Grade 11 – Book B
(CAPS Edition)

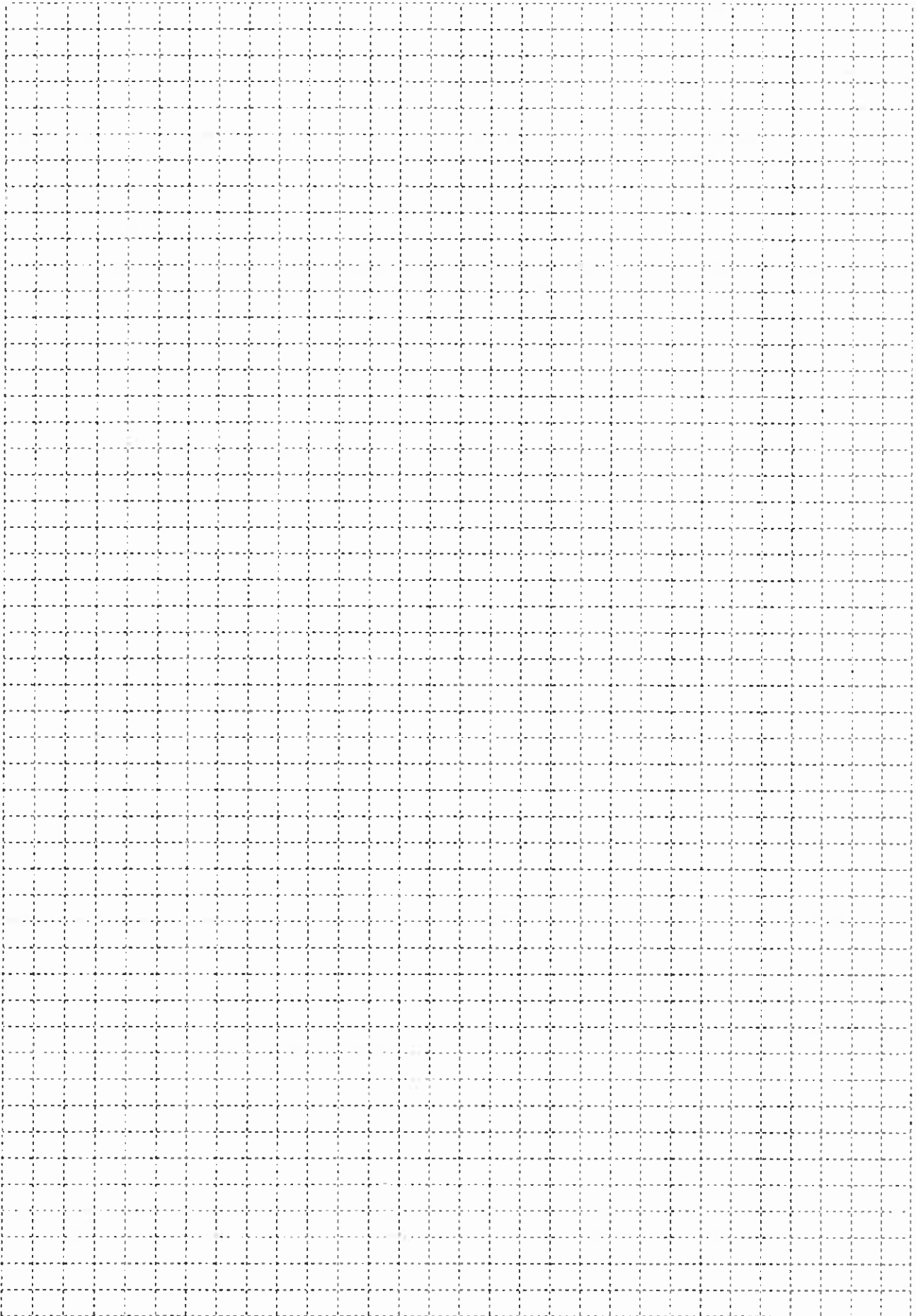# CONTENT:

Chapter B1

Functions

B1.1 Linear functions:

Revision!

Standard form: $y = mx + c$ with $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ as the gradient and c as the y-intercept.

Exercise 1:   Date:_____

(1) Draw each set of straight lines on the same system of axes and determine the point of intersection for each set:

   (a) $x - y + 1 = 0$ and $x + y = 3$ (b) $2y + 1 = x$ and $x + y = 1$

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   (c) $2x + 3 = y$ and $2y - 4x + 6 = 0$ (d) $4x + 2y = -3$ and $2y + 15 = 3x$

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(2) Determine the equation of the straight line:
   (a) through $(1 ; 3)$ and $(2 ; -1)$ (b)   through $(4 ; 0)$ and parallel to
                                                                         $3y + 6x - 2 = 0$

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(c) through $(3 ; -7)$ and $(3 : 4)$

(d) through $(0 ; 2)$ with an inclination of $135°$

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(3) The points $(3 ; 5)$, $(0 ; 4)$ and $(-1 ; m)$ is collinear. Calculate the value of m.

(4) $3x - 2y = 3$ and $px + 1 = 2y$ is perpendicular. Calculate the value of p.

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B1.2 Quadratic function (parabola):
B1.2.1 Sketching of the parabola:
B1.2.1.1 Standard form 1:

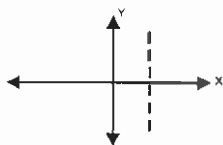$$y = ax^2 + bx + c$$

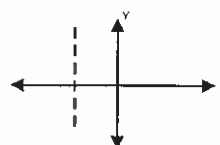**Influence of a:** [Form!]

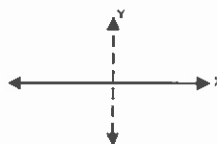If $a > 0$:   and   if $a < 0$:

Influence of b: [Symmetry-axis!]

If symm-axis $(x) = \frac{-b}{2a} > 0$ then:

If symm-axis $(x) = \frac{-b}{2a} < 0$ then:

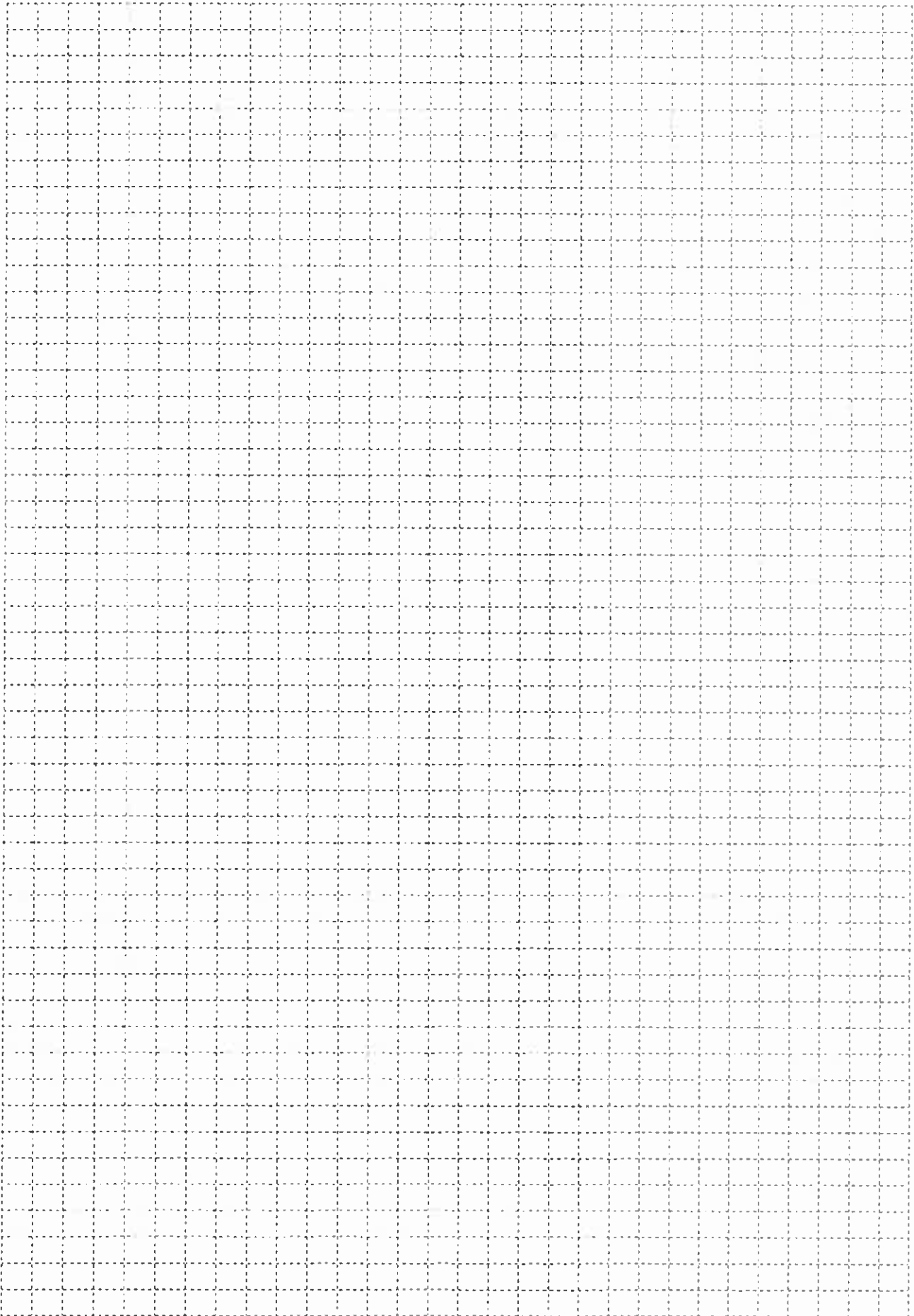If symm-axis $(x) = \frac{-b}{2a} = 0$ then:

Influence of c: [*y*-intercept!]

c represents, similar to the straight line, the y-intercept of the parabola.

E.g. 1 Sketch the following: $2y = -2x^2 + 4x + 16$
                                          ****

  <u>Step 1</u> *[Write the equation in the standard form]:* $y = -x^2 + 2x + 8$

  <u>Step 2</u> *[Interpret the form]: $a < 0$ ∴*

  <u>Step 3</u> *[Determine the y-intercept]: $c = 8$ or   substitute $x = 0$ ∴ y-int: (0 ; 8)*

  <u>Step 4</u> *[Determine the x-intercept(s)] There can be two, one, or no x-intercept(s).*
              Subst $y = 0 \rightarrow 0 = -x^2 + 2x + 8$
                         $0 = x^2 - 2x - 8$
                         $0 = (x - 4)(x + 2)$
                    ∴ $x = 4$ or $x = -2$
          ∴ x-intercepts: (4 ; 0) and (-2 ; 0)
          NB: If you do not find factors for the equation, make use of the formula!

  <u>Step 5</u> *[Determine the equation of the symmetry-axis]:   Formula $\rightarrow x = \frac{-b}{2a}$*
              From standard form: $a = -1$ and $b = 2$ $\rightarrow$ $x = \frac{-(2)}{2(-1)}$
                                                        $x = \frac{-2}{-2} = 1$

  or the symm-axis is exactly halfway between the two x-int: ∴ symm-axis $= \frac{4 + (-2)}{2} = \frac{2}{2} = 1$

  <u>Step 6</u> *[Determine the coordinates of the turning point]:*
              Subst $x = 1$ (symm-axis) in the equation of step 1
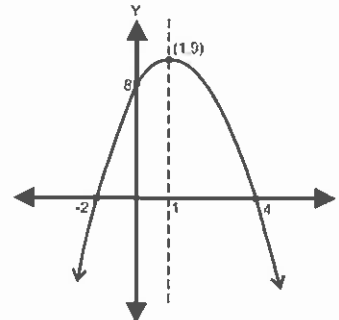   *∴ $y = -x^2 + 2x + 8$*
              ∴ $y = -(1)^2 + 2(1) + 8$
              ∴ $y = -1 + 2 + 8 = 9$ *∴ TP = (1 ; 9)*

  <u>Step 7</u> *[Draw the curve of the function]:*
              Show the x-and y-intercepts and the
              turning point clearly.

<u>Conclusions:</u>

Max value of 9
Domain: $x \in R$
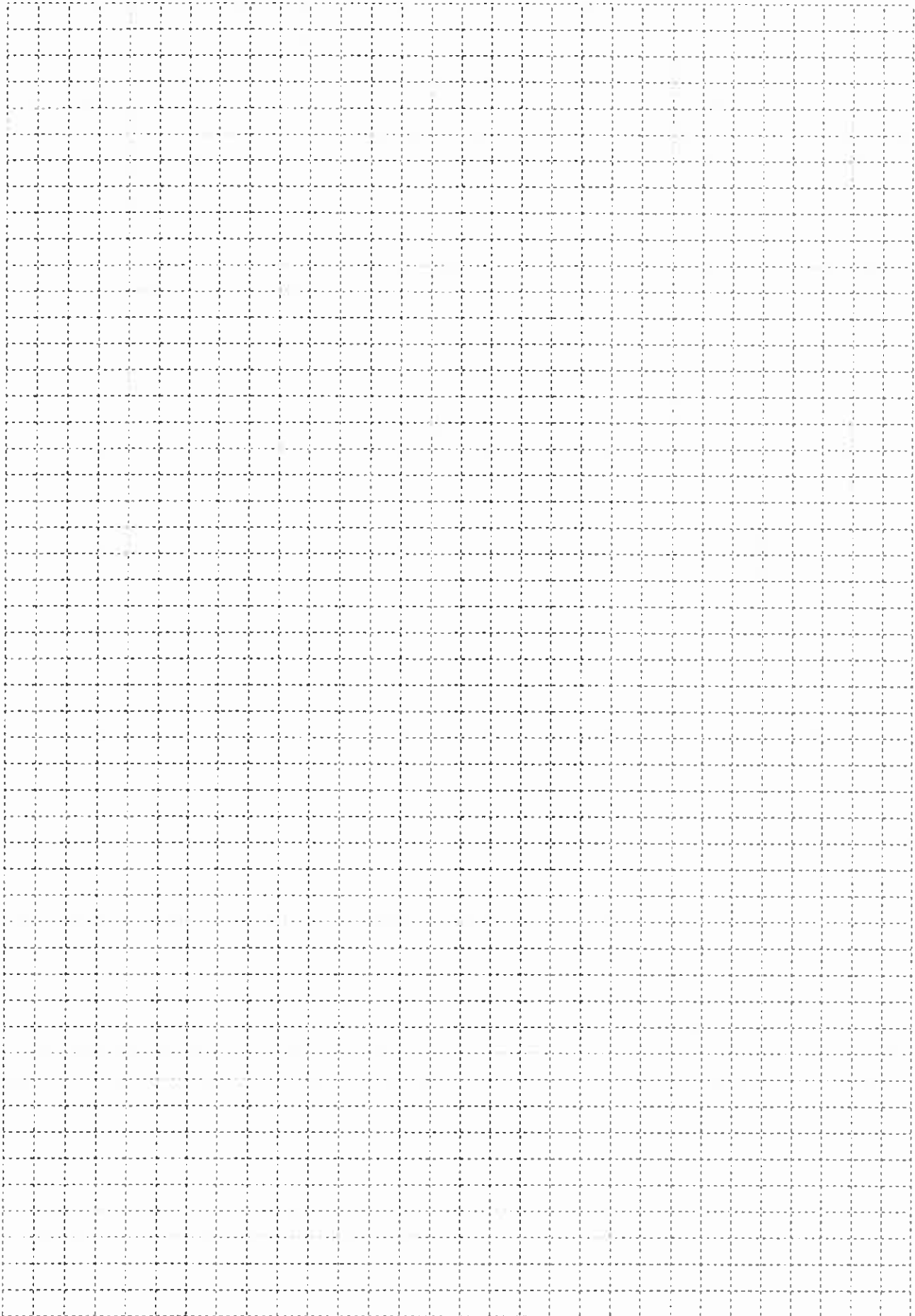**Range: $y \leq 9$**
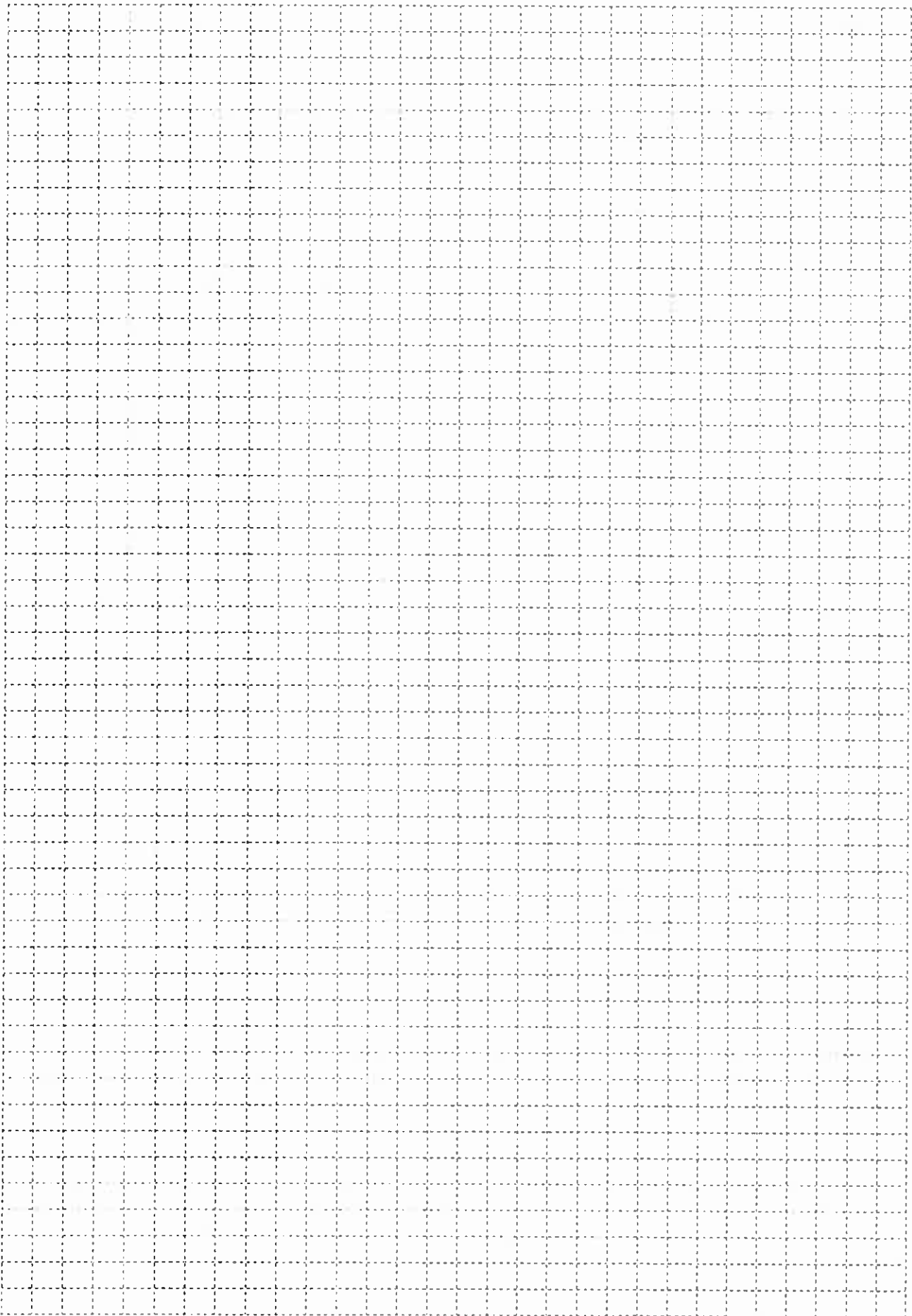

<u>Exercise 2:</u>   Date:_____
(1) Draw the following functions on different Cartesian planes: (Do drawings on the left!)
(a) $y = x^2 + 8x + 12$

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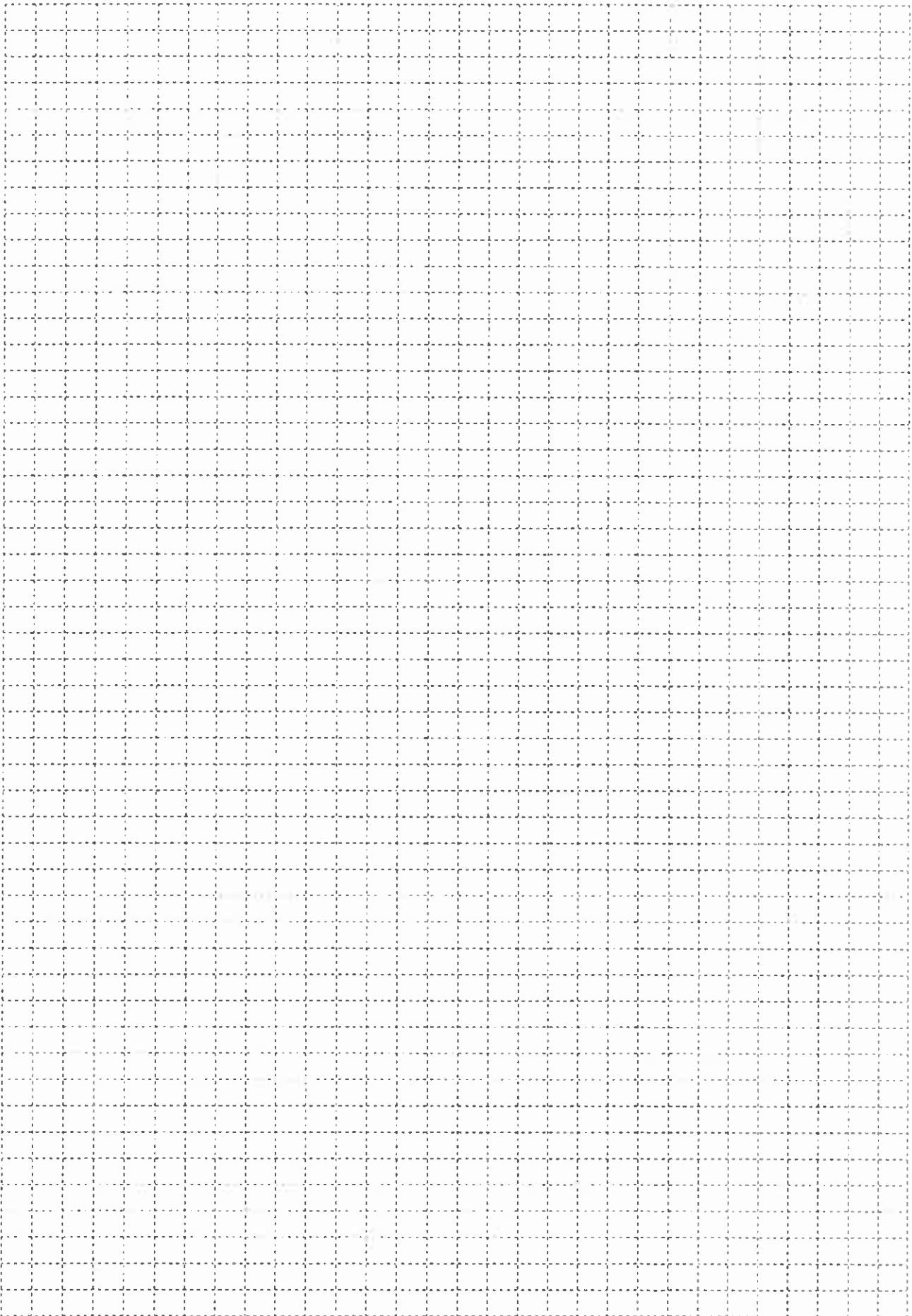
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(b) $y = -x^2 - x + 6$

(c) $2y = 2x^2 - 6x + 4$

(d) $y = x^2 + 4x + 1$

(e) $y = -x^2 - 2x - 3$

(2) Consider: $f(x) = 2x^2 - 3x + 1$
  (a) Sketch f. Show all calculations.

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  (b) Draw on the same Cartesian plane as in (a): $g: x \rightarrow -3x + 3$. Show all calculations.

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  (c) Determine the following: (i) the domain of g.
                              (ii) the range of f.
                              (iii) The equation of the symmetry-axis of f.
                              (iv) The coordinates of f ∩ g.

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(3) (a) Draw on the same Cartesian plane: $p(x) = x^2 - 2x$ and $q(x) = 4 - x^2$

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(b) Use the graph in (a) and determine the following:

    (i) Domain of p.   (iii) Min/Max value of q.

    (ii) Range of q.   (iv) x if $p(x) = q(x)$.

B1.2.1.2 Standard form 2:

$$y = a(x - p)^2 + q$$

Influence of a: [Form!]

If $a > 0$:   and   if $a < 0$:

Influence of p: [Symmetry-axis!]

The equation of the symm-axis: $x = p$

Influence of q: [Min/Max!]

q represents the y-coordinate of the turning point. $\therefore$ TP $= (p \,;\, q)$

E.g. 2 Sketch the following: $y = (x - 1)^2 - 4$

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  Step 1 *[Interpret the form]: $a > 0$* $\therefore$

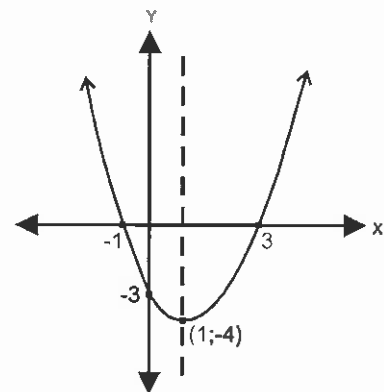 *Step 2* *[Determine the coordinates of the turning point]:* $TP = (p \,;\, q) = (1 \,;\, -4)$

  Step 3 *[Determine the x-intercept(s)]: Subst $y = 0$*

$\therefore \quad 0 = (x - 1)^2 - 4$ *or* $0 = (x - 1)^2 - 4$

$\qquad 4 = (x - 1)^2$ $0 = x^2 - 2x + 1 - 4$

$\pm\sqrt{4} = x - 1$ $0 = x^2 - 2x - 3$

$\pm 2 = x - 1$ $0 = (x - 3)(x + 1)$

$\therefore \ x = +2 + 1$ *or* $x = -2 + 1$ $x = 3$ *or* $x = -1$

$\qquad x = 3 \qquad\qquad x = -1$ $\therefore$ *x-int: (3 ; 0) and (-1 ; 0)*

  Step 4 *[Determine the y-int]: Subst $x = 0$*   *Step 5*   *[Draw the graph!]*

$\therefore y = (0 - 1)^2 - 4$

$\therefore y = (-1)^2 - 4$

$\therefore y = 1 - 4$

$\therefore y = -3$

$\therefore$ *y-int: (0 ; -3)*

Conclusions:

Min value of -4

Domain: $x \in R$

Range: $y \geq -4$

(1) Sketch the following functions on separate Cartesian planes: (Draw left!)

(a) $y = 2(x + 1)^2 - 8$ (b) $y = -(x + 2)^2 + 1$
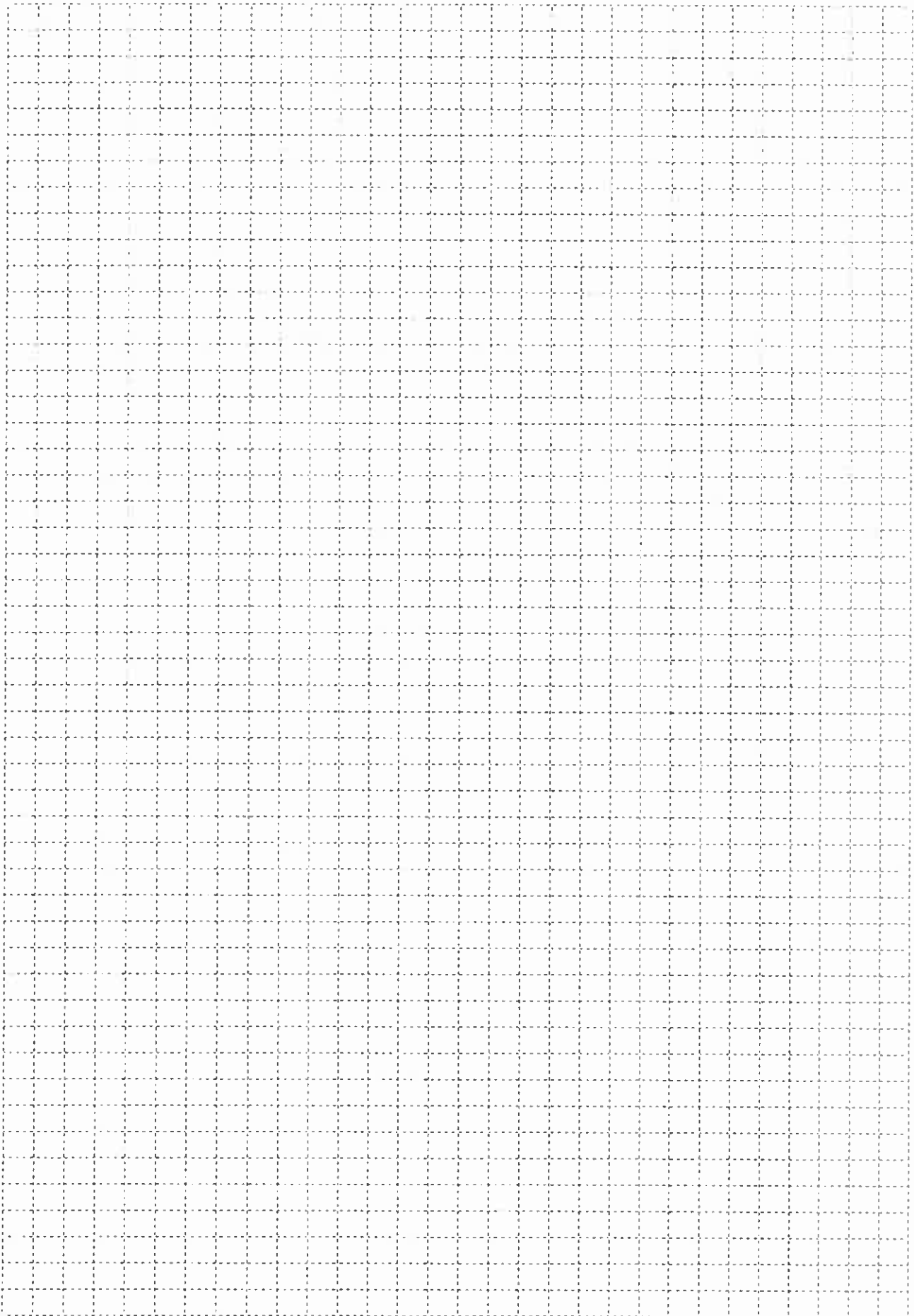
(c) $y = -3(x + 3)^2$ (d) $y = (x - 2)^2 + 2$

(2) Consider: $h{:}x \rightarrow -(x + 1)^2 - 3$

   (a) Draw h. Show all calculations.

   (b) Draw $p(x) = -x^2 - 3$ on the same Cartesian plane as (a).

(c) Describe the transformation of h to p as in (a) and (b). What is the influence of such a transformation on the equation of h to p?

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(d) Determine the equation of the straight line through the turning points of the two parabola:

(e) Write down the ranges of h and p:

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B1.2.1.3 Standard form 3:

$$y = a(x - x_1)(x - x_2)$$

Influence of a: [Form!]

If $a > 0$:   and   if $a < 0$:

Influence of x_1 and x_2 : [x-intercepts!]

Parabola intercepts the x-axis at x_1 and x_2.

E.g. 3 Sketch the following: $y = 2(x - 3)(x + 1)$
                                             ****

Step 1   *[Interpret the form]:* $a > 0$ $\therefore$

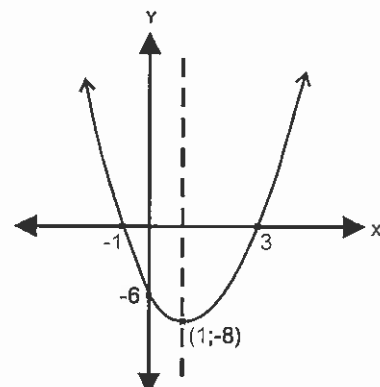**Step 2**   *[Determine the x-intercept(s)]:* $x_1 = 3$ *and* $x_2 = -1$
                                        $\therefore$ *x-int: (3 ; 0) and (-1 ; 0)*

Step 3   *[Determine the equation of the symm-axis]:* $symm\text{-}axis = \dfrac{x_1 + x_2}{2}$

$$x = \frac{3 + (-1)}{2} = 1$$

Step 3   *[Determine the coordinates of the turning point]:*
        Substitute $x = 1$ (symm-axis) in equation: $\therefore$ $y = 2(1 - 3)(1 + 1)$
                                        $\therefore$ $y = 2(-2)(2) = -8$
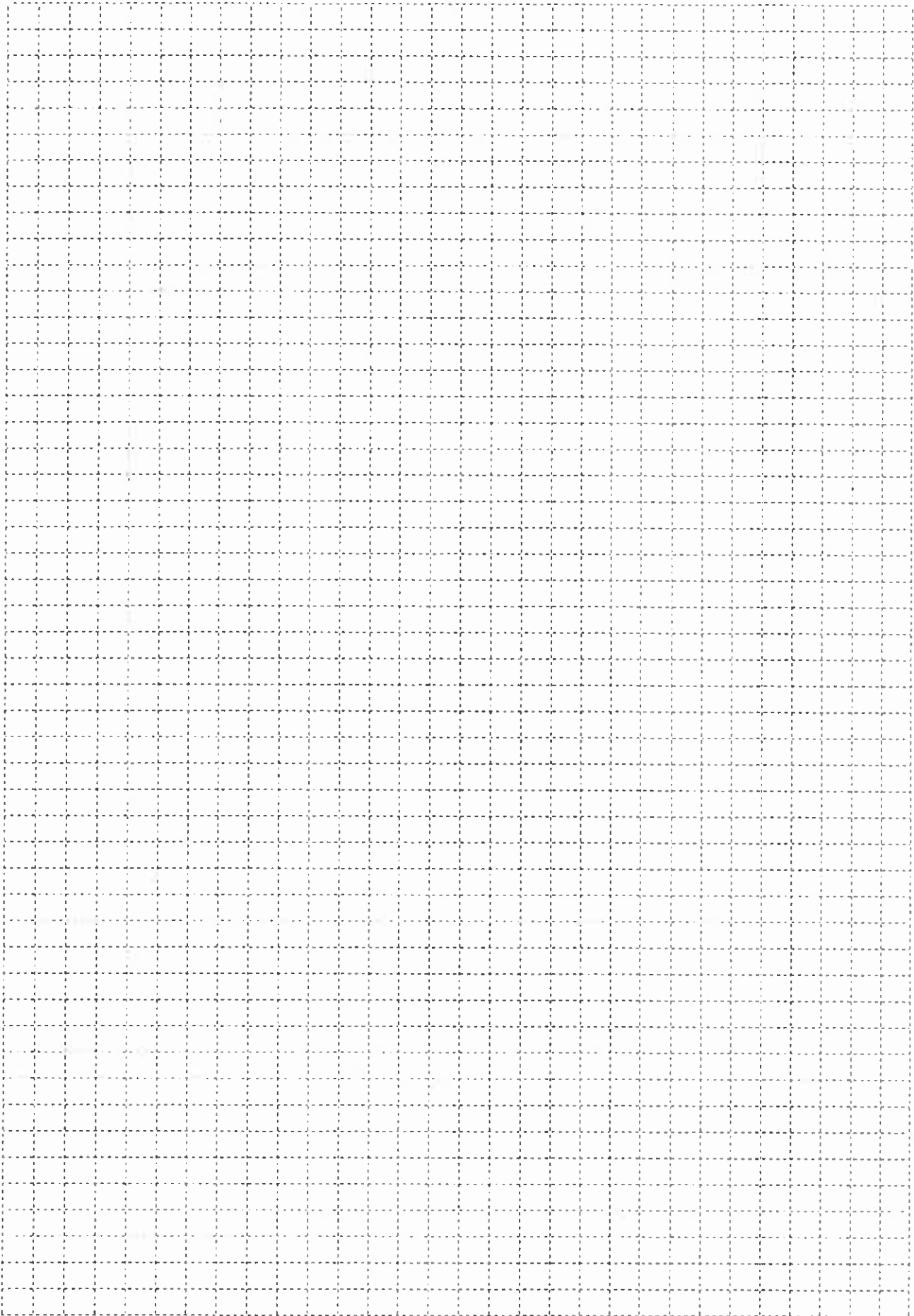 $\therefore$ *TP = (1 ; -8)*

Step 4   *[Determine the y-int]: Subst $x = 0$*   **Step 5**   *[Draw the graph!]*
        $\therefore$ $y = 2(0 - 3)(0 + 1)$
        $\therefore$ $y = 2(-3)(1)$
        $\therefore$ $y = -6$
        $\therefore$ *y-int: (0 ; -6)*

Conclusions:

Min value of -8
Domain: $x \in R$
Range: $y \geq -8$

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Exercise 4:   Date:_____

(1) Sketch the following on different Cartesian planes: (Draw left!)
(a) $y = -3(x + 1)(x - 1)$ (b) $y = (x + 2)(x - 3)$

(2) Consider the following: $f(x) = 2(x - 1)(x + 2)$ and $g(x) = 2x^2 - 2x - 4$
(a) Draw f and g on the same Cartesian plane

(b) Write g in the form $g(x) = a(x - x_1)(x - x_2)$

(c) Describe the transformation of f $\rightarrow$ g. Also explain the relation between the equations
    of f and g and the transformation.

Exercise 4: