

Grade 7 – Answer book

CONTENTS:

	<u>Page:</u>
A1. Whole numbers	3
A2. Exponents	12
A3. Fractions	21
A4. Percentage, rate and ratio	58
A5. Functions and relationships (Graphs)	79
B1. Number patterns	98
B2. Algebraic expressions and equations	107
B3. Integers	122
B4. Statistics	134
B5. Probability	161
C1. Construction and measurement	167
C2. Geometry of 2-D shapes	176
C3. Perimeter and area	196
C4. Three dimensional objects	213
C5. Transformation	224

This book was compiled and processed by E.J. Du Toit in 2013.

Revised edition 2020. Newest version 2022.

Contact: info@abcbooks.co.za

Copyright© 2013. All copyrights are reserved. No part of this publication may be reproduced in any form unless written consent is obtained.

ISBN 978-1-920505-00-4

Also visit www.abcmathsandscience.co.za for extra exercise, tests and exam papers.

Chapter A1

Whole numbers

A1.1 Natural and Whole Numbers:

Exercise 1:

Complete: * Natural numbers: $\mathbb{N} = \{1; 2; 3; 4; \dots\}$

* Whole numbers: $\mathbb{N}_0 = \{0; 1; 2; 3; 4; \dots\}$

A1.2 Characteristics of Whole Numbers – Revision grade 6:

- * Prime numbers are numbers with only two factors, namely 1 and the number itself. Therefore, the number 1 is **not** a prime number, because it has only one factor!
- * Compound numbers are numbers with more than two factors.
- * The number 0 is the identity element for addition, which means that:
 $0 + \text{any number} = \text{the number}$. E.g. $0 + 5 = 5$
- * The number 1 is the identity element for multiplication, which means that:
 $1 \times \text{any number} = \text{the number itself}$. E.g. $1 \times 7 = 7$
- * If we multiply any number with 0, the answer is 0. E.g. $0 \times 16 = 0$
- * If we divide 0 by any number, the answer is 0: E.g. $0 \div 189 = 0$
- * **If we divide by 0, the answer will be undefined.**
E.g. $24 \div 0 = \text{undefined}$
- * Factors are the numbers by which a number is dividable without a remainder.
 E.g. the factors of 6 are 1 ; 2 ; 3 and 6. We write it as: $F_6 = \{1; 2; 3; 6\}$
- * Multiples are the numbers where a number can be divided without a remainder.
 E.g. multiples of 6 are 6 ; 12 ; 18 We write it as: $M_6 = \{6; 12; 18; 24; \dots\}$
- * Commutative property: E.g. $4 + 5 = 5 + 4$ or $4 \times 5 = 5 \times 4$
- * Associative property: E.g. $(2 + 3) + 4 = 2 + (3 + 4)$ or $(2 \times 3) \times 4 = 2 \times (3 \times 4)$
- * Distributive property: E.g. $2 \times (3 + 4) = 2 \times 3 + 2 \times 4$ or $2 \times (3 - 4) = 2 \times 3 - 2 \times 4$
- * **Remember the order of operations:**
 - (1) Brackets
 - (2) Powers and roots
 - (3) Of $\rightarrow \times$
 - (4) Multiplication and division
 - (5) Plus and minus

Exercise 2:

(1) Complete:

- (a) The natural numbers smaller than 10: **1; 2; 3; 4; 5; 6; 7; 8; 9**
- (b) The five uneven numbers just before 10 000: **9 991; 9 993; 9 995; 9 997; 9 999**
- (c) The first five prime numbers: **2; 3; 5; 7; 11**
- (d) The even whole numbers between 325 and 341: **326; 328; 330; 332; 334; 336; 338; 340**
- (e) The first four natural numbers greater than 25: **26; 27; 28; 29**
- (f) The factors of 12: **1; 2; 3; 4; 6; 12**
- (g) The multiples of 12 between 20 and 80: **24; 36; 48; 60; 72**
- (h) The multiples of 9, from 18 to 54: **18; 27; 36; 45; 54**
- (i) The largest six-digit number. Write this number in words as well. **999 999**
Nine hundred and ninety-nine thousand nine hundred and ninety-nine
- (j) Add the largest five-digit number to the smallest three-digit number.

$$\begin{array}{r} 99\,999 \\ + \quad 100 \\ \hline 100\,099 \end{array}$$
 $99\,999 + 100 = 100\,099$
- (k) Subtract the largest two-digit number from the smallest four-digit number

$$\begin{array}{r} 1\,000 \\ - \quad 99 \\ \hline 901 \end{array}$$
 $1\,000 - 99 = 901$
- (l) Is the number 1 a prime number or a compound number? **Neither**
- (m) The months of the year that consist of an even number of days in the year 2007.
February; April; June; September; November
- (n) The factors of 36: **1; 2; 3; 4; 6; 9; 12; 18; 36**
- (o) The multiples of 8: **8; 16; 24; 32; ...**
- (p) The factors of 60 which are also prime numbers: **2; 3; 5**

(2) Complete the next four numbers of each of the following sequences:

- (a) 4 567⁺³; 4 570; 4 573; 4 576; **4 579; 4 582; 4 585; 4 588**
- (b) 12 346⁻¹⁰⁰; 12 246; 12 146; 12 046; **11 946; 11 846; 11 746; 11 646**
- (c) 128⁺²; 130⁺³; 133⁺⁴; 137⁺⁵; **142⁺⁶; 148⁺⁷; 155⁺⁸; 163**
- (d) 26⁺²; 28; 30; 32; **34; 36; 38; 40**
- (e) 144 578⁻⁴⁰; 144 538; 144 498; 144 458; **144 418; 144 378; 144 338; 144 298**
- (f) 2^{×2}; 4; 8; 16; **32; 64; 128; 256**
- (g) 1⁺; 4⁺; 5⁺; 9; 14; **23; 37; 60; 97**
- (h) 1⁺²; 3; 5; 7; **9; 11; 13; 15**
- (i) 1 999⁻¹⁰⁰; 1 899; 1 799; 1 699; **1 599; 1 499; 1 399; 1 299**
- (j) 1; 4; 9; 16; **25; 36; 49; 64** [$1 \times 1; 2 \times 2; 3 \times 3; 4 \times 4; \dots$]
- (k) 1; 8; 27; 64; **125; 216; 343; 512** [$1 \times 1 \times 1; 2 \times 2 \times 2; 3 \times 3 \times 3; \dots$]

(3) Determine the value of: **[Remember the order of operations!]**

$$(a) \quad 17 \div 1 = \frac{17}{1} = 17$$

$$(b) \quad 1 \times 1 \times 1 + 0 = 1 + 0 = 1$$

$$(c) \quad 2 + 2 \times 0 + 2 \times 1 = 2 + 0 + 2 = 4$$

$$(d) \quad 389 \div 0 = \frac{389}{0} = \text{undefined}$$

$$(e) \quad 0 \div 983 = \frac{0}{983} = 0$$

$$(f) \quad 64 - 0 = 64$$

$$(g) \quad \frac{0 + 5 \times 1}{7 - 7}$$

$$(h) \quad \frac{(14 - 14) \times 0}{18 \div 1 + 0}$$

$$= \frac{0 + 5}{0} = \frac{5}{0} = \text{undefined}$$

$$= \frac{0 \times 0}{18 + 0} = \frac{0}{18} = 0$$

(4) Round the following off to the nearest number, as indicated in brackets:

$$(a) \quad 3\,472 \text{ (nearest 10)}$$

$$\approx 3\,470$$

$$(b) \quad 3\,472 \text{ (nearest 5)}$$

$$\approx 3\,470$$

$$(c) \quad 3\,475 \text{ (nearest 100)}$$

$$\approx 3\,500$$

$$(d) \quad 769\,909 \text{ (nearest 10)}$$

$$\approx 769\,910$$

$$(e) \quad 769\,909 \text{ (nearest 1\,000)}$$

$$\approx 770\,000$$

$$(f) \quad 769\,909 \text{ (nearest 100)}$$

$$\approx 769\,900$$

$$(g) \quad 769\,909 \text{ (nearest 5)}$$

$$\approx 769\,910$$

$$(h) \quad 567 \text{ (nearest 10)}$$

$$\approx 570$$

$$(i) \quad 567 \text{ (nearest 5)}$$

$$\approx 565$$

$$(j) \quad 567 \text{ (nearest 100)}$$

$$\approx 600$$

(5) Write the following answers:

$$(a) \quad 34 \times 1\,000$$

$$= 34\,000$$

$$(b) \quad 50 \times 400 \quad [5 \times 4 = 20]$$

$$= 20\,000$$

$$(c) \quad 48\,000 \div 1\,000 = \frac{48\,000}{1\,000}$$

$$= 48$$

$$(d) \quad 680 \div 10 = \frac{680}{10}$$

$$= 68$$

$$(e) \quad 5\,600 \div 100 = \frac{5\,600}{100}$$

$$= 56$$

$$(f) \quad 300 \times 10\,000$$

$$= 3\,000\,000$$

$$(g) \quad 800 \times 120 \quad [8 \times 12 = 96]$$

$$= 96\,000$$

$$(h) \quad 451 \times 100$$

$$= 45\,100$$

$$(i) \quad 770 \div 110 = \frac{770}{110}$$

$$= 7$$

$$(j) \quad 350\,000 \div 50 = \frac{350\,000}{50}$$

$$= 7\,000$$

(6) Fill in: < ; = or >

- | | | | | | | | |
|-----|---------|---|----------------------------|-----|------------------------|---|----------------------------|
| (a) | 25 026 | > | 25 025 | (b) | 8 100 | = | $81 \times 100 = 8\ 100$ |
| (c) | 123 587 | < | 123 588 | (d) | 487 | > | 477 |
| (e) | 1 987 | < | 1 989 | (f) | 100 999 | < | 101 000 |
| (g) | 520 520 | > | 520 250 | (h) | 2 345 | < | 2 453 |
| (i) | 7 070 | = | $70\ 700 \div 10 = 7\ 070$ | (j) | $2\ 300 \div 10 = 230$ | > | $23\ 000 \div 1\ 000 = 23$ |

(7) Use the distributive property to calculate the following:

(a) E.g. $345 \times 16 = 345 \times (6 + 10)$

$$\begin{aligned}
 &= 345 \times 6 + 345 \times 10 \\
 &= 2\ 070 + 3\ 450 \\
 &= 5\ 520
 \end{aligned}$$

$$\begin{array}{r}
 \begin{array}{r} 2\ 3 \\ 345 \end{array} \\
 \times \quad 6 \\
 \hline
 2\ 070 \\
 + 3\ 450 \\
 \hline
 5\ 520
 \end{array}$$

(b) $257 \times 25 = 257 \times (20 + 5)$

$$\begin{aligned}
 &= 257 \times 20 + 257 \times 5 \\
 &= 5\ 140 + 1\ 285 \\
 &= 6\ 425
 \end{aligned}$$

$$\begin{array}{r}
 257 \\
 \times \quad 5 \\
 \hline
 1\ 285 \\
 + 5\ 140 \\
 \hline
 6\ 425
 \end{array}$$

(c) $1\ 234 \times 12 = 1\ 234 \times (2 + 10)$

$$\begin{aligned}
 &= 1\ 234 \times 2 + 1\ 234 \times 10 \\
 &= 2\ 468 + 12\ 340 \\
 &= 14\ 808
 \end{aligned}$$

$$\begin{array}{r}
 2\ 468 \\
 + 12\ 340 \\
 \hline
 14\ 808
 \end{array}$$

(d) $780 \times 34 = 780 \times (30 + 4)$

$$\begin{aligned}
 &= 780 \times 30 + 780 \times 4 \\
 &= 23\ 400 + 3\ 120 \\
 &= 26\ 520
 \end{aligned}$$

$$\begin{array}{r}
 \begin{array}{r} 3 \\ 780 \end{array} \\
 \times \quad 4 \\
 \hline
 3\ 120 \\
 + 23\ 400 \\
 \hline
 26\ 520
 \end{array}$$

(8) Identify the property used. Write A – associative property, C – commutative property and D – distributive property or N – if none of the properties apply.

- | | |
|---|----------|
| (a) $45 + 67 = 67 + 45$ | C |
| (b) $18 \times 12 = 18 \times (10 + 2)$ | D |
| (c) $56 - 44 = 44 - 56$ | N |
| (d) $2 \times 6 \times 18 = 2 \times (6 \times 18)$ | A |
| (e) $(29 + 35) + 15 = 29 + (35 + 15)$ | A |
| (f) $7 \times 234 = 234 \times 7$ | C |
| (g) $450 \times 36 = 450 \times 40 - 450 \times 4$ | D |
| (h) $128 \div (8 \times 8) = (128 \div 8) \times 8$ | N |

C
D
N
A
A
C
D
N

A1.3 Prime factors:

The prime factors of a number are the factors that are also prime numbers.

E.g. the factors of 12 are 1 ; 2 ; 3 ; 4 ; 6 and 12. But the prime factors of 12 are only 2 and 3.

E.g. 1 The factors of 6 are: $F_6 = \{1; 2; 3; 6\}$

∴ The prime factors of 6: **2 and 3.** (I.e. they are the factors that are prime numbers.)

E.g. 2 The factors of 20 are: $F_{20} = \{1; 2; 4; 5; 10; 20\}$

∴ The prime factors of 20: **2 and 5.**

E.g. 3 Determine the prime factors of 60:
$$\begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array} \quad \therefore 60 = 2 \times 2 \times 3 \times 5$$

Exercise 3:

Determine the prime factors of the following numbers:

$$(1) \begin{array}{r|l} 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

$$\therefore 12 = 2 \times 2 \times 3$$

$$(2) \begin{array}{r|l} 2 & 56 \\ 2 & 28 \\ 2 & 14 \\ 7 & 7 \\ & 1 \end{array}$$

$$\therefore 56 = 2 \times 2 \times 2 \times 7$$

$$(3) \begin{array}{r|l} 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$$\therefore 30 = 2 \times 3 \times 5$$

$$(4) \begin{array}{r|l} 2 & 44 \\ 2 & 22 \\ 11 & 11 \\ & 1 \end{array}$$

$$\therefore 44 = 2 \times 2 \times 11$$

$$(5) \begin{array}{r|l} 2 & 148 \\ 2 & 74 \\ 37 & 37 \\ & 1 \end{array}$$

$$\therefore 148 = 2 \times 2 \times 37$$

$$(6) \begin{array}{r|l} 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$$\therefore 27 = 3 \times 3 \times 3$$

$$(7) \begin{array}{r|l} 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$$\therefore 18 = 2 \times 3 \times 3$$

$$(8) \begin{array}{r|l} 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$$\therefore 100 = 2 \times 2 \times 5 \times 5$$

$$(9) \begin{array}{r|l} 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

$$\therefore 24 = 2 \times 2 \times 2 \times 3$$

$$(10) \begin{array}{r|l} 2 & 640 \\ 2 & 320 \\ 2 & 160 \\ 2 & 80 \\ 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ 5 & 5 \\ & 1 \end{array}$$

$$\therefore 640 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5$$

$$(11) \begin{array}{r|l} 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$$\therefore 36 = 2 \times 2 \times 3 \times 3$$

$$(12) \begin{array}{r|l} 3 & 75 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$$\therefore 75 = 3 \times 5 \times 5$$

A1.4 LCM and HCF:

LCM = Lowest common multiple.

HCF = Highest common factor.

E.g.4 The multiples of 3 are $M_3 = \{3; 6; 9; 12; 15; 18; 21; 24; \dots\}$

The multiples of 4 are $M_4 = \{4; 8; 12; 16; 20; 24; 28; \dots\}$

The common multiples of 3 and 4 are all the multiples that occur with both, namely:
12; 24; ...

Therefore the LCM of 3 and 4 is **12**, because it is the lowest common multiple!

E.g.5 The factors of 12 are $F_{12} = \{1; 2; 3; 4; 6; 12\}$

The factors of 18 are $F_{18} = \{1; 2; 3; 6; 9; 18\}$

The common factors of 12 and 18 are thus: 1; 2; 3 and 6.

Therefore the HCF of 12 and 18 is **6**, because it is the highest common factor!

Exercise 4:

Complete:

- (1) (a) The multiples of 2: **{2; 4; 6; 8; 10; 12; 14; 16; 18; ...}**
- (b) The multiples of 3: **{3; 6; 9; 12; 15; 18; 21; 24; ...}**
- (c) The common multiples of 2 and 3: **6; 12; 18; ...**
- (d) The LCM of 2 and 3: **6**
- (2) (a) $M_6 =$ **{6; 12; 18; 24; 30; 36; 42; 48; 54; 60; ...}**
- (b) $M_{15} =$ **{15; 30; 45; 60; 75; ...}**
- (c) The common multiples of 6 and 15: **30; 60; ...**
- (d) The LCM of 6 and 15: **30**
- (3) (a) $F_{12} =$ **{1; 2; 3; 4; 6; 12}**
- (b) $F_8 =$ **{1; 2; 4; 8}**
- (c) The common factors of 12 and 8: **1; 2 and 4**
- (d) HCF of 12 and 8: **4**
- (4) (a) $M_{10} =$ **{10; 20; 30; 40; 50; 60; ...}** and $F_{10} =$ **{1; 2; 5; 10}**
- (b) $M_{15} =$ **{15; 30; 45; 60; ...}** and $F_{15} =$ **{1; 3; 5; 15}**
- (c) LCM of 10 and 15: **30**
- (d) HCF of 10 and 15: **5**

- (5) (a) The multiples of 3: {3; 6; 9; 12;; 51; 54; 57; 60; 63;}
- (b) The multiples of 4: {4; 8; 12; 16;; 52; 56; 60; 64;}
- (c) The multiples of 5: {5; 10; 15; 20;; 50; 55; 60; 65;}
- (d) The LCM of 3; 4 and 5: 60
- (6) (a) $M_3 = \{3; 6; 9; 12; \dots \dots; 45; 48; 51; 54; 57; 60; 63; \dots \dots\}$
- (b) $M_5 = \{5; 10; 15; 20; \dots \dots; 45; 50; 55; 60; 65; \dots \dots\}$
- (c) $M_9 = \{9; 18; 27; 36; 45; 54; 60; \dots \dots\}$
- (d) LCM of 3; 5 and 9: 45
- (7) (a) $F_{20} = \{1; 2; 4; 5; 10; 20\}$
- (b) $F_{36} = \{1; 2; 3; 4; 6; 9; 12; 18; 36\}$
- (c) $F_{28} = \{1; 2; 4; 7; 14; 28\}$
- (d) HCF of 20; 36 and 28: 4

- (8) Paul and John are on a hike to raise funds. Paul hikes exactly 12 kilometres every day and John hikes exactly 10 kilometres every day. Calculate how many days it will take them to hike exactly the same number of kilometres. How many kilometres in total did each of them hike at that stage?

Total after	①	②	③	④	⑤	⑥	⑦ days
Paul:	12	24	36	48	60	72	84	
John:	10	20	30	40	50	60	70	

After Paul has hiked for 5 days and John for 6 days, they both hiked a total of 60 km.

- (9) The product of two numbers is 588. The HCF of the numbers is 14. Give all the possible combinations of numbers to which these conditions apply.

$$14 \times 42 = 588, \text{ because } 14 \text{ and } 42 \text{ are both divisible by } 14.$$

The HCF of 14 and 42 is equal to 14.

$28 \times 21 = 588$, but 21 is not divisible by 14, therefore 28×21 is not a possible answer, because both conditions are not applicable.

Or $12 \times 49 = 588$, but neither 12 nor 49 is divisible by 14, therefore 12×49 is not a possible answer because both conditions are not applicable.

© The ancient Romans used certain symbols to represent their numbers. Do research to find out which symbols they used for each of the following:

1 → **I**

5 → **V**

10 → **X**

50 → **L**

100 → **C**

500 → **D**

1 000 → **M**

(1) Which number is represented by the following: MCXLVI ? **1 146**

(2) Present the following number as a Roman numeral: 3 914 **MMMCMXIV**

A1.5 REVISION EXERCISE:

(1) Complete:

(a) The uneven compound numbers, larger than 10 and smaller than 20: **15**

(b) The factors of 12, which are also multiples of 2: $F_{12} = \{1; 2; 3; 4; 6; 12\} \rightarrow \{2; 4; 6; 12\}$

(c) All even prime numbers: **2**

(d) The five whole numbers just greater than 9 998:

9 999 ; 10 000 ; 10 001 ; 10 002 ; 10 003

(e) Write 2 344 298 in words:

Two million three hundred forty-four thousand two hundred and ninety eight

(f) The first four whole numbers just smaller than 12: **11 ; 10 ; 9 ; 8**

(2) Complete:

(a) $M_6 = \{6; 12; 18; 24; 30; 36; 42; 48; \dots \dots \dots\}$

(b) $M_8 = \{8; 16; 24; 32; 40; 48; 56; \dots \dots \dots\}$

(c) $F_{18} = \{1; 2; 3; 6; 9; 18\}$

(d) $F_{24} = \{1; 2; 3; 4; 6; 8; 12; 24\}$

(3) Use your answer in (2) and determine the LCM of 6 and 8: **24**

(4) Use your answer in (2) and determine the HCF of 18 and 24: **6**

(5) Complete the next five numbers of each of the following sequences:

$$(a) \quad \overset{-1}{97} ; \overset{-2}{96} ; \overset{-3}{94} ; \overset{-4}{91} ; \overset{-5}{87} ; \overset{-6}{82} ; \overset{-7}{76} ; \overset{-8}{69} ; \overset{-9}{61} ; 52$$

$$(b) \quad 14 ; \overset{+3}{17} ; 20 ; 23 ; 26 ; 29 ; 32 ; 35 ; 38 ; 41$$

$$(c) \quad 144 ; \overset{-12}{132} ; 120 ; 108 ; 96 ; 84 ; 72 ; 60 ; 48$$

$$(d) \quad 3 ; \overset{\times 2}{6} ; 12 ; 24 ; 48 ; 96 ; 192 ; 384 ; 768 ; 1536$$

(6) Determine the prime factors of the following numbers:

$$(a) \quad \begin{array}{r|l} 5 & 315 \\ 3 & 63 \\ 3 & 21 \\ 7 & 7 \\ & 1 \end{array}$$

$$(b) \quad \begin{array}{r|l} 2 & 144 \\ 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$$(c) \quad \begin{array}{r|l} 2 & 98 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

$$(d) \quad \begin{array}{r|l} 5 & 525 \\ 5 & 105 \\ 3 & 21 \\ 7 & 7 \\ & 1 \end{array}$$

$$\therefore 315 = 5 \times 3 \times 3 \times 7$$

$$\therefore 98 = 2 \times 7 \times 7$$

$$\therefore 525 = 5 \times 5 \times 3 \times 7$$

$$\therefore 144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

Chapter A2

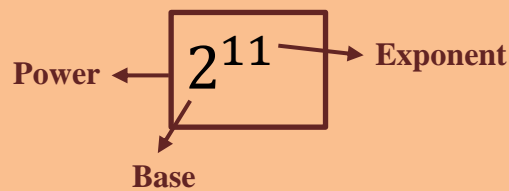
Exponents

A2.1 Numbers in exponential format:

* Exponential format is a way to write large numbers in a shorter format.

E.g. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{11} = 2\,048$

* We read 2^{11} as “two to the power of 11”, where 2 is called the base number and 11 the exponent:



* We read 2^2 as “two to the power of two” or, “2 squared” which means that it is $2 \times 2 = 4$, where 4 is a perfect square.

* Also $2^3 = 2 \times 2 \times 2 = 8$, where 8 is therefore a perfect cube.

* Any number to the power of 1, is equal to the number itself. E.g. $6^1 = 6$.

** Any number to the power of 0, is equal to 1. E.g. $6^0 = 1$. (Only for enrichment!)

E.g. 1 (a) Write in exponential format:

$$\begin{aligned} & 5 \times 2 \times 2 \times 2 \times 5 \times 5 \times 2 \\ &= 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \\ &= 2^4 \times 5^3 \end{aligned}$$

(b) Write in expanded format: $3^5 = 3 \times 3 \times 3 \times 3 \times 3$

(c) Calculate: Remember the order of operations!

(i) $2^3 + 7^2 - 9^1 = 2 \times 2 \times 2 + 7 \times 7 - 9 = 8 + 49 - 9 = 48$

(ii) $(4 \times 10^4) + (7 \times 10^3) + (2 \times 10^2) + (6 \times 10^1) + (5 \times 1)$
 $= 40\,000 + 7\,000 + 200 + 60 + 5$
 $= 47\,265$

Because $[4 \times 10^4 = 4 \times 10\,000 = 40\,000]$ and $[7 \times 10^3 = 7 \times 1\,000 = 7\,000]$ and $[2 \times 10^2 = 2 \times 100 = 200]$ and $[6 \times 10^1 = 6 \times 10 = 60]$ and $[5 \times 1 = 5]$

Exercise 1:

(1) Complete the table. Mark only with \checkmark in each applicable block.

Number:	1	6	8	9	12	16	25	27	30	36	64	80	100	125
Perfect square	\checkmark			\checkmark		\checkmark	\checkmark			\checkmark	\checkmark		\checkmark	
Perfect cube	\checkmark		\checkmark					\checkmark			\checkmark			\checkmark

(2) Write in exponential format:

(a) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7$

(b) $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^8$

(c) $10 \times 10 \times 7 \times 10 \times 10 \times 10 = 7^1 \times 10^5$

(d) $5 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5 \times 2 = 2^4 \times 5^4$

(3) Write in expanded format:

(a) $6^8 = 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$

(b) $3^4 = 3 \times 3 \times 3 \times 3$

(4) Calculate: (Without using a calculator!)

(a) $4^2 - 3^2$
 $= 16 - 9 = 7$

(b) $1^3 + 1^2 + 2^2$
 $= 1 + 1 + 4 = 6$

(c) $5^2 - 4^2 - 3^2$
 $= 25 - 16 - 9$
 $= 0$

(d) $4^3 \times 10^2 - 5^3$
 $= 64 \times 100 - 125$
 $= 6275$

(e) $5^3 \div 5^2$
 $= 125 \div 25 = \frac{125}{25}$
 $= 5$

(f) $12^2 - 11^2$
 $= 144 - 121$
 $= 23$

(g) $5^1 + 1^5$
 $= 5 + 1 = 6$

(h) $7^2 - 2^3$
 $= 49 - 8 = 41$

(i) $2 \times 6^2 + 8^2$
 $= 2 \times 36 + 64$
 $= 72 + 64 = 136$

(j) $(20 - 2 \times 4)^2$
 $= (20 - 8)^2$
 $= (12)^2 = 144$

(k) $2^2 \times 2^3$
 $= 4 \times 8 = 32$

(l) $3^3 - 3^2$
 $= 27 - 9 = 18$

$$\begin{aligned}
 \text{(m)} \quad & 10^3 \div 10^3 \\
 & = 1\,000 \div 1\,000 \\
 & = 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(n)} \quad & 9^2 \times 1^2 \div 3^2 \\
 & = 81 \times 1 \div 9 \\
 & = 81 \div 9 \\
 & = 9
 \end{aligned}$$

$$\begin{aligned}
 \text{(o)} \quad & (4^2 + 2^2) \times (1^2 + 1^3) \\
 & = (16 + 4) \times (1 + 1) \\
 & = 20 \times 2 \\
 & = 40
 \end{aligned}$$

$$\begin{aligned}
 \text{(p)} \quad & (7 - 3)^3 + (4 + 1)^2 \\
 & = (4)^3 + (5)^2 \\
 & = 64 + 25 \\
 & = 89
 \end{aligned}$$

$$\begin{aligned}
 \text{*(q)} \quad & (3^3 \times 10^5)^0 + 10^2 \times 10^3 \\
 & = (27 \times 100\,000)^0 + 100 \times 1\,000 \\
 & = 1 + 100\,000 \\
 & = 100\,001
 \end{aligned}$$

$$\begin{aligned}
 \text{(r)} \quad & 4^2 + (8 - 3)^3 + (8 + 3)^2 \\
 & = 16 + (5)^3 + (11)^2 \\
 & = 16 + 125 + 121 \\
 & = 262
 \end{aligned}$$

(5) Calculate:

$$\begin{aligned}
 \text{(a)} \quad & (4 \times 10^3) + (3 \times 10^2) + (2 \times 10^1) + (1 \times 10^0) \\
 & = 4\,000 + 300 + 20 + 1 \\
 & = 4\,321
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & (7 \times 10^6) + (2 \times 10^5) + (2 \times 10^4) + (1 \times 10^3) + (2 \times 10^2) + (6 \times 10) + (9 \times 10^0) \\
 & = 7\,000\,000 + 200\,000 + 20\,000 + 1\,000 + 200 + 60 + 9 \\
 & = 7\,221\,269
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & (5 \times 10^5) + (6 \times 10^3) + (6 \times 10) \\
 & = 500\,000 + 6\,000 + 60 \\
 & = 506\,060
 \end{aligned}$$

- ☺ The American government decides to donate one trillion \$ to Africa for the treatment and prevention of AIDS. If one \$ is equal to R12, calculate how many rand will be donated to Africa. Write your answer in shortened format by using exponential notation.

Remember: 1 billion = 1 thousand million
 1 trillion = 1 million million

$$\begin{aligned}
 \text{Donation} &= 1 \text{ trillion dollar} \times \text{R12} \\
 &= 1 \text{ million million dollar} \times \text{R12} \\
 &= 1\,000\,000\,000\,000 \text{ dollar} \times \text{R12} \\
 &= \text{R12}\,000\,000\,000\,000 \\
 &= \text{R12} \times 10^{12}
 \end{aligned}$$

A2.2 Square roots and cube roots:

The opposite calculation of powers is called the calculation of roots.

E.g. if $5^2 = 25$, then $\sqrt{25} = 5$. We read it as: the square root of 25 is equal to 5.

Or if $2^3 = 8$, then $\sqrt[3]{8} = 2$. We read it as: the cube root of 8 is equal to 2.

E.g. 2 Calculate: (a) $\sqrt{100} - \sqrt{64} = 10 - 8 = 2$

(b) $\sqrt{100 - 64} = \sqrt{36} = 6$

(c) $\sqrt[3]{4^2 - 8} = \sqrt[3]{16 - 8} = \sqrt[3]{8} = 2$

Exercise 2:

(1) Complete the following table and study it!

(a)	$1^2 = 1 \quad \therefore \sqrt{1} = 1$
(b)	$2^2 = 4 \quad \therefore \sqrt{4} = 2$
(c)	$3^2 = 9 \quad \therefore \sqrt{9} = 3$
(d)	$4^2 = 16 \quad \therefore \sqrt{16} = 4$
(e)	$5^2 = 25 \quad \therefore \sqrt{25} = 5$
(f)	$6^2 = 36 \quad \therefore \sqrt{36} = 6$
(g)	$7^2 = 49 \quad \therefore \sqrt{49} = 7$
(h)	$8^2 = 64 \quad \therefore \sqrt{64} = 8$
(i)	$9^2 = 81 \quad \therefore \sqrt{81} = 9$

(j)	$10^2 = 100 \quad \therefore \sqrt{100} = 10$
(k)	$11^2 = 121 \quad \therefore \sqrt{121} = 11$
(l)	$12^2 = 144 \quad \therefore \sqrt{144} = 12$
(m)	$1^3 = 1 \quad \therefore \sqrt[3]{1} = 1$
(n)	$2^3 = 8 \quad \therefore \sqrt[3]{8} = 2$
(o)	$3^3 = 27 \quad \therefore \sqrt[3]{27} = 3$
(p)	$4^3 = 64 \quad \therefore \sqrt[3]{64} = 4$
(q)	$5^3 = 125 \quad \therefore \sqrt[3]{125} = 5$
(r)	$10^3 = 1\,000 \quad \therefore \sqrt[3]{1\,000} = 10$

(2) Calculate:

$$(a) \quad \sqrt{36} - \sqrt{4}$$

$$= 6 - 2 = 4$$

$$(b) \quad \sqrt{100 - 64}$$

$$= \sqrt{36} = 6$$

$$(c) \quad \sqrt[3]{8} \times \sqrt{100}$$

$$= 2 \times 10 = 20$$

$$(d) \quad \sqrt{64} - \sqrt[3]{64}$$

$$= 8 - 4 = 4$$

$$(e) \quad 5^2 + \sqrt{25}$$

$$= 25 + 5 = 30$$

$$(f) \quad 11^2 - \sqrt{121}$$

$$= 121 - 11 = 110$$

$$(g) \quad \sqrt{9} - \sqrt[3]{27}$$

$$= 3 - 3 = 0$$

$$(h) \quad (\sqrt[3]{125})^2$$

$$= (5)^2 = 25$$

$$(i) \quad 2^3 + \sqrt[3]{8}$$

$$= 8 + 2 = 10$$

$$(j) \quad \sqrt{9 - 2^3}$$

$$= \sqrt{9 - 8} = \sqrt{1} = 1$$

$$(k) \quad \sqrt{49 - 13}$$

$$= \sqrt{36} = 6$$

$$(l) \quad 6^2 + 4^3$$

$$= 36 + 64 = 100$$

$$(m) \quad \sqrt{4^3}$$

$$= \sqrt{64} = 8$$

$$(n) \quad 7 \times 7 \times 7 - 7^3$$

$$= 7^3 - 7^3 = 0 \quad \text{or} \quad 343 - 343 = 0$$

$$(o) \quad \sqrt{10^2 - 8^2}$$

$$= \sqrt{100 - 64}$$

$$= \sqrt{36} = 6$$

$$(p) \quad \sqrt[3]{1\,000} \times \sqrt{144}$$

$$= 10 \times 12$$

$$= 120$$

$$(q) \quad (\sqrt{12})^2 \rightarrow (\sqrt{\quad})^2$$

$$= 12$$

$$(r) \quad \sqrt{1} + 7^2 - \sqrt[3]{8}$$

$$= 1 + 49 - 2 = 48$$

(3) The area of a square is 121 cm^2 . Calculate the length of each side of the square.

$$\text{Area} = L \times L = L^2 = 121 = 11^2$$

$$\therefore L = 11$$

The length of each side is equal to 11 cm.

A2.3 Square roots and cube roots – using prime factors:

E.g.3 Determine the following by using prime factors:

(a) $\sqrt{784}$

(b) $\sqrt[3]{3375}$

$$\begin{array}{r|l} 2 & 784 \\ 2 & 392 \\ 2 & 196 \\ 2 & 98 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

$$\begin{array}{r|l} 3 & 3375 \\ 3 & 1125 \\ 3 & 375 \\ 5 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$$\begin{aligned} \therefore 784 &= 2 \times 2 \times 2 \times 2 \times 7 \times 7 \\ &= 2^2 \times 2^2 \times 7^2 \end{aligned}$$

$$\begin{aligned} \therefore 3375 &= 3 \times 3 \times 3 \times 5 \times 5 \times 5 \\ &= 3^3 \times 5^3 \end{aligned}$$

$$\begin{aligned} \therefore \sqrt{784} &= 2 \times 2 \times 7 \\ &= \mathbf{28} \end{aligned}$$

$$\begin{aligned} \therefore \sqrt[3]{3375} &= 3 \times 5 \\ &= \mathbf{15} \end{aligned}$$

Exercise 3:

Calculate: (by using prime factors)

(1) $\sqrt{225}$

(2) $\sqrt[3]{2744}$

$$\begin{array}{r|l} 5 & 225 \\ 5 & 45 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 2744 \\ 2 & 1372 \\ 2 & 686 \\ 7 & 343 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

$$\begin{aligned} \therefore 225 &= 5 \times 5 \times 3 \times 3 \\ &= 5^2 \times 3^2 \end{aligned}$$

$$\begin{aligned} \therefore 2744 &= 2 \times 2 \times 2 \times 7 \times 7 \times 7 \\ &= 2^3 \times 7^3 \end{aligned}$$

$$\begin{aligned} \therefore \sqrt{225} &= 5 \times 3 \\ &= \mathbf{15} \end{aligned}$$

$$\begin{aligned} \therefore \sqrt[3]{2744} &= 2 \times 7 \\ &= \mathbf{14} \end{aligned}$$

(3) $\sqrt{1\,225}$

$$\begin{array}{r|l} 5 & 1\,225 \\ 5 & 245 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

$$\begin{aligned} \therefore 1\,225 &= 5 \times 5 \times 7 \times 7 \\ &= 5^2 \times 7^2 \end{aligned}$$

$$\begin{aligned} \therefore \sqrt{1\,225} &= 5 \times 7 \\ &= \mathbf{35} \end{aligned}$$

(4) $\sqrt{4\,624}$

$$\begin{array}{r|l} 2 & 4\,624 \\ 2 & 2\,312 \\ 2 & 1\,156 \\ 2 & 578 \\ 17 & 289 \\ 17 & 17 \\ & 1 \end{array}$$

$$\begin{aligned} \therefore 4\,624 &= 2 \times 2 \times 2 \times 2 \times 17 \times 17 \\ &= 2^2 \times 2^2 \times 17^2 \end{aligned}$$

$$\begin{aligned} \therefore \sqrt{4\,624} &= 2 \times 2 \times 17 \\ &= \mathbf{68} \end{aligned}$$

(5) $\sqrt[3]{343}$

$$\begin{array}{r|l} 7 & 343 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

$$\begin{aligned} \therefore 343 &= 7 \times 7 \times 7 \\ &= 7^3 \end{aligned}$$

$$\therefore \sqrt[3]{343} = \mathbf{7}$$

(6) $\sqrt[3]{1\,728}$

$$\begin{array}{r|l} 2 & 1\,728 \\ 2 & 864 \\ 2 & 432 \\ 2 & 216 \\ 2 & 108 \\ 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$$\begin{aligned} \therefore 1\,728 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\ &= 2^3 \times 2^3 \times 3^3 \end{aligned}$$

$$\begin{aligned} \therefore \sqrt[3]{1\,728} &= 2 \times 2 \times 3 \\ &= \mathbf{12} \end{aligned}$$

(7) $\sqrt[3]{1\,000}$

$$\begin{array}{r|l} 2 & 1\,000 \\ 2 & 500 \\ 2 & 250 \\ 5 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$$\begin{aligned} \therefore 1\,000 &= 2 \times 2 \times 2 \times 5 \times 5 \times 5 \\ &= 2^3 \times 5^3 \end{aligned}$$

$$\begin{aligned} \therefore \sqrt[3]{1\,000} &= 2 \times 5 \\ &= \mathbf{10} \end{aligned}$$

(8) $\sqrt{576}$

$$\begin{array}{r|l} 2 & 576 \\ 2 & 288 \\ 2 & 144 \\ 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$$\begin{aligned} \therefore 576 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \\ &= 2^2 \times 2^2 \times 2^2 \times 3^2 \end{aligned}$$

$$\begin{aligned} \therefore \sqrt{576} &= 2 \times 2 \times 2 \times 3 \\ &= \mathbf{24} \end{aligned}$$

A2.4 REVISION EXERCISE:

(1) Calculate:

$$\begin{aligned} \text{(a)} \quad & 2^3 + 7^2 - \sqrt{25} \\ & = 8 + 49 - 5 \\ & = \mathbf{52} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & 6^2 \times \sqrt[3]{1} + 0 \\ & = 36 \times 1 + 0 \\ & = 36 + 0 = \mathbf{36} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & \sqrt{5^3 - 2^2} \\ & = \sqrt{125 - 4} \\ & = \sqrt{121} = \mathbf{11} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad & \sqrt[3]{729} \\ & = \sqrt[3]{3^3 \times 3^3} \\ & = 3 \times 3 = \mathbf{9} \end{aligned}$$

$$\begin{array}{r|l} 3 & 729 \\ 3 & 243 \\ 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$$\begin{aligned} \text{(e)} \quad & \sqrt{100} + 7^2 \\ & = 10 + 49 \\ & = \mathbf{59} \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad & 3^3 + 11^2 \\ & = 27 + 121 \\ & = \mathbf{148} \end{aligned}$$

$$\begin{aligned} \text{(g)} \quad & (\sqrt{49} + 1)^2 \\ & = (7 + 1)^2 \\ & = (8)^2 = \mathbf{64} \end{aligned}$$

$$\begin{aligned} \text{(h)} \quad & \sqrt{2\,025} \\ & = \sqrt{5^2 \times 3^2 \times 3^2} \\ & = 5 \times 3 \times 3 = \mathbf{45} \end{aligned}$$

$$\begin{array}{r|l} 5 & 2\,025 \\ 5 & 405 \\ 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$$\begin{aligned} \text{(i)} \quad & \sqrt{144} \div 4^1 \\ & = 12 \div 4 \\ & = \mathbf{3} \end{aligned}$$

$$\begin{aligned} \text{(j)} \quad & (\sqrt{7})^2 \\ & = \mathbf{7} \end{aligned}$$

$$\begin{aligned} \text{(k)} \quad & \sqrt{\sqrt{81}} \\ & = \sqrt{9} \\ & = \mathbf{3} \end{aligned}$$

$$\begin{aligned} \text{(l)} \quad & (5 - 2)^2 + \sqrt{36} \\ & = (3)^2 + 6 \\ & = 9 + 6 = \mathbf{15} \end{aligned}$$

$$\begin{aligned} \text{(m)} \quad & 3^2 \times 0 + (9 - 2)^2 \\ & = 9 \times 0 + (7)^2 \\ & = 0 + 49 = \mathbf{49} \end{aligned}$$

$$\begin{aligned} \text{(n)} \quad & \sqrt{121} - \sqrt[3]{125} \\ & = 11 - 5 \\ & = \mathbf{6} \end{aligned}$$

(2) The sum of the first three prime numbers is squared. What will the answer be?

$$\begin{aligned} & (2 + 3 + 5)^2 \\ &= (10)^2 \\ &= \mathbf{100} \end{aligned}$$

(3) Calculate: $3 \times 10^5 + 2 \times 10^4 + 7 \times 10^3 + 2 \times 10 + 8 \times 10^0 + 3 \times 10^2$

$$\begin{aligned} &= 300\,000 + 20\,000 + 7\,000 + 20 + 8 + 300 \\ &= 300\,000 + 20\,000 + 7\,000 + 300 + 20 + 8 \\ &= \mathbf{327\,328} \end{aligned}$$

(4) Are the following statements true or false?

(a) $8^2 = 8 \times 2$

False $\rightarrow 8^2 = 8 \times 8$

(b) The square root of 4 is 16.

False \rightarrow The square root of 16 is 4. or The square root of 4 is 2.

(c) $1^3 + 3^1 = 3 + 1 = 4$

True

(5) Complete the next five numbers in each sequence:

(a) 9 ; 16 ; 25 ; 36 ; **49 ; 64 ; 81 ; 100 ; 121**

(b) 2×3^2 ; 3×4^3 ; 4×5^4 ; **5×6^5 ; 6×7^6 ; 7×8^7 ; 8×9^8 ; 9×10^9**
