Grade 10 – Book A

(Revised edition – CAPS)

TEACHER GUIDELINES

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This book was compiled and processed by E.J. Du Toit in 2011.
Revised edition 2023.

Website: www.abcbooks.co.za

ISBN 978-1-919957-09-8

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Chapter A1

Number systems

NO CALCULATOR MAY BE USED IN THIS CHAPTER!

A1.1 Number systems:

Exercise 1: Date: _____

Complete:

* Natural numbers: $\mathbb{N} = \{1\ ;\ 2\ ;\ 3\ ;\ \ldots\ldots..\}$
* Whole numbers: $\mathbb{N}_0 = \{0\ ;\ 1\ ;\ 2\ ;\ 3\ ;\ \ldots\ldots..\}$
* Integers: $\mathbb{Z} = \{\ldots\ ;\ -2\ ;\ -1\ ;\ 0\ ;\ 1\ ;\ 2\ ;\ 3\ ;\ \ldots\}$
* Rational numbers: $\mathbb{Q} = \left\{\frac{a}{b}\ /\ a, b \in \mathbb{Z}; b \neq 0\right\}$

A1.2 Rational numbers:

A1.2.1 Equivalent fractions:

E.g.1 Write down two equivalent fractions for $\frac{2}{3}$:

$\frac{2\times3}{3\times3} = \frac{6}{9}$ **or** $\frac{2\times5}{3\times5} = \frac{10}{15}$

Exercise 2: Date: _____

These are possible answers!

(1) Write down three equivalent fractions for each of the following rational numbers:

(a) $\frac{-1}{4} = \frac{-2}{8} = \frac{-3}{12} = \frac{-10}{40}$

(b) $\frac{3}{7} = \frac{6}{14} = \frac{9}{21} = \frac{15}{35}$

(c) $\frac{1}{6} = \frac{2}{12} = \frac{10}{60} = \frac{100}{600}$

(d) $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{20}{30}$

(e) $\frac{12}{14} = \frac{6}{7} = \frac{24}{28} = \frac{120}{140}$

(f) $\frac{-36}{-9} = \frac{-4}{-1} = \frac{4}{1} = \frac{12}{3}$

(g) $2\frac{6}{11} = \frac{28}{11} = 2\frac{12}{22} = 2\frac{60}{110}$

(h) $5 = \frac{5}{1} = \frac{10}{2} = \frac{25}{5}$

(2) Are the following equivalent fractions or not? (Answer yes or no only.)

(a) $\frac{12}{5} = \frac{24}{10}$? : **Yes**

(b) $\frac{7}{3} = \frac{3}{7}$? : **No**

(c) $\frac{3}{-2} = \frac{6}{4}$? : **No**

(d) $\frac{3}{-5} = \frac{-9}{15}$? : **Yes**

(e) $\frac{2}{3} = \frac{4}{9}$? : **No**

(f) $\frac{3}{1} = \frac{48}{16}$? : **Yes**

(g) $\frac{4}{3} = \frac{-12}{-9}$? : **Yes**

(h) $\frac{25}{10} = \frac{5}{2}$? : **Yes**

(i) $\frac{5}{4} = \frac{4}{3}$? : **No**

A1.2.2 Order of rational numbers:

E.g.2 (a) Arrange the following fractions in ascending order: $\frac{1}{2}$; $\frac{3}{4}$ and $\frac{2}{3}$:

$\frac{1}{2} = \frac{6}{12}$; $\frac{3}{4} = \frac{9}{12}$ and $\frac{2}{3} = \frac{8}{12}$

$\therefore \frac{1}{2} < \frac{2}{3} < \frac{3}{4}$

(b) Write down a rational number between $\frac{3}{4}$ and $\frac{1}{3}$:

$\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{3} = \frac{4}{12}$

$\therefore \frac{1}{3} < \frac{5}{12}$ or $\frac{6}{12}$ or $\frac{7}{12}$ or $\frac{8}{12} < \frac{3}{4}$

Exercise 3: Date: _____

(1) Arrange the following fractions in ascending order:

(a) $\frac{3}{4}$; $\frac{2}{3}$ and $\frac{4}{5}$: $\frac{3}{4} = \frac{45}{60}$; $\frac{2}{3} = \frac{40}{60}$; $\frac{4}{5} = \frac{48}{60}$ $\therefore \frac{2}{3} < \frac{3}{4} < \frac{4}{5}$

(b) $\frac{2}{3}$; $\frac{5}{7}$ and $\frac{4}{6}$: $\frac{2}{3} = \frac{28}{42}$; $\frac{5}{7} = \frac{30}{42}$; $\frac{4}{6} = \frac{28}{42}$ $\therefore \frac{2}{3} = \frac{4}{6} < \frac{5}{7}$

(2) Arrange the following fractions in descending order:

(a) $\frac{5}{8}$; $\frac{2}{3}$ and $\frac{3}{4}$: $\frac{5}{8} = \frac{15}{24}$; $\frac{2}{3} = \frac{16}{24}$; $\frac{3}{4} = \frac{18}{24}$ $\therefore \frac{3}{4} > \frac{2}{3} > \frac{5}{8}$

(b) $-1\frac{1}{2}$; $-1\frac{2}{3}$ and $\frac{-7}{5}$: $\frac{-3}{2} = \frac{-45}{30}$; $\frac{-5}{3} = \frac{-50}{30}$; $\frac{-7}{5} = \frac{-42}{30}$ $\therefore \frac{-7}{5} > -1\frac{1}{2} > -1\frac{2}{3}$

(3) Place a rational number between each of the following numbers:

(a) $\frac{-1}{3}$ and $\frac{-3}{5}$: $\frac{-1}{3} = \frac{-5}{15}$ and $\frac{-3}{5} = \frac{-9}{15}$ $\therefore \frac{-3}{5} < \frac{-8}{15}$ or $\frac{-7}{15}$ or $\frac{-6}{15} < \frac{-1}{3}$

(b) $\frac{3}{4}$ and $\frac{7}{10}$: $\frac{3}{4} = \frac{15}{20} = \frac{30}{40}$ and $\frac{7}{10} = \frac{14}{20} = \frac{28}{40}$ $\therefore \frac{7}{10} < \frac{29}{40} < \frac{3}{4}$

A1.2.3 Conversion of common fractions to decimal fractions:

E.g.3 Express the following as decimal fractions, without using a calculator:

(a) $\frac{3}{8} = \frac{3,000\ldots}{8} = \frac{3,^{3}0^{6}0^{4}0}{8} = 0,375$ **(b)** $1\frac{2}{3} = 1\frac{2,000\ldots}{3} = 1\frac{2,^{2}0^{2}0^{2}0\ldots}{3} = 1,66\ldots = 1,\dot{6}$

Exercise 4: Date: _____

Express the following as decimal fractions, without using a calculator:

(1) $\frac{22}{7} = \frac{22{,}000\ldots}{7} = \frac{22,{}^{1}0\,{}^{3}0\,{}^{2}0}{7} = 3,142\ldots$

(2) $4\frac{2}{3} = 4\frac{2,{}^{2}0\,{}^{2}0\,{}^{2}0}{3} = 4,66\ldots \approx 4,\dot{6}$

(3) $\frac{1}{8} = \frac{1{,}000\ldots}{8} = \frac{1,{}^{1}0\,{}^{2}0\,{}^{4}0}{8} = 0,125$

(4) $\frac{7}{9} = \frac{7{,}000\ldots}{9} = \frac{7,{}^{7}0\,{}^{7}0\,{}^{7}0}{9} = 0,77\ldots \approx 0,\dot{7}$

(5) $\frac{17}{25} = \frac{17{,}000\ldots}{25} = \frac{17,{}^{17}0\,{}^{20}0}{25} = 0,68$

(6) $\frac{5}{100} = \frac{5{,}000\ldots}{100} = \frac{5,{}^{5}0\,{}^{50}0}{100} = 0,05$

(7) $\frac{4}{11} = \frac{4,{}^{4}0\,{}^{7}0\,{}^{4}0\,{}^{7}0\,{}^{4}0}{11} = 0,3636\ldots \approx 0,\dot{3}\dot{6}$

(8) $-2\frac{6}{7} = -2\frac{6,{}^{6}0\,{}^{4}0\,{}^{5}0}{7} = -2,857\ldots$

(9) $-5\frac{5}{6} = -5\frac{5,{}^{5}0\,{}^{2}0\,{}^{2}0}{6} = -5,833\ldots \approx -5,8\dot{3}$

(10) $\frac{33}{8} = \frac{33{,}000\ldots}{8} = \frac{33,{}^{1}0\,{}^{2}0\,{}^{4}0}{8} = 4,125$

A1.2.4 Rounding off decimal fractions:

E.g.4 Round off the following fractions correct to the number of decimals indicated in brackets:

(a) 4,34712 (to 3 dec)

= 4,34712 Consider the underlined number

≈ 4,347

(b) 290,09832 (to 2 dec)

= 290,09832

≈ 290,10

Exercise 5: Date: _____

(1) Round off the following fractions correct to the number of decimals indicated in brackets:

(a) 3,573 (to 2 dec) ≈ 3,57

(b) 12,00873 (to 3 dec) ≈ 12,009

(c) 0,00384 (to 5 dec) = 0,00384

(d) 7,3226 (to 1 dec) ≈ 7,3

(e) 8,39999 (to 1 dec) ≈ 8,4

(f) 90,9023 (to the nearest integer) ≈ 91

(g) 0,433218 (to 4 dec) ≈ 0,4332

(h) 1 456,6799 (to 3 dec) ≈ 1 456,680

(i) 66,666 (to 2 dec) ≈ 66,67

(j) 13,00034 (to 3 dec) ≈ 13,000

(2) Consider the following and choose the correct way of rounding off in brackets:

(a) 3,47653 ≈ 3,477 correct to the nearest (tenth, hundredth or **thousandth**)

(b) 96 995,31956 ≈ 96 995,32 correct to the nearest (tenth, **hundredth** or thousandth)

A1.2.5 Conversion of decimal fractions to common fractions:

E.g.5 Express the following as common fractions in its simplest form:

(a) $4,5 = 4\frac{5}{10}\left(\div\frac{5}{5}\right) = 4\frac{1}{2}$ **(b)** $-0,12 = -\frac{12}{100}\left(\div\frac{4}{4}\right) = -\frac{3}{25}$

Exercise 6: Date: _____

Express the following as common fractions in its simplest form:

(1) $0,125 = \frac{125}{1\,000} = \frac{125}{1\,000} \div \frac{125}{125}$ $= \frac{1}{8}$

(2) $1,25 = 1\frac{25}{100} = 1\frac{25}{100} \div \frac{25}{25}$ $= 1\frac{1}{4}$

(3) $14,6 = 14\frac{6}{10} = 14\frac{6}{10} \div \frac{2}{2}$ $= 14\frac{3}{5}$

(4) $-0,5 = -\frac{5}{10} = -\frac{5}{10} \div \frac{5}{5}$ $= -\frac{1}{2}$

(5) $-1,2 = -1\frac{2}{10} = -1\frac{2}{10} \div \frac{2}{2}$ $= -1\frac{1}{5}$

(6) $23,5 = 23\frac{5}{10} = 23\frac{5}{10} \div \frac{5}{5}$ $= 23\frac{1}{2}$

(7) $3,04 = 3\frac{4}{100} = 3\frac{4}{100} \div \frac{4}{4}$ $= 3\frac{1}{25}$

(8) $7,3 = 7\frac{3}{10}$

(9) $100,75 = 100\frac{75}{100} = 100\frac{75}{100} \div \frac{25}{25}$ $= 100\frac{3}{4}$

(10) $0,00005 = \frac{5}{100\,000} = \frac{5}{100\,000} \div \frac{5}{5}$ $= \frac{1}{20\,000}$

A1.2.6 Conversion of recurring fractions to common fractions:

E.g.6 Convert the following to common fraction in its simplest form:

(a) $0,\dot{1} = \frac{1}{9}$; $0,\dot{3} = \frac{3}{9} = \frac{1}{3}$; $0,\dot{5} = \frac{5}{9}$; $0,\dot{8} = \frac{8}{9}$

(b) $3,\dot{2}\dot{4} = 3\frac{24}{99} = 3\frac{8}{33}$; $0,\dot{4}\dot{2}\dot{1} = \frac{421}{999}$; $15,\dot{1}\dot{6}\dot{5}\dot{3} = 15\frac{1653}{9999} = 15\frac{551}{3333}$

(c) $0,0\dot{3} = 0,\dot{3} \div 10 = \frac{3}{9} \div \frac{10}{1} = \frac{3}{9} \times \frac{1}{10} = \frac{3}{90} = \frac{1}{30}$

(d) $0,00\dot{4}\dot{6} = 0,\dot{4}\dot{6} \div 100 = \frac{46}{99} \div \frac{100}{1} = \frac{46}{99} \times \frac{1}{100} = \frac{46}{9900} = \frac{23}{4950}$

(e) $0,5\dot{7} = 0,5 + 0,0\dot{7} = 0,5 + 0,\dot{7} \div 10 = \frac{5}{10} + \frac{7}{9} \times \frac{1}{10} = \frac{5 \times 9}{10 \times 9} + \frac{7}{90} = \frac{45+7}{90} = \frac{52}{90} = \frac{26}{45}$

Exercise 7: Date: _____

Convert the following to common fractions in its simplest form: (Without a calculator.)

(1) $3,\dot{6}$

$= 3\frac{6}{9}$

$= 3\frac{6}{9} \div \frac{3}{3}$

$= 3\frac{2}{3}$

(2) $0,\dot{1}\dot{3}$

$= \frac{13}{99}$

(3) $22,3\dot{9}$

$= 22 + 0,3 + 0,0\dot{9}$

$= 22 + \frac{3}{10} + 0,\dot{9} \div 10$

$= 22 + \frac{3}{10} + \frac{9}{9} \times \frac{1}{10}$

$= 22 + \frac{3}{10} + 1 \times \frac{1}{10}$

$= 22 + \frac{3}{10} + \frac{1}{10}$

$= 22 + \frac{3+1}{10} = 22\frac{4}{10}$

$= 22\frac{2}{5}$

(4) $-1,\dot{1}\dot{3}\dot{5}$ or $-1,\overline{135}$

$= -1\frac{135}{999}$

$= -1\frac{135}{999} \div \frac{27}{27}$

$= -1\frac{5}{37}$

(5) $0,\dot{7}$

$= \frac{7}{9}$

(6) $0,00\dot{3}$

$= 0,\dot{3} \div 100$

$= \frac{3}{9} \times \frac{1}{100} = \frac{3}{900}$

$= \frac{1}{300}$

(7) $1,\overline{214}$

$= 1\frac{214}{999}$

(8) $3,2\dot{5}\dot{8}$

$= 3 + 0,2 + 0,0\dot{5}\dot{8}$

$= 3 + \frac{2}{10} + 0,\dot{5}\dot{8} \div 10$

$= 3 + \frac{2}{10} + \frac{58}{99} \times \frac{1}{10}$

$= 3 + \frac{2}{10} \times \frac{99}{99} + \frac{58}{990}$

$= 3 + \frac{198}{990} + \frac{58}{990} = 3\frac{256}{990}$

$= 3\frac{128}{495}$

☺ **Calculate the following without using a calculator:** $0,\dot{4} + \frac{2}{3}$

$$0,\dot{4} + \frac{2}{3} = \frac{4}{9} + \frac{2}{3}$$
$$= \frac{4}{9} + \frac{2}{3} \times \frac{3}{3}$$
$$= \frac{4}{9} + \frac{6}{9} = \frac{10}{9} = 1\frac{1}{9}$$

A1.3 Irrational and Real numbers:

Irrational numbers cannot be expressed as a ratio between two integers. These numbers are non-terminating and non-recurring decimals.

E.g. 7 Irrational numbers:

- $\sqrt{2}$ or $\sqrt{7}$ or $\sqrt{\frac{3}{4}}$ etc. because 2 ; 7 and 3 are not perfect squares!
- $\sqrt[3]{12}$ or $\sqrt[3]{100}$ etc. because 12 and 100 are not perfect cubes!

Whereas the following numbers are rational numbers:

- $\sqrt{4}$ or $\sqrt{0,01}$ or $\sqrt{\frac{25}{9}}$ etc. because 4 ; 0,01 ; 25 and 9 are perfect squares!
- $\sqrt[3]{27}$ or $\sqrt[3]{125}$ etc. because 27 and 125 are perfect cubes!

The real numbers, $\mathbb{R}$ consist of the rational numbers, $\mathbb{Q}$ and the irrational numbers, $\mathbb{Q}'$. Remember that all terminating and recurring decimals are rational numbers.

E.g.8 Determine the two integers between which the irrational number $\sqrt{7}$ lies.

****

Choose the two perfect squares on either side of 7:

$$\sqrt{4} < \sqrt{7} < \sqrt{9}$$
$$\therefore 2 < \sqrt{7} < 3$$

Exercise 8: Date: _____

(1) Which of the numbers are Rational numbers ($\mathbb{Q}$) and which are Irrational numbers ($\mathbb{Q}'$)?

(a) 14 : $\mathbb{Q}$ (b) $\frac{1}{5}$: $\mathbb{Q}$ (c) $\sqrt{81} = 9$: $\mathbb{Q}$

(d) 0,12 : $\mathbb{Q}$ (e) $\sqrt{18}$: $\mathbb{Q}'$ (f) $12,\dot{2}\dot{3}$: $\mathbb{Q}$

(g) $-\sqrt{\frac{12}{3}} = -\sqrt{4} = -2$: $\mathbb{Q}$ (h) 0,2945 ... : $\mathbb{Q}'$ (i) $\sqrt[3]{64} = 4$: $\mathbb{Q}$

(j) π : $\mathbb{Q}'$ (k) $\sqrt[5]{32} = 2$: $\mathbb{Q}$ (l) $\frac{11}{7}$: $\mathbb{Q}$

(2) Between which two integers do the following irrational numbers lie?

(a) $-\sqrt{12}$

$-\sqrt{16} < -\sqrt{12} < -\sqrt{9}$

$-4 < -\sqrt{12} < -3$

(b) $\sqrt{66}$

$\sqrt{64} < \sqrt{66} < \sqrt{81}$

$8 < \sqrt{66} < 9$

(c) $\sqrt[3]{5}$

$\sqrt[3]{1} < \sqrt[3]{5} < \sqrt[3]{8}$

$1 < \sqrt[3]{5} < 2$

(d) $\sqrt[5]{2}$

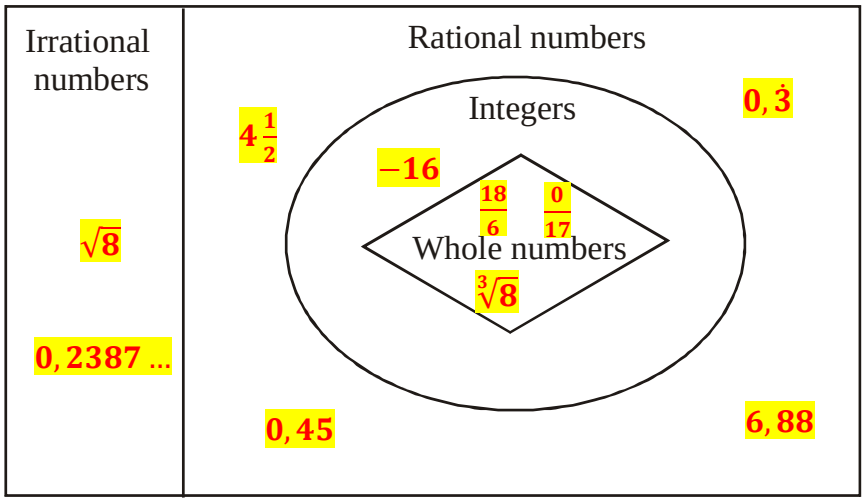
$\sqrt[5]{1} < \sqrt[5]{2} < \sqrt[5]{32}$

$1 < \sqrt[5]{2} < 2$

(3) The diagram below is a summary of all the numbers that are used on school level.
Place the following numbers in the right place on the table; simplify the number if necessary:

$4\frac{1}{2}$; $\sqrt[3]{8} = 2$; $\sqrt{8}$; -16 ; $0{,}45$; $0{,}\dot{3}$; $\frac{18}{6} = 3$; $0{,}2387\ldots$; $\frac{0}{17} = 0$; $6{,}88$

Real numbers:



☺ **(1) Except for the real numbers we also have the non-real numbers.**
Give an example of a non-real number. $\sqrt{-1}$; $\sqrt[3]{-2}$; ... ...

(2) What is the set called that contain all real and non-real numbers?

Complex numbers

A1.4 Representation of sets of numbers:

Sets of numbers can be represented or written in the following ways:

A1.4.1 Set builder notation:

E.g. 9 Write the following sets of numbers in set builder notation:

(a) All natural numbers greater than 6: $\{x \,/\, x > 6\,; x \in \mathbb{N}\}$

(b) All real numbers between -2 and 5: $\{m : -2 < m < 5\ ;\ m \in \mathbb{R}\}$

A1.4.2 Interval notation:

Only sets that form part of real numbers can be represented using interval notation!

E.g. 10 Write the following in interval notation:

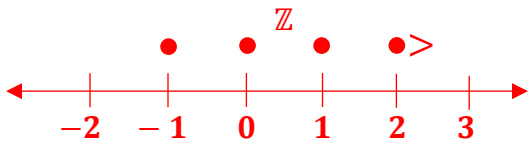
(a) The real numbers between -2 and 4, including 4: $x \in (-2\,; 4]$ **Open, closed interval!**

(b) $\{m \,/\, m > 7;\ m \in \mathbb{R}\}$: $m \in (7\,; \infty)$ **Open interval!**

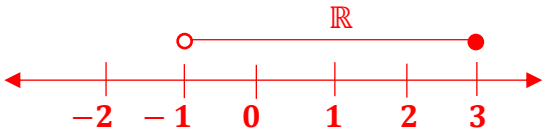
A1.4.3 Number lines:

E.g. 11 Represent the following on a number line:

(a) $\{-1\,;\ 0\,;\ 1\,;\ 2\,;\ \ldots\ldots\ldots\}$



(b) $\{x : -1 < x \leq 3\ ;\ x \in \mathbb{R}\}$



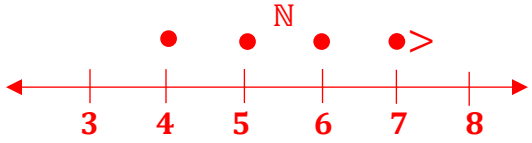
A1.4.4 Solving of linear inequalities:

E.g. 12 Solve for x in each of the following and represent the solution on a number line:

(a) $x - 2 \geq 2$ **if** $x \in \mathbb{N}$

$x \geq 2 + 2$

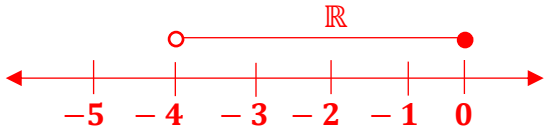
$x \geq 4$



(b) $-3 < x + 1 \leq 1$ **if** $x \in \mathbb{R}$

$-3 - 1 < x \leq 1 - 1$

$-4 < x \leq 0$

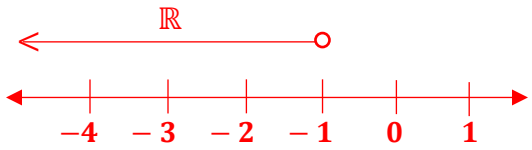


Exercise 9: Date: _____

(1) Write the following in interval notation (if applicable) and represent it on a number line:

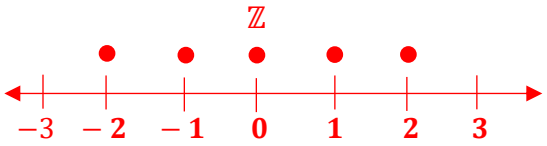
(a) $\{x : x < -1;\ x \in \mathbb{R}\}$

$x \in (-\infty\ ;\ -1)$



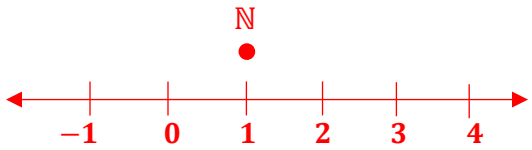
(b) $\{x : -3 < x < 3;\ x \in \mathbb{Z}\}$

No interval notation



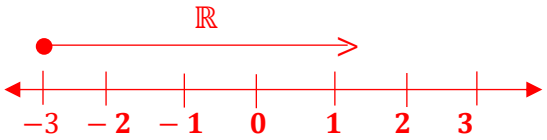
(c) $\{y : y < 2\ ;\ y \in \mathbb{N}\}$

No interval notation



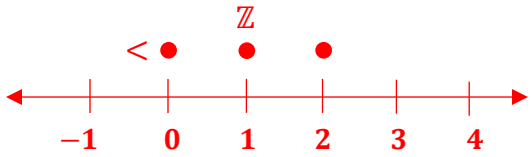
(d) $\{x : x \geq -3;\ x \in \mathbb{R}\}$

$x \in [-3\ ;\ \infty)$



(e) $\{x\ /\ x < 3;\ x \in \mathbb{Z}\}$

No interval notation

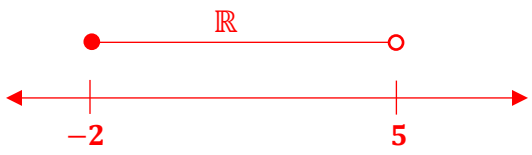


(f) $\{p\ /\ p \geq \frac{-5}{2};\ p \in \mathbb{R}\}$

$p \in [-\frac{5}{2}\ ;\ \infty)$

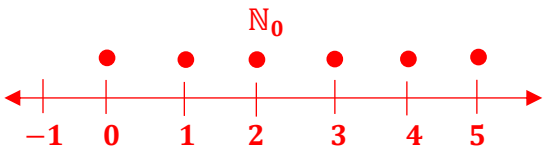
(g) $\{m : -2 \leq m < 5;\ m \in \mathbb{R}\}$

$m \in [-2\ ;\ 5)$



(h) $\{x : x \leq 5;\ x \in \mathbb{N}_0\}$

No interval notation

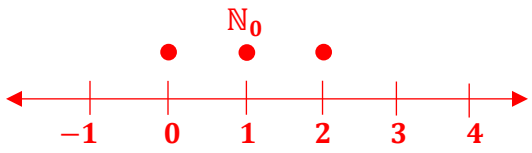


(2) Solve for x in each of the following and represent the solution on a number line:

(a) $x + 1 \leq 3;\quad x \in \mathbb{N}_0$

$x \leq 3 - 1$

$x \leq 2$



(b) $2x \geq -8;\quad x \in \mathbb{R}$

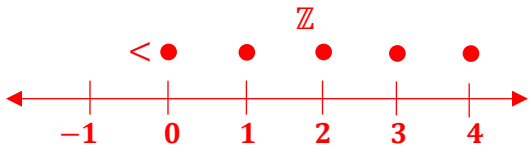
$x \geq \frac{-8}{2}$

$x \geq -4$

(c) $x - 4 \leq 0; \quad x \in \mathbb{Z}$

$x \leq 0 + 4$

$x \leq 4$

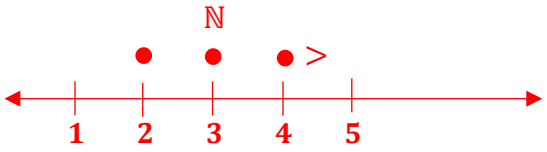


(d) $2x + 3 > 7; \quad x \in \mathbb{N}$

$2x > 7 - 3$

$x \geq \frac{4}{2}$

$x \geq 2$



(e) $-6 < x - 1 \leq 6; \quad x \in \mathbb{R}$

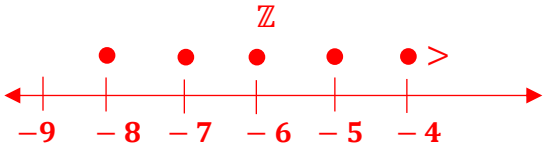
$-6 + 1 < x \leq 6 + 1$

$-5 < x \leq 7$

(f) $x + 7 \geq -1; \quad x \in \mathbb{Z}$

$x \geq -1 - 7$

$x \geq -8$



A1.5 REVISION EXERCISE:

Date: _____

(1) Convert the following to common fractions in its simplest form: (Without a calculator.)

(a) $14,1\dot{7}$

$= 14 + 0,1 + 0,0\dot{7}$

$= 14 + \frac{1}{10} + 0,\dot{7} \div 10$

$= 14 + \frac{1}{10} + \frac{7}{9} \times \frac{1}{10}$

$= 14 + \frac{1}{10} + \frac{7}{90}$

$= 14 + \frac{1 \times 9}{10 \times 9} + \frac{7}{90}$

$= 14 + \frac{9}{90} + \frac{7}{90} = 14\frac{16}{90}$

$= 14\frac{8}{45}$

(b) $0,\overline{1234}$

$= \frac{1\,234}{9\,999}$

(c) $4,68$

$= 4\frac{68}{100}$

$= 4\frac{68}{100} \div \frac{4}{4}$

$= 4\frac{17}{25}$

(d) $5,\dot{1}$

$= 5\frac{1}{9}$

(2) Indicate, by using a ✓, all the rational numbers between 0 and 10:

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
$\sqrt{9} = 3$	-1	$\sqrt{8}$	$\frac{6}{3} = 2$	$\sqrt[3]{16}$	π	$\frac{0}{3} = 0$	$\sqrt{144} = 12$	4,124….	$\sqrt{\frac{24}{6}} = \sqrt{4} = 2$
✓			✓						✓

(3) Round off the following fractions correct to the number of decimals indicated in brackets:

(a) 7,199 (to 1 dec)

$\approx 7,2$

(b) 0,048561 (to 4 dec)

$\approx 0,0486$

(c) 234,34 (to 1 dec)

$\approx 234,3$

(d) 1 001,1989 (to the nearest integer)

$\approx 1\ 001$

(e) 3,997 (to 2 dec)

$\approx 4,00$

(f) 23,712 (to the nearest integer)

≈ 24

(4) Place any two irrational numbers between 2 and 3.

$2 = \sqrt{4}$ and $3 = \sqrt{9}$

$\therefore\ 2 < \sqrt{5} < \sqrt{6} < \sqrt{7} < \sqrt{8} < 3$

Any 2!

(5) Between which two integers do the following irrational numbers lie?

(a) $\sqrt{\frac{1}{2}}$

$\sqrt{\frac{1}{1}} > \sqrt{\frac{1}{2}} > \sqrt{\frac{1}{4}}$

$1 > \sqrt{\frac{1}{2}} > \frac{1}{2}$

or $\frac{1}{2} < \sqrt{\frac{1}{2}} < 1$

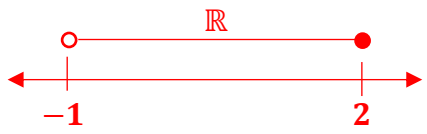
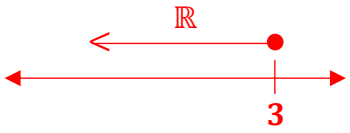
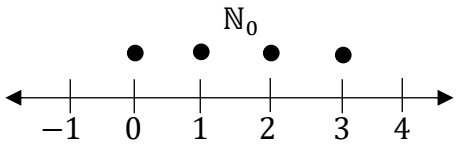
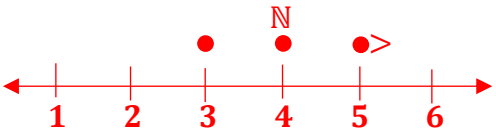
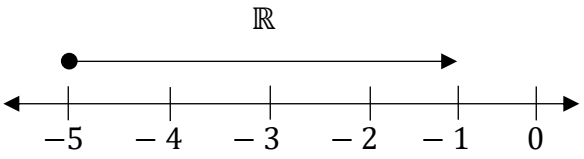
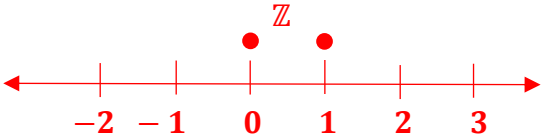
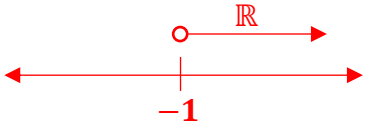
(b) $\sqrt[3]{57}$

$\sqrt[3]{27} < \sqrt[3]{57} < \sqrt[3]{64}$

$3 < \sqrt[3]{57} < 4$

$4 > \sqrt[3]{57} > 3$

(6) Complete the missing representations in the table below:

	Set builder notation:	Interval notation:	Number line:
(1)	$\{x \,/\, -1 < x \leq 2;\ x \in \mathbb{R}\}$	$x \in (-1\,;\,2]$	
(2)	$\{x : -2 \leq x \leq 5\,;x \in \mathbb{R}\}$	$x \in [-2\,;5]$	
(3)	$\{y \,/\, y \leq 3\,;y \in \mathbb{R}\}$	$y \in (-\infty\,;3]$	
(4)	$\{x \,/\, x \leq 3\,;x \in \mathbb{N}_0\}$	N/A	
(5)	$\{y \,/\ y \geq 3;\ y \in \mathbb{N}\}$	N/A	
(6)	$\{m : 0 < m \leq 4\,;m \in \mathbb{R}\}$	$m \in (0\,;4]$	
(7)	$\{t : t \geq -5\,;t \in \mathbb{R}\}$	$t \in [-5\,;\infty)$	
(8)	$\{m : m \leq 6;\ m \in \mathbb{R}\}$	$m \in (-\infty\,;6]$	
(9)	$\{x \,/\, -1 < x < 2;\ x \in \mathbb{Z}\}$	N/A	
(10)	$\{x \,/\, x > -1\,;x \in \mathbb{R}\}$	$x \in (-1;\ \infty)$	

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Chapter A2

Algebraic expressions

A2.1 Products:

A2.1.1 The law of distribution:

E.g. 1 Determine the following products by using the law of distribution:

(a) $(x - 2)(x + 2)$ (b) $(3m + n)(2m + 5n)$

****

(a) $x(x + 2) - 2(x + 2)$

$= x^2 + 2x - 2x - 4$

$= x^2 - 4$

(b) $(3m + 1n)(2m + 5n)$

$= 6m^2 + 15mn + 2mn + 5n^2$

$= 6m^2 + 17mn + 5n^2$

Exercise 1: Date: _____

Determine the following products:

(1) $(y - 4)(y + 3)$

$= y^2 + 3y - 4y - 12$

$= y^2 - y - 12$

(2) $(p - 2)(p - 7)$

$= p^2 - 7p - 2p + 14$

$= p^2 - 9p + 14$

(3) $(2x + 1)(x - 5)$

$= 2x^2 - 10x + 1x - 5$

$= 2x^2 - 9x - 5$

(4) $(x - 2y)(2x - y)$

$= 2x^2 - 1xy - 4xy + 2y^2$

$= 2x^2 - 5xy + 2y^2$

(5) $(4ab + 1)(2ab - 3)$

$= 8a^2b^2 - 12ab + 2ab - 3$

$= 8a^2b^2 - 10ab - 3$

(6) $(5 - 7m)(2 - 3m)$

$= 10 - 15m - 14m + 21m^2$

$= 10 - 29m + 21m^2$

(7) $(2a - 4b)(3a + 2b)$

$= 6a^2 + 4ab - 12ab - 8b^2$

$= 6a^2 - 8ab - 8b^2$

(8) $(m + n)(2m - 1)$

$= 2m^2 - 1m + 2mn - n$

(9) $(d - 12)(12 + d)$

$= 12d + d^2 - 144 - 12d$

$= d^2 - 144$

(10) $(a^2 + 4)(a^2 + 2)$

$= a^4 + 2a^2 + 4a^2 + 8$

$= a^4 + 6a^2 + 8$

(11) $\left(\frac{1}{2}m - 6\right)(8m - 3)$

$= 4m^2 - \frac{3}{2}m - 48m + 18$

$= 4m^2 - 49\frac{1}{2}m + 18$

(12) $(-2k - 5)(5 + 3k)$

$= -10k - 6k^2 - 25 - 15k$

$= -6k^2 - 25k - 25$

(13) $\left(p + \frac{1}{p}\right)\left(8p - \frac{4}{p}\right)$

$= 8p^2 - \frac{4p}{p} + \frac{8p}{p} - \frac{4}{p^2}$

$= 8p^2 - 4 + 8 - \frac{4}{p^2}$

$= 8p^2 + 4 - \frac{4}{p^2}$

(14) $(abc - 2ac)(abc + 3bc)$

$= a^2b^2c^2 + 3ab^2c^2 - 2a^2bc^2 - 6abc^2$

(15) $(3r^3 + 2)(2r^2 - 5)$

$= 6r^5 - 15r^3 + 4r^2 - 10$

(16) $2x(x - 5y)(3x + 2y)$

$= 2x(3x^2 + 2xy - 15xy - 10y^2)$

$= 2x(3x^2 - 13xy - 10y^2)$

$= 6x^3 - 26x^2y - 20xy^2$

(17) $\left(\frac{1}{p^3q^2} - \frac{2}{p^2q}\right)\left(\frac{1}{p} + \frac{2}{q}\right)$

$= \frac{1}{p^4q^2} + \frac{2}{p^3q^3} - \frac{2}{p^3q} - \frac{4}{p^2q^2}$

(18) $\left(\frac{m^2n}{3} - \frac{6}{mn}\right)\left(\frac{mn}{2} - \frac{3}{mn^2}\right)$

$= \frac{m^3n^2}{6} - \frac{3m^2n}{3mn^2} - \frac{6mn}{2mn} + \frac{18}{m^2n^3}$

$= \frac{m^3n^2}{6} - \frac{m}{n} - 3 + \frac{18}{m^2n^3}$

E.g. 2 Simplify:

(a) $(2a + 1)(2a - 1) = 4a^2 - 2a + 2a - 1 = 4a^2 - 1$

(b) $(m^2 - 5n)(m^2 + 5n) = m^4 + 5m^2n - 5m^2n - 25n^2 = m^4 - 25n^2$

Or shorter

(c) $(xy + 3)(xy - 3) = x^2y^2 - 9$

(d) $\left(\frac{ab}{4} - \frac{1}{7}\right)\left(\frac{ab}{4} + \frac{1}{7}\right) = \frac{a^2b^2}{16} - \frac{1}{49}$

Exercise 2: Date: _____

Simplify:

(1) $(abc - 2)(abc + 2)$

$= a^2b^2c^2 - 4$

(2) $\left(\frac{1}{3} + 5t\right)\left(\frac{1}{3} - 5t\right)$

$= \frac{1}{9} - 25t^2$

(3) $(p - 9q)(9q + p)$

$= (p - 9q)(p + 9q)$

$= p^2 - 81q^2$

(4) $(n + 7k)(7n - k)$

$= 7n^2 - 1kn + 49kn - 7k^2$

$= 7n^2 + 48kn - 7k^2$

(5) $(-a + 4b)(-a - 4b)$

$= a^2 + 4ab - 4ab - 16b^2$

$= a^2 - 16b^2$

(6) $-x\left(\frac{1}{x} - x\right)\left(\frac{1}{x} + x\right)$

$= -x\left(\frac{1}{x^2} - x^2\right)$

$= \frac{-x}{x^2} + x^3\left(\frac{1}{x^2} - x^2\right)$

$= \frac{-1}{x} + x^3$

(7) $(x^{2m} - 8)(x^{2m} + 8)$

$= x^{2m+2m} - 64$

$= x^{4m} - 64$

(8) $(0{,}3 + 3q)(0{,}3 - 3q)$

$= 0{,}09 - 9q^2$

(9) $(b^6c^3 + 6)(b^6c^3 + 6)$

$= b^{12}c^6 + 6b^6c^3 + 6b^6c^3 + 36$

$= b^{12}c^6 + 12b^6c^3 + 36$

(10) $(4xk^5 - 7)(7 + 4xk^5)$

$= (4xk^5 - 7)(4xk^5 + 7)$

$= 16x^2k^{10} - 49$

(11) $(m - 2n)^2(m + 2n)^2$

$= [(m - 2n)(m + 2n)]^2$

$= [m^2 - 4n^2]^2$

$= m^4 - 8m^2n^2 + 16n^4$

(12) $\left(\frac{m}{n} + 2\right)\left(\frac{m^2}{n^2} + 4\right)\left(\frac{m}{n} - 2\right)$

$= \left(\frac{m}{n} + 2\right)\left(\frac{m}{n} - 2\right)\left(\frac{m^2}{n^2} + 4\right)$

$= \left(\frac{m^2}{n^2} - 4\right)\left(\frac{m^2}{n^2} + 4\right)$

$= \frac{m^4}{n^4} - 16$

A2.1.2 Squaring of a binomial:

E.g. 3 Determine the following products:

(a) $(2x + 1)^2$

****

$= (2x + 1)(2x + 1)$

$= 4x^2 + 2x + 2x + 1$

$= 4x^2 + 4x + 1$

(b) $\left(m - \frac{1}{m}\right)^2$

$= \left(m - \frac{1}{m}\right)\left(m - \frac{1}{m}\right)$

$= m^2 - \frac{m}{m} - \frac{m}{m} + \frac{1}{m^2}$

$= m^2 - 2 + \frac{1}{m^2}$

Exercise 3: Date: _____

Determine the following squares:

(1) $(y - 11)^2$

$= (y - 11)(y - 11)$

$= y^2 - 11y - 11y + 121$

$= y^2 - 22y + 121$

(2) $(3p + 2q)^2$

$= (3p + 2q)(3p + 2q)$

$= 9p^2 + 6pq + 6pq + 4q^2$

$= 9p^2 + 12pq + 4q^2$

(3) $(-4 + 5c)^2$

$= (-4 + 5c)(-4 + 5c)$

$= 16 - 20c - 20c + 25c^2$

$= 16 - 40c + 25c^2$

(4) $(mn + 3)^2$

$= (mn + 3)(mn + 3)$

$= m^2n^2 + 3mn + 3mn + 9$

$= m^2n^2 + 6mn + 9$

(5) $(k^2 + 1)^2$

$= (k^2 + 1)(k^2 + 1)$

$= k^4 + 1k^2 + 1k^2 + 1$

$= k^4 + 2k^2 + 1$

(6) $(8 - 3b)^2$

$= (8 - 3b)(8 - 3b)$

$= 64 - 24b - 24b + 9b^2$

$= 64 - 48b + 9b^2$

(7) $\left(x - \frac{1}{2}\right)^2$

$= \left(x - \frac{1}{2}\right)\left(x - \frac{1}{2}\right)$

$= x^2 - \frac{1}{2}x - \frac{1}{2}x + \frac{1}{4}$

$= x^2 - 1x + \frac{1}{4}$

(8) $\left(\frac{y}{5} - 3\right)^2$

$= \left(\frac{y}{5} - 3\right)\left(\frac{y}{5} - 3\right)$

$= \frac{y^2}{25} - \frac{3y}{5} - \frac{3y}{5} + 9$

$= \frac{y^2}{25} - \frac{6y}{5} + 9$

(9) $(5p - 2p^2)^2$

$= (5p - 2p^2)(5p - 2p^2)$

$= 25p^2 - 10p^3 - 10p^3 + 4p^4$

$= 25p^2 - 20p^3 + 4p^4$

(10) $\left(4 + \frac{3}{n}\right)^2$

$= \left(4 + \frac{3}{n}\right)\left(4 + \frac{3}{n}\right)$

$= 16 + \frac{12}{n} + \frac{12}{n} + \frac{9}{n^2}$

$= 16 + \frac{24}{n} + \frac{9}{n^2}$

(11) $(0{,}2 + 6y)^2$

$= (0{,}2 + 6y)(0{,}2 + 6y)$

$= 0{,}04 + 1{,}2y + 1{,}2y + 36y^2$

$= 0{,}04 + 2{,}4y + 36y^2$

(12) $\left(\frac{2m}{p} + \frac{p^2}{3m}\right)^2$

$= \left(\frac{2m}{p} + \frac{p^2}{3m}\right)\left(\frac{2m}{p} + \frac{p^2}{3m}\right)$

$= \frac{4m^2}{p^2} + \frac{2mp^2}{3mp} + \frac{2mp^2}{3mp} + \frac{p^4}{9m^2}$

$= \frac{4m^2}{p^2} + \frac{4p}{3} + \frac{p^4}{9m^2}$

E.g. 4 Simplify the following: (Shorter method!)

(a) $(m + 5n)^2 = (m)^2 + 2(m)(5n) + (5n)^2 = m^2 + 10mn + 25n^2$

(b) $(pq - 2)^2 = p^2q^2 - 4pq + 4$

(c) $\left(\frac{1}{3} + 3x\right)^2 = \frac{1}{9} + 2x + 9x^2$

Exercise 4: Date: _____

Simplify (Use the shorter method!)

(1) $(x - 3)^2$

$= x^2 - 6x + 9$

(2) $(6m - 1)^2$

$= 36m^2 - 12m + 1$

(3) $(3y + 7)^2$

$= 9y^2 + 42y + 49$

(4) $(3 + pq)^2$

$= 9 + 6pq + p^2q^2$

(5) $(5t^2 + 8)^2$

$= 25t^4 + 80t^2 + 64$

(6) $\left(\frac{2}{3} - 6y\right)^2$

$= \frac{4}{9} - 8y + 36y^2$

(7) $(-2k - 5)^2$

$= 4k^2 + 20k + 25$

(8) $\left(\frac{3p - 2q}{5m}\right)^2$

$= \frac{9p^2 - 12pq + 4q^2}{25m^2}$

(9) $(4x^2 + 10y^2)^2$

$= 16x^4 + 80x^2y^2 + 100y^4$

(10) $(2mn + 7)(7 + 2mn)$

$= (2mn + 7)(2mn + 7)$

$= 4m^2n^2 + 28mn + 49$

(11) $(8 - 3y)(8 + 3y)$

$= 64 - 9y^2$

(12) $-2(abc - 11)^2$

$= -2(a^2b^2c^2 - 22abc + 121)$

$= -2a^2b^2c^2 + 44abc - 242$

A2.1.3 Binomials and trinomials:

E.g. 5 Simplify the following products: $(4y + 1)(y^2 - y + 5)$

$= 4y^3 - 4y^2 + 20y + 1y^2 - 1y + 5$

$= 4y^3 - 3y^2 + 19y + 5$

Exercise 5: Date: _____

Simplify the following products:

(1) $(2a - 3)(a^2 + 5a - 4)$

$= 2a^3 + 10a^2 - 8a - 3a^2 - 15a + 12$

$= 2a^3 + 7a^2 - 23a + 12$

(2) $(m + 7)(2m^2 + 3m + 3)$

$= 2m^3 + 3m^2 + 3m + 14m^2 + 21m + 21$

$= 2m^3 + 17m^2 + 24m + 21$

(3) $(1 + x)(1 - x + x^2)$

$= 1 - 1x + 1x^2 + 1x - 1x^2 + x^3$

$= 1 + x^3$

(4) $(3y - 2)(9y^2 + 6y + 4)$

$= 27y^3 + 18y^2 + 12y - 18y^2 - 12y - 8$

$= 27y^3 - 8$

(5) $\left(2m + \frac{1}{2}\right)\left(\frac{m^2}{4} + 4 - 4m\right)$

$= \frac{2m^3}{4} + 8m - 8m^2 + \frac{m^2}{8} + 2 - \frac{4m}{2}$

$= \frac{m^3}{2} + 8m - \frac{8m^2}{1} + \frac{1m^2}{8} + 2 - 2m$

$= \frac{m^3}{2} + 6m - 7\frac{7}{8}m^2 + 2$

(6) $(m^2n^2 - 5)(25 + 5m^2n^2 + m^4n^4)$

$= 25m^2n^2 + 5m^4n^4 + m^6n^6 - 125$
$- 25m^2n^2 - 5m^2n^2$

$= m^6n^6 - 125$

A2.1.4 The sum and difference of two cubes:

E.g. 6 Consider the following:

Product:

(a) $(x - 3)(x^2 + 3x + 9) = x^3 + 3x^2 + 9x - 3x^2 - 9x - 27 = x^3 - 27$

(b) $(y + 5)(y^2 - 5y + 25) = y^3 - 5y^2 + 25y + 5y^2 - 25y + 125 = y^3 + 125$

Pattern:

(c) $(4m - 1)(16m^2 + 4m + 1) = (4m - 1)[(4m)^2 + (4m)(1) + (1)^2] = 64m^3 - 1$

Pattern:

(d) $(n^2 + 2)(n^4 - 2n^2 + 4) = (n^2 + 2)[(n^2)^2 - (2)(n^2) + (2)^2] = n^6 + 8$

Exercise 6: Date: _____

Write down the following products directly, if possible:

(1) $(a + 3)(a^2 - 3a + 9)$

$= a^3 + 27$

(2) $(2y^3 + 4)(4y^6 - 8y^3 + 16)$

$= 8y^9 + 64$

(3) $\left(\frac{x}{3} - 1\right)\left(\frac{1}{9}x^2 + \frac{1}{3}x + 1\right)$

$= \frac{x^3}{27} - 1$

(4) $\left(6a^2 - \frac{1}{2}\right)\left(36a^4 + 3a^2 + \frac{1}{4}\right)$

$= 216a^6 - \frac{1}{8}$

(5) $(5q + 7)(25q^2 - 35q + 49)$

$= 125q^3 + 343$

(6) $(8 - 3m)(9m^2 + 24m + 64)$

$= (8 - 3m)(64 + 24m + 9m^2)$

$= 512 - 27m^3$

(7) $(x - 5)(x^2 - 5x + 25)$

$= x^3 - 5x^2 + 25x - 5x^2 + 25x - 125$

$= x^3 - 10x^2 + 50x - 125$

(8) $(0{,}1 + 0{,}2y)(0{,}01 - 0{,}02y + 0{,}04y^2)$

$= 0{,}001 + 0{,}008y^3$

(9) $(9a^4 + 6a^2b + 4b^2)(3a^2 - 2b)$

$= (3a^2 - 2b)(9a^4 + 6a^2b + 4b^2)$

$= 27a^6 - 8b^3$

(10) $2(-1 + 5m)(25m^2 + 5m + 1)$

$= 2(5m - 1)(25m^2 + 5m + 1)$

$= 2(125m^3 - 1)$

$= 250m^3 - 2$