<u>Grade 12 – Book D</u>

(First edition – CAPS)

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Chapter D1
Analytical Geometry

D1.1 Revision grade 10 and 11:

Gradient of the straight line through points P and Q: $m_{PQ} = \dfrac{y_Q - y_P}{x_Q - x_P}$

Applications of gradient:

* Parallel lines have the same gradient → ($m_1 = m_2$).
* The product of the gradients of perpendicular lines is – 1 → ($m_1 \times m_2 = -1$).
* The angle of inclination of a straight line is calculated by $\tan\theta = m$.
* Lines with positive gradients all lie in one direction and are increasing (angle of inclination is an acute angle) and lines with a negative gradient lie in another direction and are decreasing (angle of inclination is an obtuse angle)!
* Collinear points lie on the same straight line and thus have the same gradient.

Distance between two points: d(PQ) $= \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$

Midpoint of line PQ: $M(x_M; y_M) = \left(\dfrac{x_P + x_Q}{2} \; ; \; \dfrac{y_P + y_Q}{2}\right)$

Equation of a straight line: $y = mx + c$ **or** $y - y_1 = m(x - x_1)$

Exercise 1: Date: ______________________

(1) Given: R$(1 \; ; \; 1)$, S$(-1 \; ; \; 0)$, T$(2 \; ; \; -2)$ and V$(4 \; ; \; -1)$

(a) Prove that RSTV is a parallelogram by using the gradient formula.

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(b) Calculate the coordinates of the point of intersection of the diagonals of the parallelogram.

(c) Determine the ratio between the side lengths of the parallelogram.

(d) Calculate the size $S\hat{R}V$, correct to one decimal.

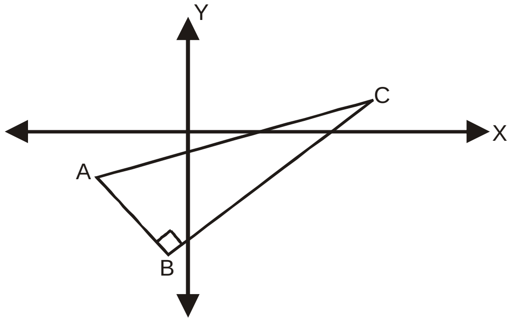
(2) $A(-1\,;\ 3)$, $B(3\,;\ 5)$ and $C(7\,;\ 7)$

(a) Are A, B and C collinear? Show all workings.

(b) Show that AB = BC.

(c) Is B the midpoint of line AC? Motivate your answer.

(3) A$(-4\,;\,-2)$, B$(-1\,;\,-5)$ and C$(x\,;\,2)$



(a) Calculate the gradient of AB.

(b) Write down the gradient of BC.

(c) Calculate the value of x.

(d) If BC $= \sqrt{98}$, calculate the perimeter and area of ΔABC. Leave your answer in simplest surd form.

(4) D(1 ; 1), E(7 ; 3), F(6 ; 6) and G(0 ; 4)

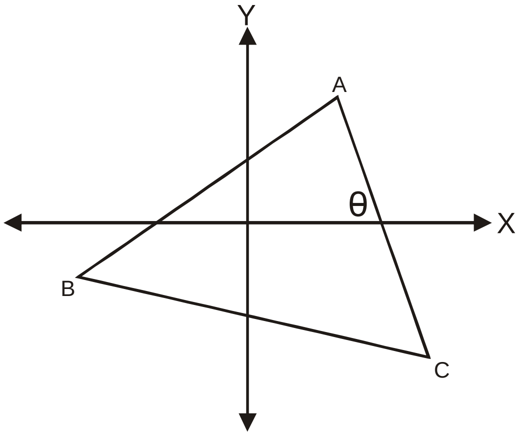
(a) Show that DEFG is a rectangle.

(b) Determine the equation of EG.

(c) Determine the equation of EF.

(d) Calculate the area of rectangle DEFG.

(5) A$(2\,;\,4)$, B$(-4\,;\,-2)$ and C$(4\,;\,-4)$ are the vertices of Δ ABC.



(a) Calculate the coordinates of P and Q if P and Q are the the midpoints of respectively AB and AC.

(b) Prove that PQ // BC.

(c) Prove that PQ $= \frac{1}{2}$ BC.

(d) Calculate the size of θ correct to the nearest degree.

(6) $P(0\ ;\ 2)$, $Q(2\ ;\ 5)$, $R(-1\ ;\ 3)$ and S are the vertices of parallelogram PQRS.

(a) Calculate the gradient of line QR.

(b) Determine the equation of line PS.

(c) Show that PQRS is a rhombus.

(d) Calculate the coordinates of the point where the diagonals of PQRS intersect one another.

(7) TWK is an isosceles triangle with TW = WK.
$T(7\ ;\ 8)$, $W(1\ ;\ 6)$ and $K(-1\ ;\ y)$.

(a) Calculate the length of TW.

(b) Calculate the value(s) of y.

(c) Determine the equation of the:

(i) median through T for

(ii) altitude through W for

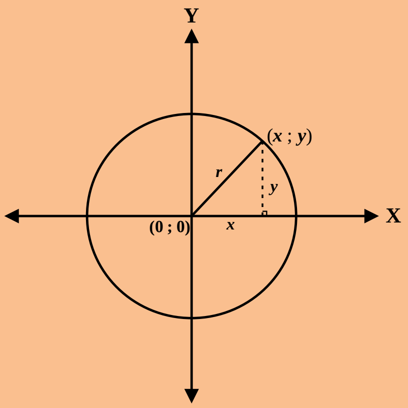
D1.2 <u>Circles</u>:

D1.2.1 <u>Circles with the origin as midpoint</u>:

The equation of the circle with midpoint $(0\,;\,0)$:

$x^2 + y^2 = r^2$

with $(x\,;\,y)$ any point on the circumference of the circle and r the radius of the circle.



D1.2.2 Other circles:

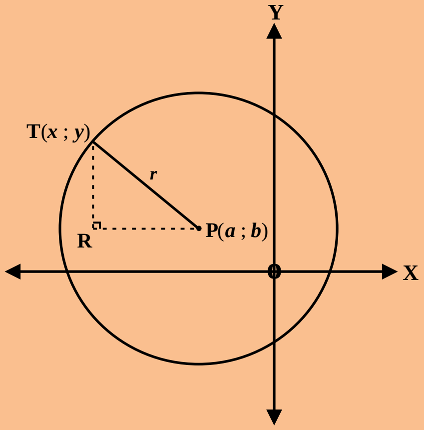
The equation of the circle with midpoint $(a\,;\,b)$:

$(x-a)^2+(y-b)^2=r^2$

with $(x\,;\,y)$ any point on the circumference of the circle and r the radius of the circle.

The radius can be calculated with the distance formula:

d(PT) = $\sqrt{(x_P-x_T)^2+(y_P-y_T)^2}$



Ex. 1 Determine the coordinates of the midpoint and the length of the radius of the following circle:

$\boldsymbol{x^2+6x+y^2-4y=12}$

Use completing of the square to write the equation in standard form $[(x-a)^2+(y-b)^2=r^2]$:

$$x^2+6x+y^2-4y=12$$

$$x^2+6x+\left(\frac{6}{2}\right)^2+y^2-4y+\left(\frac{-4}{2}\right)^2=12+\left(\frac{6}{2}\right)^2+\left(\frac{-4}{2}\right)^2$$

$$x^2+6x+(3)^2+y^2-4y+(-2)^2=12+9+4$$

$$(x+3)^2+(y-2)^2=25$$

$\therefore$ **MP = $(-3\,;\,2)$** **and** $r^2=25$

$\therefore r=5$

Ex. 2 Determine the equation of the circle with midpoint $(-2\,;\,5)$ through the point $(1\,;\,-1)$.

$(x-a)^2+(y-b)^2=r^2$ **with MP =** $(\overset{a}{-2}\,;\,\overset{b}{5})$

$\therefore (x-(-2))^2+(y-5)^2=r^2$

$\therefore (1+2)^2+(-1-5)^2=r^2$ **through** $(\overset{x}{1}\,;\,\overset{y}{-1})$

$\therefore (3)^2+(-6)^2=r^2$

$\therefore r^2=9+36=45$

$\therefore (x+2)^2+(y-5)^2=45$

Exercise 2: Date: ______________________

(1) Determine the equations of the following circles:

(a) with midpoint $(5\,;\ 2)$ and radius 6.

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(b) with midpoint $(-1\,;\ 3)$ and radius $\sqrt{12}$.

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(c) with midpoint $(4\,;\ -2)$ and through the point $(-2\,;\ 0)$.

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(d) with midpoint $(-2\,;\ -3)$ and through the point $(2\,;\ -1)$.

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(2) Determine the coordinates of the midpoints and the length of the radius of the following circles:

(a) $(x-4)^2+(y-2)^2=36$ ______________________

(b) $x^2+y^2-10y=6$

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(c) $(x+3)^2+(y+6)^2=20$ ______________________

(d) $(x+5)^2+(y+5)^2=280$ ______________________

(e) $(x-1)^2+(y+2)^2=9$ ______________________

(f) $x^2 + (y - 4)^2 = 100$ ____________________

(g) $x^2 - 8x + y^2 - 6y = 12$

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(h) $\left(x + \frac{1}{2}\right)^2 + (y + 2)^2 = 48$ ____________________

(i) $(x - 6)^2 + y^2 = 1$ ____________________

(j) $2x^2 + 2y^2 - 4x - y = 2$

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(3) Determine whether the point $(3\,;\,-2)$ lies on the circle with midpoint $(-1\,;\,5)$.
The radius of the circle is 8.

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(4) The equation of the circle through the point $(-3\,;\,-1)$ is $x^2 + 10x + y^2 - 2y + p = 0$.

(a) Determine the coordinates of the midpoint of the circle.

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(b) Calculate the value of p.

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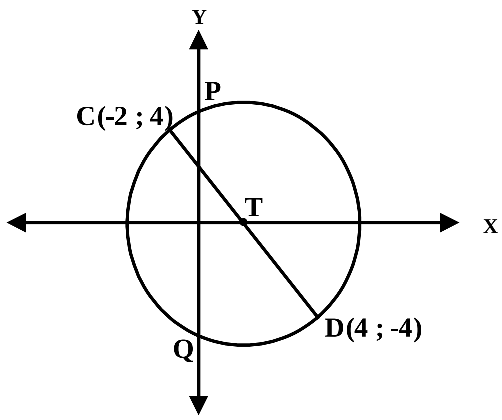
(5) Determine the equation of the circle with midpoint $(-4\ ;\ -3)$ and diameter 18.

(6) CD is the diameter of a circle with T as the midpoint of CD. Calculate:



(a) the coordinates of the midpoint of the circle.

(b) the equation of the circle.

(c) the length of PQ if P and Q are the y-intercepts of the circle.

(d) The equation of diameter CD.