		15	20	42
$-2m$	-2	-4		-12			

(c)

t	1	2	3		7		12
$2t - 1$	1	3		11		21	

(d)

k	-3	-2	-1	0	1	7	9
$k + 2$							

(e)

n							
$n \div 2$	1	2	3	7	13	34	50

(f)

y	1	2	3		8	12	
$3y + 2$	5			23			62

(g)

x	1	2	3	4		8	9
$-x^3$	-1	-8			-125		

(h)

n	1	2	3			12
$n^2 + 3$	4		12	28	67	

(i)

n	-3	-2	0		12	33	50
$5 - n$	8	7		-3			

(j)

n	2	4		8		12
$-3 \times n$	-6		-18		-33	

(2) Write the relation between n and T_n in the following tables and then complete the tables:

(a)

n	1	2	3	4	5	6	10
T_n	-2	-4	-6				

(b)

n	1	2	3	5	7	11	20
T_n		4	7	13			

(c)

n	1	2	3	7	8	11	12
T_n	5	8	13	53			

(d)

n	1	2	3	4	5	6	
T_n	-3	-5	-7				-21

(e)

n	1	2	3	4	5	
T_n	17	20	23			176

(f)

n	1	3	5	7			30
T_n	7	9	11		15	20	

(g)

n	1	2	3	6	8	10	15
T_n	25	50	75				

(h)

n	1	2	3	4	5	7	11
T_n	-1	-8	-27	-64			

(i)

n	1	2	3	4	5	6	8
T_n	2	6	12	20			

(j)

n	1	2	3	4	6		11
T_n	-3	-6	-9	-12		-21	

(3) Complete the following table and answer the questions:

(a)

Position in sequence	1	2	3	4		10	15	20
Term	-5	-8	-11		-23			

(b) Describe the pattern in words.
(c) Determine T_n, the general term.
(d) Determine the 50^{th} term.
(e) Which term in the sequence is equal to -101?

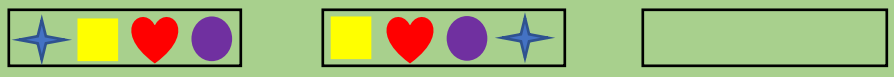
(4) Complete the following table and answer the questions:

(a)

Position in sequence	1	2	3	4	10		15	20
Term	4	7	12	19		147		

(b) Describe the pattern in words.
(c) Determine T_n, the general term.
(d) Determine the 80^{th} term.
(e) Which term in the sequence is equal to 228?

☺ Complete the next pattern in each sequence:

(1) 

(2) 

(3) 

A3.3 Other number patterns:

Exercise 4:

(1) The general term for a sequence is $T_n = n^3 + 1$

(a) Determine the first seven terms of the sequence.

(b) Is the first difference constant?

(c) Is the second difference constant?

(d) Which difference is constant – if applicable?

(e) Determine the 20^{th} term of the sequence.

(f) Determine the general term of the second difference of the above sequence as T_n = …….

(2) Determine the general rule for each of the following sequences. Give special attention to the difference between the terms as well as the second differences (difference between the first differences).

(a) (i) 2; 6; 12; 20; ……

(ii) 0; 2; 6; 12; ……

(iii) 3; 7; 13; 21; ……

(b) What is the similarity between the sequences in (a)?

(3) Consider the following number patterns:

(i) Sequence 1

(ii) Sequence 2

(iii) Sequence 3 = $3^2 + 4^2 + 12^2 = 13^2$

(iv) Sequence 4 = $4^2 + 5^2 + 20^2 = 21^2$

(v) Sequence 5 = $5^2 + 6^2 + 30^2 = 31^2$

(vi) Sequence 6 = $6^2 + 7^2 + 42^2 = 43^2$

(a) Complete sequence 1 and sequence 2.

(b) Describe the pattern in words.

(c) Complete sequence 7 and sequence 8 of the pattern.

(d) Determine the general term as T_n = …...

(4)

Pattern 1

Pattern 2

Pattern 3

Pattern 4

Pattern x

(a) Use the patterns above and complete the following table:

Pattern	1	2	3	4	5	6	x
Number of black Δ^s							
Number of white Δ^s							

(b) Describe, in words, the sequences for the number of black Δ^s and the number of white Δ^s.

(c) Determine the value of x.

A3.4 REVISION EXERCISE:

(1) Complete the next four terms in each of the following sequences. Also describe the pattern of the sequence:

(a) $102\,400;\ 25\,600;\ 6\,400;\ \ldots\ldots$

(b) $-12;\ -23;\ -34;\ \ldots\ldots$

(c) $6;\ 9;\ 14;\ 21;\ \ldots\ldots$

(d) $1;\ 25;\ 3;\ 19;\ 5;\ 13;\ \ldots\ldots$

(2) Consider the following:

$$\begin{aligned} 11 \times 1 - 1 &= 10 \\ 11 \times 2 - 2 &= 20 \\ 11 \times 3 - 3 &= 30 \\ &\Downarrow \\ 11 \times 9 - 9 &= \text{(i)} \\ 11 \times 12 - 12 &= \text{(ii)} \\ 11 \times 18 - 18 &= \text{(iii)} \end{aligned}$$

(a) Determine (i) - (iii), without using a calculator.
(b) Determine the general rule of the n^{th} term.
(c) Describe the pattern in words. Explain this pattern.

(3) The fifth, seventh and ninth terms of a sequence are $14;\ 22$ and 30 respectively.

(a) Does this sequence have a constant difference? If so, what is the constant difference?
(b) Complete the sequence from term 1 up to term 10.
(c) Determine T_n, the general term.
(d) Which term in the sequence will be equal to 94?

(4) Consider the following sequence:

$\ldots\ldots\ldots\ldots\ldots\ldots;\ 8;\ 13;\ 21;\ 34;\ 55;\ \ldots\ldots\ldots\ldots\ldots\ldots$

(a) What is the name of this number pattern?
(b) Complete the five terms before the 8 as well as the next five terms after the 55.

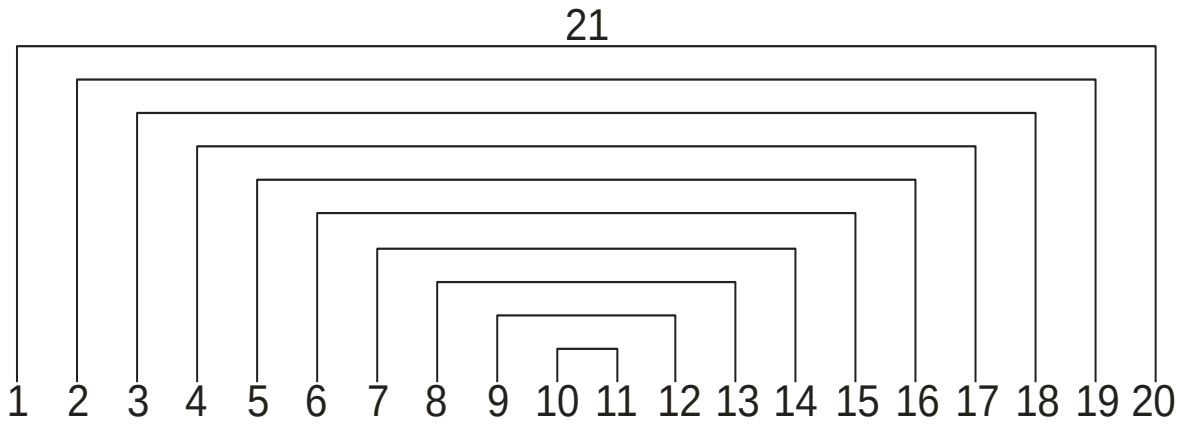
(5) Complete the following table and answer the questions:

(a)

Position in sequence	1	2	3	7	11		35
Term	-5	-8	-11			-86	

(b) Describe the pattern in words.
(c) Determine T_n, the general term.
(d) Determine the 100^{th} term.

(6)



The sum of two terms is determined each time in the diagram above.
For example: $1 + 20 = 21 \rightarrow$ as indicated in the diagram.

(a) Find the missing sums in the diagram – thus add term 2 and the second last term, term 3 and the third last term, etc.

(b) Without using a calculator, calculate the sum of all the natural numbers from 1 up to 20. Try making use of the answers in (a).

(7) Consider the following number pattern:

$9^2 = 81$
$99^2 =$ (i)
$999^2 =$ (ii)
$9999^2 =$ (iii)
$\Downarrow$
$99999^2 =$ (iv)
$99999999^2 =$ (v)

(a) Complete (i) - (iii) by using a calculator.
(b) Determine the values of (iv) and (v) without using a calculator.

(8) Complete the missing numbers and then determine the general term T_n:

(a) ____; ____; 5; 11; 17; 23; ……

(b) 8; ____; ____; -4; ……

(c) ____; 16; ____; 0; ____; -16; ……

(d) ____; 326; 346; 366; 386; ……

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