Grade 12 – Book D
(First edition – CAPS)

TEACHER'S GUIDELINES

CONTENTS:

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Chapter D1
Analytical Geometry

D1.1 Revision grade 10 and 11:

Gradient of the straight line through points P and Q: $m_{PQ} = \dfrac{y_Q - y_P}{x_Q - x_P}$

Applications of gradient:

* Parallel lines have the same gradient → ($m_1 = m_2$).
* The product of the gradients of perpendicular lines is – 1 → ($m_1 \times m_2 = -1$).
* The angle of inclination of a straight line is calculated by $\tan\theta = m$.
* Lines with positive gradients all lie in one direction and are increasing (angle of inclination is an acute angle) and lines with a negative gradient lie in another direction and are decreasing (angle of inclination is an obtuse angle)!
* Collinear points lie on the same straight line and thus have the same gradient.

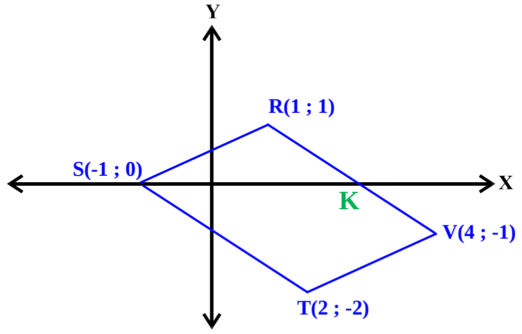
Distance between two points: $d(PQ) = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$

Midpoint of line PQ: $M(x_M; y_M) = \left(\dfrac{x_P + x_Q}{2} \; ; \; \dfrac{y_P + y_Q}{2}\right)$

Equation of a straight line: $y = mx + c$ **or** $y - y_1 = m(x - x_1)$

Exercise 1: Date: ______________________

(1) Given: R(1 ; 1), S(−1 ; 0), T(2 ; −2) and V(4 ; −1)



(a) Prove that RSTV is a parallelogram by using the gradient formula.

$m_{RS} = \dfrac{y_S - y_R}{x_S - x_R} = \dfrac{0-1}{-1-1} = \dfrac{-1}{-2} = \dfrac{1}{2}$

$m_{TV} = \dfrac{y_V - y_T}{x_V - x_T} = \dfrac{-1-(-2)}{4-2} = \dfrac{-1+2}{2} = \dfrac{1}{2}$

$m_{RV} = \dfrac{y_V - y_R}{x_V - x_R} = \dfrac{-1-1}{4-1} = \dfrac{-2}{3} = -\dfrac{2}{3}$

$m_{ST} = \dfrac{y_T - y_S}{x_T - x_S} = \dfrac{-2-0}{2-(-1)} = \dfrac{-2}{2+1} = -\dfrac{2}{3}$

∴ **RS // TV → Gradients are equal**

∴ **RV // ST → Gradients are equal**

∴ **RSTV is a parallelogram, because both pairs of opposite sides are parallel.**

(b) Calculate the coordinates of the point of intersection of the diagonals of the parallelogram.

The coordinates of the intersection of the diagonals → midpoint of VS or TR

$$M_{VS} = \left(\frac{x_V + x_S}{2} ; \frac{y_V + y_S}{2}\right)$$

$$\therefore\ M_{VS} = \left(\frac{4 + (-1)}{2} ; \frac{-1 + 0}{2}\right)$$

$$\therefore\ M_{VS} = \left(\frac{3}{2} ; -\frac{1}{2}\right)$$

(c) Determine the ratio between the side lengths of the parallelogram.

$$d(VT) = \sqrt{(x_V - x_T)^2 + (y_V - y_T)^2}$$

$$\therefore\ d(VT) = \sqrt{(4 - 2)^2 + (-1 - (-2))^2}$$

$$\therefore\ d(VT) = \sqrt{(2)^2 + (1)^2}$$

$$\therefore\ d(VT) = \sqrt{4 + 1} = \sqrt{5}$$

$$d(VR) = \sqrt{(x_V - x_R)^2 + (y_V - y_R)^2}$$

$$\therefore\ d(VR) = \sqrt{(4 - 1)^2 + (-1 - 1)^2}$$

$$\therefore\ d(VR) = \sqrt{(3)^2 + (-2)^2}$$

$$\therefore\ d(VR) = \sqrt{9 + 4} = \sqrt{13}$$

VT = RS → Opposite sides of parm ← VR = TS

$$\therefore\ \frac{VT}{VR} = \frac{\sqrt{5}}{\sqrt{13}} = \frac{5}{13} \quad \text{or} \quad \frac{VR}{VT} = \frac{13}{5}$$

(d) Calculate the size $S\hat{R}V$, correct to one decimal.

$$\tan R\hat{K}X = m_{RV} = -\frac{2}{3} \quad \rightarrow \quad \text{See (a) and K on sketch}$$

$$\therefore\ R\hat{K}X = 180° - 33,69\ldots°$$

$$\therefore\ R\hat{K}X = 146,309\ldots°$$

$$\tan K\hat{S}R = m_{RS} = \frac{1}{2} \quad \rightarrow \quad \text{See (a)}$$

$$\therefore\ K\hat{S}R = 26,565\ldots°$$

$$\therefore\ S\hat{R}V = R\hat{K}X - K\hat{S}R \quad \rightarrow \quad \text{Exterior } \angle \text{ of } \Delta$$

$$\therefore\ S\hat{R}V = 146,309\ldots° - 26,565\ldots°$$

$$\therefore\ S\hat{R}V \approx 119,74°$$

(2) A(−1 ; 3), B(3 ; 5) and C(7 ; 7)

(a) Are A, B and C collinear? Show all workings.

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{5 - 3}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}$$

$$m_{CB} = \frac{y_B - y_C}{x_B - x_C} = \frac{5 - 7}{3 - 7} = \frac{-2}{-4} = \frac{1}{2}$$

$\therefore$ **A, B and C are collinear because $m_{AB} = m_{CB}$**

(b) Show that AB = BC. → $A(-1\,;\ 3)$, $B(3\,;\ 5)$ and $C(7\,;\ 7)$

$d(AB) = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$ | $d(CB) = \sqrt{(x_B - x_C)^2 + (y_B - y_C)^2}$

$\therefore\ d(AB) = \sqrt{(3-(-1))^2 + (5-3)^2}$ | $\therefore\ d(CB) = \sqrt{(3-7)^2 + (5-7)^2}$

$\therefore\ d(AB) = \sqrt{(4)^2 + (2)^2}$ | $\therefore\ d(CB) = \sqrt{(-4)^2 + (-2)^2}$

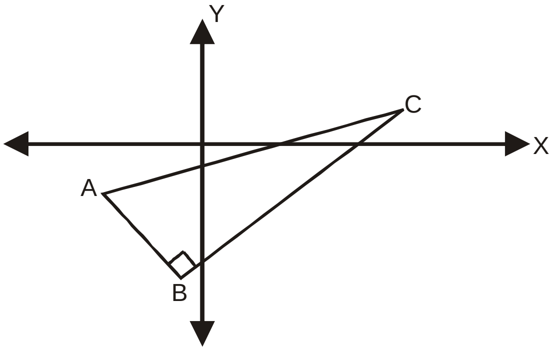
$\therefore\ d(AB) = \sqrt{16+4} = \sqrt{20}$ | $\therefore\ d(CB) = \sqrt{16+4} = \sqrt{20}$

$\therefore$ **AB = BC**

(c) Is B the midpoint of line AC? Motivate your answer.

Yes, because A, B and C lie on a straight line (are collinear) and AB = BC.

(3) $A(-4\,;\ -2)$, $B(-1\,;\ -5)$ and $C(x\,;\ 2)$



(a) Calculate the gradient of AB.

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{-5-(-2)}{-1-(-4)} = \frac{-5+2}{-1+4} = \frac{-3}{3}$$

$\therefore\ m_{AB} = -1$

(b) Write down the gradient of BC.

$m_{BC} = 1$ → $AB \perp BC$

(c) Calculate the value of x.

$$m_{BC} = \frac{-5-2}{-1-x} = 1 \quad \rightarrow \quad \therefore\ -7 = -1 - x$$

$$\therefore\ x = -1 + 7 \quad \rightarrow \quad \therefore\ x = 6$$

(d) If $BC = \sqrt{98}$, calculate the perimeter and area of ΔABC. Leave your answer in simplest surd form.

$d(AB) = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$ → $A(-4\,;\ -2)$ and $B(-1\,;\ -5)$

$\therefore\ d(AB) = \sqrt{(-1-(-4))^2 + (-5-(-2))^2}$

$\therefore\ d(AB) = \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = 8$

$\therefore\ AC^2 = AB^2 + BC^2$ → Pythagoras

$\therefore\ AC^2 = \left(\sqrt{18}\right)^2 + \left(\sqrt{98}\right)^2 = 18 + 98 = 116$

$\therefore\ AC = \sqrt{116}$

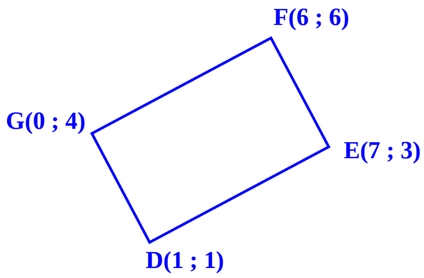
$\text{Perimeter} = \sqrt{18} + \sqrt{98} + \sqrt{116} = 3\sqrt{2} + 7\sqrt{2} + 2\sqrt{29}$

$\therefore\ \text{Perimeter} = 10\sqrt{2} + 2\sqrt{29}$

$\text{Area} = \frac{1}{2} \times \text{Base} \times \perp \text{Height} = \frac{1}{2} \times \sqrt{18} \times \sqrt{98} = \frac{1}{2} \times 3\sqrt{2} \times 7\sqrt{2}$

$\therefore\ \text{Area} = \frac{1}{2} \times 21 \times 2$

$\therefore\ \text{Area} = 21$



(4) D(1 ; 1), E(7 ; 3), F(6 ; 6) and G(0 ; 4)

(a) Show that DEFG is a rectangle.

$m_{DE} = \frac{y_E - y_D}{x_E - x_D} = \frac{3-1}{7-1} = \frac{2}{6} = \frac{1}{3}$

$m_{EF} = \frac{y_F - y_E}{x_F - x_E} = \frac{6-3}{6-7} = \frac{3}{-1} = -3$

$m_{FG} = \frac{y_G - y_F}{x_G - F} = \frac{4-6}{0-6} = \frac{-2}{-6} = \frac{1}{3}$

$m_{GD} = \frac{y_D - y_G}{x_D - x_G} = \frac{1-4}{1-0} = \frac{-3}{1} = -3$

∴ **DE // FG and EF // GD → Gradients are equal**

But $m_{DE} \times m_{EF} = \frac{1}{3} \times 3 = -1$ → **DE ⊥ EF**

∴ DEFG is a rectangle, because both pairs of opposite sides are parallel and the adjacent sides are perpendicular to one another → all angles are equal to 90°

(b) Determine the equation of EG.

$m_{EG} = \frac{y_G - y_E}{x_G - x_E} = \frac{4-3}{0-7} = \frac{1}{-7}$ **through the point G(0 ; 4) →** y**-intercept because** $x = 0$

∴ $y = mx + c$

∴ $y = -\frac{1}{7}x + 4$

(c) Determine the equation of EF.

$m_{EF} = -3$ → **See (a)**

∴ $y - y_1 = m(x - x_1)$ **through the points E(7 ; 3) and F(6 ; 6)**

∴ $y - 3 = -3(x - 7)$ **or** $y - 6 = -3(x - 6)$

∴ $y = -3x + 21 + 3$ $\qquad$ $y = -3x + 18 + 6$

∴ $y = -3x + 24$ $\qquad$ $y = -3x + 24$

(d) Calculate the area of rectangle DEFG.

$d(DE) = \sqrt{(x_E - x_D)^2 + (y_E - y_D)^2}$ **or** $d(FG) = \sqrt{(x_G - x_F)^2 + (y_G - y_F)^2}$

∴ $d(DE) = \sqrt{(7-1)^2 + (3-1)^2}$ $\qquad$ $d(FG) = \sqrt{(0-6)^2 + (4-6)^2}$

∴ $d(DE) = \sqrt{(6)^2 + (2)^2}$ $\qquad$ $d(FG) = \sqrt{(-6)^2 + (-2)^2}$

∴ $d(DE) = \sqrt{36+4} = \sqrt{40}$ $\qquad$ $d(FG) = \sqrt{36+4} = \sqrt{40}$

$d(DG) = \sqrt{(x_G - x_D)^2 + (y_G - y_D)^2}$ **or** $d(EF) = \sqrt{(x_E - x_F)^2 + (y_E - y_F)^2}$

∴ $d(DG) = \sqrt{(0-1)^2 + (4-1)^2}$ $\qquad$ $d(EF) = \sqrt{(7-6)^2 + (3-6)^2}$

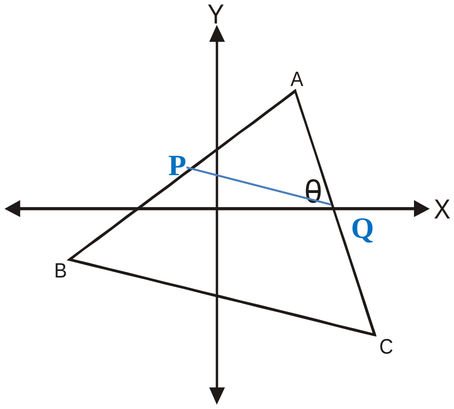
∴ $d(DG) = \sqrt{(-1)^2 + (3)^2}$ $\qquad$ $d(EF) = \sqrt{(1)^2 + (-3)^2}$

∴ $d(DG) = \sqrt{1+9} = \sqrt{10}$ $\qquad$ $d(EF) = \sqrt{1+9} = \sqrt{10}$

∴ **Area rectangle DEFG = L × B =** $\sqrt{40} \times \sqrt{10} = \sqrt{400}$

∴ Area rectangle DEFG = 20 square units

(5) A(2 ; 4), B(−4 ; −2) and C(4 ; −4) are the vertices of Δ ABC.



(a) Calculate the coordinates of P and Q if P and Q are the the midpoints of respectively AB and AC.

$$\mathbf{P = M_{AB} = \left(\frac{x_A + x_B}{2}\,;\,\frac{y_A + y_B}{2}\right)}$$

$$\therefore\ \mathbf{P = M_{AB} = \left(\frac{2-4}{2}\,;\,\frac{4-2}{2}\right) = \left(\frac{-2}{2}\,;\,\frac{2}{2}\right)}$$

$$\therefore\ \mathbf{P = (-1\,;1)}$$

$$\mathbf{Q = M_{AC} = \left(\frac{x_A + x_C}{2}\,;\,\frac{y_A + y_C}{2}\right)}$$

$$\therefore\ \mathbf{Q = M_{AC} = \left(\frac{2+4}{2}\,;\,\frac{4-4}{2}\right) = \left(\frac{6}{2}\,;\,\frac{0}{2}\right)}$$

$$\therefore\ \mathbf{Q = (3\,;0)}$$

(b) Prove that PQ // BC.

$$m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P} = \frac{0-1}{3-(-1)} = \frac{-1}{4}$$

$$m_{BC} = \frac{y_C - y_B}{x_C - x_B} = \frac{-4-(-2)}{4-(-4)} = \frac{-4+2}{4+4} = \frac{-2}{8} = \frac{-1}{4}$$

∴ **PQ // BC → Gradients are equal**

(c) Prove that $PQ = \frac{1}{2}BC$.

$$\mathbf{d(PQ)} = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2} \quad \text{and} \quad \mathbf{d(BC)} = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2}$$

$$\therefore\ \mathbf{d(PQ)} = \sqrt{(3-(-1))^2 + (0-1)^2} \qquad \mathbf{d(BC)} = \sqrt{(4-(-4))^2 + (-4-(-2))^2}$$

$$\therefore\ \mathbf{d(PQ)} = \sqrt{(4)^2 + (-1)^2} \qquad \mathbf{d(BC)} = \sqrt{(8)^2 + (-2)^2}$$

$$\therefore\ \mathbf{d(PQ)} = \sqrt{16+1} = \sqrt{17} \qquad \mathbf{d(BC)} = \sqrt{64+4} = \sqrt{68}$$

But $\sqrt{68} = \sqrt{4} \times \sqrt{17} = 2\sqrt{17}$

$$\therefore\ \mathbf{BC = 2\,PQ} \ \rightarrow\ \mathbf{PQ = \frac{1}{2}BC}$$

(d) Calculate the size of θ correct to the nearest degree.

$$\tan A\hat{Q}X = m_{AC}$$

$$\therefore\ \tan A\hat{Q}X = \frac{-4-4}{4-2} = \frac{-8}{2} = -4$$

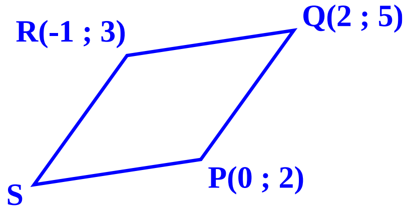
$$\therefore\ A\hat{Q}X = 180° - 76° \approx 104°$$

But $\theta = 180° - A\hat{Q}X$

$$\therefore\ \theta = 180° - 104°$$

$$\therefore\ \theta = 76°$$

(6) P(0 ; 2), Q(2 ; 5), R(−1 ; 3) and S are the vertices of parallelogram PQRS.



(a) Calculate the gradient of line QR.

$m_{QR} = \frac{y_R - y_Q}{x_R - x_Q} = \frac{3-5}{-1-2} = \frac{-2}{-3}$

$\therefore\ m_{QR} = \frac{2}{3}$

(b) Determine the equation of line PS.

$y = mx + c$ → Through point $P(0\,;2)$ → y-intercept because $x = 0$

$\therefore\ y = \frac{2}{3}x + 2$ → $m_{PS} = m_{QR}$ Lines parallel (Opposite sides of parm)

(c) Show that PQRS is a rhombus.

$d(QR) = \sqrt{(x_R - x_Q)^2 + (y_R - y_Q)^2}$ and $d(PQ) = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$

$\therefore\ d(QR) = \sqrt{(-1-2)^2 + (3-5)^2}$ $d(PQ) = \sqrt{(2-0)^2 + (5-2)^2}$

$\therefore\ d(QR) = \sqrt{(-3)^2 + (-2)^2}$ $d(PQ) = \sqrt{(2)^2 + (3)^2}$

$\therefore\ d(QR) = \sqrt{9+4}$ $d(PQ) = \sqrt{4+9}$

$\therefore\ d(QR) = \sqrt{13}$ $d(PQ) = \sqrt{13}$

$\therefore\ QR = PQ$

$\therefore$ **PQRS is a rhombus because the adjacent sides of parallelogram PQRS are equal**

(d) Calculate the coordinates of the point where the diagonals of PQRS intersect one another.

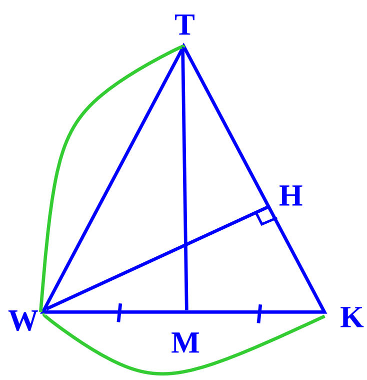
Diagonals intersect at midpoint of PR.

$\therefore\ M_{PR} = \left(\frac{x_P + x_R}{2}\,;\frac{y_P + y_R}{2}\right)$

$\therefore\ M_{PR} = \left(\frac{0-1}{2}\,;\frac{2+3}{2}\right)$

$\therefore\ M_{PR} = \left(-\frac{1}{2}\,;\frac{5}{2}\right)$

(7) TWK is an isosceles triangle with TW = WK.
T(7 ; 8), W(1 ; 6) and K(−1 ; y).



(a) Calculate the length of TW.

$d(TW) = \sqrt{(x_W - x_T)^2 + (y_W - y_T)^2}$

$\therefore\ d(TW) = \sqrt{(1-7)^2 + (6-8)^2}$

$\therefore\ d(TW) = \sqrt{(-6)^2 + (-2)^2}$

$\therefore\ d(TW) = \sqrt{36+4}$

$\therefore\ d(TW) = \sqrt{40}$

(b) Calculate the value(s) of y.

$$\mathbf{d(KW)} = \sqrt{(x_W - x_K)^2 + (y_W - y_K)^2}$$

$\therefore$ $$\mathbf{d(KW)} = \sqrt{(1-(-1))^2 + (6-y)^2}$$

But KW = TW → Given

$\therefore$ $$\sqrt{40} = \sqrt{(1+1)^2 + (6-y)^2}$$

$\therefore$ $$40 = 4 + 36 - 12y + y^2$$

$\therefore$ $$0 = y^2 - 12y$$

$\therefore$ $$y(y-12) = 0$$

$\therefore$ $y = 0$ **or** $y = 12$

(c) Determine the equation of the:

(i) median through T for $y > 0$. → **TM with M as the midpoint of WK**

$$M_{WK} = \left(\frac{x_W + x_K}{2} ; \frac{y_W + y_K}{2}\right) = \left(\frac{1-1}{2} ; \frac{6+12}{2}\right) \rightarrow y > 0 \rightarrow y = 12$$

$\therefore$ $$M = (0 ; 9)$$

$$m_{TM} = \frac{y_M - y_T}{x_M - x_T} = \frac{9-8}{0-7} = \frac{1}{-7} = -\frac{1}{7}$$ **through M = (0 ; 9) → y-intercept because $x = 0$**

$\therefore$ $y = -\frac{1}{7}x + 9$

(ii) altitude through W for $y \leq 0$. → **WH with WH ⊥ TK**

$$m_{TK} = \frac{y_K - y_T}{x_K - x_T} = \frac{0-8}{-1-7} = \frac{-8}{-8} = 1 \rightarrow y \leq 0 \rightarrow y = 0 \rightarrow K(-1 ; 0)$$

$\therefore$ $m_{WH} = -1$ → $m_1 \times m_2 = -1$ **because WH ⊥ TK**

$\therefore$ $y - y_1 = m(x - x_1)$ **through W(1 ; 6)**

$\therefore$ $$y - 6 = -1(x-1)$$

$\therefore$ $y = -1x + 1 + 6$ → $\therefore$ $y = -x + 7$

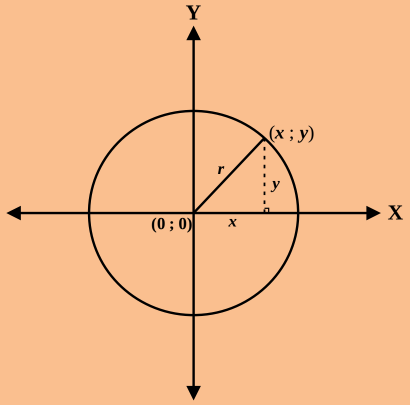
D1.2 Circles:

D1.2.1 Circles with the origin as midpoint:

The equation of the circle with midpoint (0 ; 0):

$$x^2 + y^2 = r^2$$

with $(x ; y)$ any point on the circumference of the circle and r the radius of the circle.



D1.2.2 Other circles:

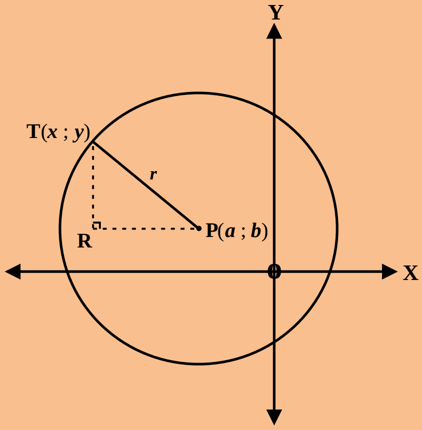
The equation of the circle with midpoint $(a\,;\,b)$:

$(x - a)^2 + (y - b)^2 = r^2$

with $(x\,;\,y)$ any point on the circumference of the circle and r the radius of the circle.

The radius can be calculated with the distance formula:

d(PT) = $\sqrt{(x_P - x_T)^2 + (y_P - y_T)^2}$



Ex. 1 Determine the coordinates of the midpoint and the length of the radius of the following circle:

$\boldsymbol{x^2 + 6x + y^2 - 4y = 12}$

Use completing of the square to write the equation in standard form [$(x - a)^2 + (y - b)^2 = r^2$]:

$$x^2 + 6x + y^2 - 4y = 12$$

$$x^2 + 6x + \left(\frac{6}{2}\right)^2 + y^2 - 4y + \left(\frac{-4}{2}\right)^2 = 12 + \left(\frac{6}{2}\right)^2 + \left(\frac{-4}{2}\right)^2$$

$$x^2 + 6x + (3)^2 + y^2 - 4y + (-2)^2 = 12 + 9 + 4$$

$$(x + 3)^2 + (y - 2)^2 = 25$$

$\therefore$ **MP =** $(-3\,;\,2)$ **and** $r^2 = 25$

$\therefore\ r = 5$

Ex. 2 Determine the equation of the circle with midpoint $(-2\,;\,5)$ **through the point** $(1\,;\,-1)$.

$(x - a)^2 + (y - b)^2 = r^2$ **with MP =** $(\overset{a}{-2}\,;\,\overset{b}{5})$

$\therefore (x - (-2))^2 + (y - 5)^2 = r^2$

$\therefore (1 + 2)^2 + (-1 - 5)^2 = r^2$ **through** $(\overset{x}{1}\,;\,\overset{y}{-1})$

$\therefore (3)^2 + (-6)^2 = r^2$

$\therefore r^2 = 9 + 36 = 45$

$\therefore (x + 2)^2 + (y - 5)^2 = 45$

Exercise 2: Date: ______________________

(1) Determine the equations of the following circles:

(a) with midpoint (5 ; 2) and radius 6.

$(x - a)^2 + (y - b)^2 = r^2$

$\therefore (x - 5)^2 + (y - 2)^2 = 6^2$

$\therefore (x - 5)^2 + (y - 2)^2 = 36$

(b) with midpoint $(-1\,;\,3)$ and radius $\sqrt{12}$.

$(x - a)^2 + (y - b)^2 = r^2$

$\therefore (x - (-1))^2 + (y - 3)^2 = (\sqrt{12})^2$

$\therefore (x + 1)^2 + (y - 3)^2 = 12$

(c) with midpoint $(4\,;\,-2)$ and through the point $(-2\,;\,0)$.

$(x - a)^2 + (y - b)^2 = r^2 \quad \rightarrow \quad (-2 - 4)^2 + (0 + 2)^2 = r^2$

$\therefore (x - 4)^2 + (y - (-2))^2 = r^2 \quad \therefore r^2 = (-6)^2 + (2)^2 = 36 + 4 = 40$

$\therefore (x - 4)^2 + (y + 2)^2 = 40$

(d) with midpoint $(-2\,;\,-3)$ and through the point $(2\,;\,-1)$.

$(x - a)^2 + (y - b)^2 = r^2 \quad \rightarrow \quad (2 + 2)^2 + (-1 + 3)^2 = r^2$

$\therefore (x - (-2))^2 + (y - (-3))^2 = r^2 \quad \therefore r^2 = (4)^2 + (2)^2 = 16 + 4 = 20$

$\therefore (x + 2)^2 + (y + 3)^2 = 20$

(2) Determine the coordinates of the midpoints and the length of the radius of the following circles:

(a) $(x - 4)^2 + (y - 2)^2 = 36 \quad \rightarrow$ MP$(4\,;2)$ with $r^2 = 36 \rightarrow r = 6$

(b) $x^2 + y^2 - 10y = 6$

$\therefore x^2 + y^2 - 10y + 25 = 6 + 25$

$\therefore x^2 + (y - 5)^2 = 31 \quad \rightarrow$ MP$(0\,;5)$ with $r^2 = 31 \rightarrow r = \sqrt{31}$

(c) $(x + 3)^2 + (y + 6)^2 = 20 \quad \rightarrow$ MP$(-3\,;-6)$ with $r^2 = 20 \rightarrow r = \sqrt{20}$

(d) $(x + 5)^2 + (y + 5)^2 = 280 \quad \rightarrow$ MP$(-5\,;-5)$ with $r^2 = 280 \rightarrow r = \sqrt{280}$

(e) $(x - 1)^2 + (y + 2)^2 = 9 \quad \rightarrow$ MP$(1\,;-2)$ with $r^2 = 9 \rightarrow r = 3$

(f) $x^2 + (y - 4)^2 = 100$ $\rightarrow$ **MP(0 ; 4)** **with** $r^2 = 100$ $\rightarrow$ $r = 10$

(g) $x^2 - 8x + y^2 - 6y = 12$

$\therefore\ x^2 - 8x + 16 + y^2 - 6y + 9 = 12 + 6 + 9$

$\therefore\ (x-4)^2 + (y-3)^2 = 27$ $\rightarrow$ **MP(4 ; 3)** **with** $r^2 = 36$ $\rightarrow$ $r = 6$

(h) $\left(x + \frac{1}{2}\right)^2 + (y + 2)^2 = 48$ $\rightarrow$ **MP**$\left(-\frac{1}{2}; -2\right)$ **with** $r^2 = 48$ $\rightarrow$ $r = \sqrt{48}$

(i) $(x-6)^2 + y^2 = 1$ $\rightarrow$ **MP(6 ; 0)** **with** $r^2 = 1$ $\rightarrow$ $r = 1$

(j) $2x^2 + 2y^2 - 4x - y = 2$

$x^2 + y^2 - 2x - \frac{1}{2}y = 1$ $\rightarrow$ $\div$ **through 2**

$\therefore\ x^2 - 2x + 1 + y^2 - \frac{1}{2}y + \frac{1}{4} = 1 + 1 + \frac{1}{4}$

$\therefore\ (x-1)^2 + \left(y - \frac{1}{2}\right)^2 = 2\frac{1}{4} = \frac{9}{4}$ $\rightarrow$ **MP**$\left(1; \frac{1}{2}\right)$ **with** $r^2 = \frac{9}{4}$ $\rightarrow$ $r = \frac{3}{2}$

(3) Determine whether the point $(3\,;\,-2)$ lies on the circle with midpoint $(-1\,;\,5)$. The radius of the circle is 8.

$(x - a)^2 + (y - b)^2 = r^2$

$\therefore\ (x - (-1))^2 + (y - 5)^2 = 8^2$

$\therefore\ (x - (-1))^2 + (y - 5)^2 = 8^2$

$\therefore\ (x+1)^2 + (y - 5)^2 = 64$

$\therefore$ LHS $= (3+1)^2 + (-2 - 5)^2 = (4)^2 + (-7)^2 = 16 + 49 = 65$

$\therefore$ **No, the point $(3\,;-2)$ does not lie on the circle but outside the circle because $65 > r^2 = 64$**

(4) The equation of the circle through the point $(-3\,;\,-1)$ is $x^2 + 10x + y^2 - 2y + p = 0$.

(a) Determine the coordinates of the midpoint of the circle.

$\therefore\ x^2 + 10x + 25 + y^2 - 2y + 1 = -p + 25 + 1$

$\therefore\ (x+5)^2 + (y-1)^2 = -p + 26$ $\rightarrow$ **MP(−5 ; 1)**

(b) Calculate the value of p.

$(x+5)^2 + (y-1)^2 = -p + 26$

$\therefore\ (-3+5)^2 + (-1-1)^2 = -p + 26$ $\rightarrow$ **through point (−3 ; −1)**

$\therefore\ (2)^2 + (-2)^2 = -p + 26$

$\therefore\ 4 + 4 = -p + 26$

$\therefore\ p = 26 - 4 - 4$

$\therefore\ p = 18$

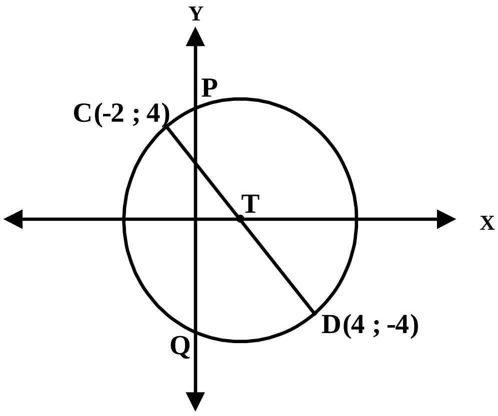
(5) Determine the equation of the circle with midpoint $(-4\,;\,-3)$ and diameter 18.

Diameter 18 → Radius = 9

$\therefore\ (x-(-4))^2 + (y-(-3))^2 = 9^2$

$\therefore\ (x+4)^2 + (y+3)^2 = 81$

(6) CD is the diameter of a circle with T as the midpoint of CD. Calculate:



(a) the coordinates of the midpoint of the circle.

$M_{CD} = \left(\frac{x_C + x_D}{2}\,;\,\frac{y_C + y_D}{2}\right)$

$\therefore\ M_{CD} = \left(\frac{-2+4}{2}\,;\,\frac{4-4}{2}\right)$

$\therefore\quad T = (1\,;\,0)$

(b) the equation of the circle.

$(x-a)^2 + (y-b)^2 = r^2$ → **With midpoint** $T(1\,;\,0)$

$\therefore\ (x-1)^2 + (y-0)^2 = r^2$

$\therefore\ (-2-1)^2 + (4-0)^2 = r^2$ → **Through** $C(-2\,;\,4)$ **or** $D(4\,;\,-4)$

$\therefore\quad r^2 = (-3)^2 + (4)^2 = 9 + 16 = 25$

$\therefore\quad (x-1)^2 + y^2 = 25$

(c) the length of PQ if P and Q are the y-intercepts of the circle.

$(0-1)^2 + y^2 = 25$ → $x = 0$ **for** y**-intercepts**

$\therefore\quad y^2 = 25 - 1 = 24$

$\therefore\quad y = \pm\sqrt{24}$

$\therefore\quad PQ = 2\sqrt{24}$

(d) The equation of diameter CD.

$m_{CD} = \frac{y_D - y_C}{x_D - x_C} = \frac{-4-4}{4-(-2)} = \frac{-8}{6} = -\frac{4}{3}$

$y - y_1 = m(x - x_1)$

$\therefore\ y - 4 = -\frac{4}{3}(x-(-2))$ → **Through** $C(-2\,;\,4)$ **or** $D(4\,;\,-4)$

$\therefore\ y - 4 = -\frac{4}{3}(x+2)$

$\therefore\quad y = -\frac{4}{3}x - \frac{8}{3} + 4$

$\therefore\quad y = -\frac{4}{3}x + \frac{4}{3}$

or

$\therefore\ y-(-4) = -\frac{4}{3}(x-4)$

$\therefore\quad y + 4 = -\frac{4}{3}x + \frac{16}{3}$

$\therefore\quad y = -\frac{4}{3}x + \frac{16}{3} - 4$

$\therefore\quad y = -\frac{4}{3}x + \frac{4}{3}$