

Graad 10 – Boek A (Hersiene KABV uitgawe)

ONDERWYSERS HANDLEIDING

INHOUD:

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Hoofstuk A1

Getalle stelsels

GEEN SAKREKENAAR MAG IN HIERDIE HOOFSTUK GEBRUIK WORD NIE!

A1.1 Getallestelsels:

Oefening 1:

Datum: _____

Voltooi:	* Natuurlike getalle:	\mathbb{N}	=	$\{1; 2; 3; \dots\}$
	* Telgetalle:	\mathbb{N}_0	=	$\{0; 1; 2; 3; \dots\}$
	* Heelgetalle:	\mathbb{Z}	=	$\{\dots; -2; -1; 0; 1; 2; 3; \dots\}$
	* Rasionale getalle:	\mathbb{Q}	=	$\left\{\frac{a}{b} / a, b \in \mathbb{Z}; b \neq 0\right\}$

A1.2 Rasionale getalle:

A1.2.1 Ekwivalente breuke:

Vb.1 Skryf twee ekwivalente breuke neer vir $\frac{2}{3}$:

$$\frac{2 \times 3}{3 \times 3} = \frac{6}{9} \quad \text{of} \quad \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

Oefening 2:

Datum: _____

Hierdie is moontlike antwoorde!

(1) Skryf drie ekwivalente breuke neer vir elk van die volgende rasionale getalle:

(a) $\frac{-1}{4} = \frac{-2}{8} = \frac{-3}{12} = \frac{-10}{40}$

(b) $\frac{3}{7} = \frac{6}{14} = \frac{9}{21} = \frac{15}{35}$

(c) $\frac{1}{6} = \frac{2}{12} = \frac{10}{60} = \frac{100}{600}$

(d) $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{20}{30}$

(e) $\frac{12}{14} = \frac{6}{7} = \frac{24}{28} = \frac{120}{140}$

(f) $\frac{-36}{-9} = \frac{-4}{-1} = \frac{4}{1} = \frac{12}{3}$

(g) $2\frac{6}{11} = \frac{28}{11} = 2\frac{12}{22} = 2\frac{60}{110}$

(h) $5 = \frac{5}{1} = \frac{10}{2} = \frac{25}{5}$

(2) Is die volgende ekwivalente breuke of nie? (Antwoord net ja of nee.)

(a) $\frac{12}{5} = \frac{24}{10}$? : **Ja**

(b) $\frac{7}{3} = \frac{3}{7}$? : **Nee**

(c) $\frac{3}{-2} = \frac{6}{4}$? : **Nee**

(d) $\frac{3}{-5} = \frac{-9}{15}$? : **Ja**

(e) $\frac{2}{3} = \frac{4}{9}$? : **Nee**

(f) $\frac{3}{1} = \frac{48}{16}$? : **Ja**

(g) $\frac{4}{3} = \frac{-12}{-9}$? : **Ja**

(h) $\frac{25}{10} = \frac{5}{2}$? : **Ja**

(i) $\frac{5}{4} = \frac{4}{3}$? : **Nee**

A1.2.2 Ordering van rationale getalle:

Vb.2 (a) Rangskik die volgende breuke in stygende volgorde: $\frac{1}{2}$; $\frac{3}{4}$ en $\frac{2}{3}$:

$$\frac{1}{2} = \frac{6}{12} \quad ; \quad \frac{3}{4} = \frac{9}{12} \quad \text{en} \quad \frac{2}{3} = \frac{8}{12}$$

$$\therefore \frac{1}{2} < \frac{2}{3} < \frac{3}{4}$$

(b) Skryf 'n rationale getal tussen $\frac{3}{4}$ en $\frac{1}{3}$:

$$\frac{3}{4} = \frac{9}{12} \quad \text{en} \quad \frac{1}{3} = \frac{4}{12}$$

$$\therefore \frac{1}{3} < \frac{5}{12} \text{ of } \frac{6}{12} \text{ of } \frac{7}{12} \text{ of } \frac{8}{12} < \frac{3}{4}$$

Oefening 3:

Datum: _____

(1) Rangskik die volgende breuke in stygende volgorde:

(a) $\frac{3}{4}$; $\frac{2}{3}$ en $\frac{4}{5}$: $\frac{3}{4} = \frac{45}{60}$; $\frac{2}{3} = \frac{40}{60}$; $\frac{4}{5} = \frac{48}{60}$

$$\therefore \frac{2}{3} < \frac{3}{4} < \frac{4}{5}$$

(b) $\frac{2}{3}$; $\frac{5}{7}$ en $\frac{4}{6}$: $\frac{2}{3} = \frac{28}{42}$; $\frac{5}{7} = \frac{30}{42}$; $\frac{4}{6} = \frac{28}{42}$

$$\therefore \frac{2}{3} = \frac{4}{6} < \frac{5}{7}$$

(2) Rangskik die volgende breuke in dalende volgorde:

(a) $\frac{5}{8}$; $\frac{2}{3}$ en $\frac{3}{4}$: $\frac{5}{8} = \frac{15}{24}$; $\frac{2}{3} = \frac{16}{24}$; $\frac{3}{4} = \frac{18}{24}$

$$\therefore \frac{3}{4} > \frac{2}{3} > \frac{5}{8}$$

(b) $-1\frac{1}{2}$; $-1\frac{2}{3}$ en $\frac{-7}{5}$: $\frac{-3}{2} = \frac{-45}{30}$; $\frac{-5}{3} = \frac{-50}{30}$; $\frac{-7}{5} = \frac{-42}{30}$

$$\therefore \frac{-7}{5} > -1\frac{1}{2} > -1\frac{2}{3}$$

(3) Plaas telkens 'n rationale getal tussen die volgende getalle:

(a) $\frac{-1}{3}$ en $\frac{-3}{5}$: $\frac{-1}{3} = \frac{-5}{15}$ en $\frac{-3}{5} = \frac{-9}{15}$

$$\therefore \frac{-3}{5} < \frac{-8}{15} \text{ of } \frac{-7}{15} \text{ of } \frac{-6}{15} < \frac{-1}{3}$$

(b) $\frac{3}{4}$ en $\frac{7}{10}$: $\frac{3}{4} = \frac{15}{20} = \frac{30}{40}$ en $\frac{7}{10} = \frac{14}{20} = \frac{28}{40}$

$$\therefore \frac{7}{10} < \frac{29}{40} < \frac{3}{4}$$

A1.2.3 Herleiding van gewone breuke na desimale breuke:

Vb.3 Druk die volgende uit as desimale breuke, sonder om 'n sakrekenaar te gebruik:

(a) $\frac{3}{8} = \frac{3,000\dots}{8} = \frac{3,306040}{8} = 0,375$ (b) $1\frac{2}{3} = 1\frac{2,000\dots}{3} = 1\frac{2,202020\dots}{3} = 1,66\dots = 1,6\dot{6}$

Oefening 4:

Datum: _____

Druk die volgende uit as desimale breuke, sonder om 'n sakrekenaar te gebruik:

(1) $\frac{22}{7} = \frac{22,000...}{7} = \frac{22,10^30^20}{7} = 3,142...$

(2) $4\frac{2}{3} = 4\frac{2,20^20^20}{3} = 4,66... \approx 4,6$

(3) $\frac{1}{8} = \frac{1,000...}{8} = \frac{1,10^20^40}{8} = 0,125$

(4) $\frac{7}{9} = \frac{7,000...}{9} = \frac{7,70^70^70}{9} = 0,77... \approx 0,7$

(5) $\frac{17}{25} = \frac{17,000...}{25} = \frac{17,170^200}{25} = 0,68$

(6) $\frac{5}{100} = \frac{5,000...}{100} = \frac{5,50^500}{100} = 0,05$

(7) $\frac{4}{11} = \frac{4,40^70^40^70^40}{11} = 0,3636... \approx 0,36$

(8) $-2\frac{6}{7} = -2\frac{6,60^40^50}{7} = -2,857...$

(9) $-5\frac{5}{6} = -5\frac{5,50^20^20}{6} = -5,833... \approx -5,83$

(10) $\frac{33}{8} = \frac{33,000...}{8} = \frac{33,10^20^40}{8} = 4,125$

A1.2.4 Afronding van desimale breuke:**Vb.4 Rond die volgende breuke af tot die aantal desimale plekke aangedui in hakies:**

(a) 4,34712 (tot 3 des)

$= 4,347\underline{12}$

$\approx 4,347$

Beskou onderstreepte getal

(b) 290,09832 (tot 2 des)

$= 290,098\underline{32}$

$\approx 290,10$

Oefening 5:

Datum: _____

(1) Rond die volgende breuke af tot die aantal desimale plekke aangedui in hakies:

(a) 3,573 (tot 2 des)

$\approx 3,57$

(b) 12,00873 (tot 3 des)

$\approx 12,009$

(c) 0,00384 (tot 5 des)

$= 0,00384$

(d) 7,3226 (tot 1 des)

$\approx 7,3$

(e) 8,39999 (tot 1 des)

$\approx 8,4$

(f) 90,9023 (tot die naaste heelgetal)

≈ 91

(g) 0,433218 (tot 4 des)

$\approx 0,4332$

(h) 1 456,6799 (tot 3 des)

$\approx 1 456,680$

(i) 66,666 (tot 2 des)

$\approx 66,67$

(j) 13,00034 (tot 3 des)

$\approx 13,000$

(2) Beskou die volgende afronding en kies die korrekte woord in die hakies:

(a) $3,47653 \approx 3,477$ korrek tot die naaste (tiende, honderdste of **duisendste**)(b) $96\ 995,31956 \approx 96\ 995,32$ korrek tot die naaste (tiende, **honderdste** of duisendste)

A1.2.5 Herleiding van desimale breuke na gewone breuke:

Vb.5 Druk die volgende uit as gewone breuke in eenvoudigste vorm:

$$(a) \quad 4,5 = 4 \frac{5}{10} \left(\div \frac{5}{5} \right) = 4 \frac{1}{2}$$

$$(b) \quad -0,12 = -\frac{12}{100} \left(\div \frac{4}{4} \right) = -\frac{3}{25}$$

Oefening 6:

Datum: _____

Druk die volgende uit as gewone breuke in eenvoudigste vorm:

$$(1) \quad 0,125 = \frac{125}{1\,000} = \frac{125}{1\,000} \div \frac{125}{125} \\ = \frac{1}{8}$$

$$(2) \quad 1,25 = 1 \frac{25}{100} = 1 \frac{25}{100} \div \frac{25}{25} \\ = 1 \frac{1}{4}$$

$$(3) \quad 14,6 = 14 \frac{6}{10} = 14 \frac{6}{10} \div \frac{2}{2} \\ = 14 \frac{3}{5}$$

$$(4) \quad -0,5 = -\frac{5}{10} = -\frac{5}{10} \div \frac{5}{5} \\ = -\frac{1}{2}$$

$$(5) \quad -1,2 = -1 \frac{2}{10} = -1 \frac{2}{10} \div \frac{2}{2} \\ = -1 \frac{1}{5}$$

$$(6) \quad 23,5 = 23 \frac{5}{10} = 23 \frac{5}{10} \div \frac{5}{5} \\ = 23 \frac{1}{2}$$

$$(7) \quad 3,04 = 3 \frac{4}{100} = 3 \frac{4}{100} \div \frac{4}{4} \\ = 3 \frac{1}{25}$$

$$(8) \quad 7,3 = 7 \frac{3}{10}$$

$$(9) \quad 100,75 = 100 \frac{75}{100} = 100 \frac{75}{100} \div \frac{25}{25} \\ = 100 \frac{3}{4}$$

$$(10) \quad 0,00005 = \frac{5}{100\,000} = \frac{5}{100\,000} \div \frac{5}{5} \\ = \frac{1}{20\,000}$$

A1.2.6 Herleiding van repeterende breuke na gewone breuke:

Vb.6 Herlei die volgende na 'n gewone breuk in eenvoudigste vorm:

$$(a) \quad 0,\dot{1} = \frac{1}{9} \quad ; \quad 0,\dot{3} = \frac{3}{9} = \frac{1}{3} \quad ; \quad 0,\dot{5} = \frac{5}{9} \quad ; \quad 0,\dot{8} = \frac{8}{9}$$

$$(b) \quad 3,2\dot{4} = 3 \frac{24}{99} = 3 \frac{8}{33} \quad ; \quad 0,4\dot{2}1 = \frac{421}{999} \quad ; \quad 15,1\dot{6}5\dot{3} = 15 \frac{1653}{9999} = 15 \frac{551}{3333}$$

$$(c) \quad 0,0\dot{3} = 0,\dot{3} \div 10 = \frac{3}{9} \div \frac{10}{1} = \frac{3}{9} \times \frac{1}{10} = \frac{3}{90} = \frac{1}{30}$$

$$(d) \quad 0,004\dot{6} = 0,4\dot{6} \div 100 = \frac{46}{99} \div \frac{100}{1} = \frac{46}{99} \times \frac{1}{100} = \frac{46}{9900} = \frac{23}{4950}$$

$$(e) \quad 0,5\dot{7} = 0,5 + 0,0\dot{7} = 0,5 + 0,\dot{7} \div 10 = \frac{5}{10} + \frac{7}{9} \times \frac{1}{10} = \frac{5 \times 9}{10 \times 9} + \frac{7}{90} = \frac{45+7}{90} = \frac{52}{90} = \frac{26}{45}$$

Oefening 7:

Datum: _____

Herlei die volgende na gewone breuke in eenvoudigste vorm: (Sonder 'n sakrekenaar.)

(1) $3,6\dot{6}$

$$= 3\frac{6}{9}$$

$$= 3\frac{6}{9} \div \frac{3}{3}$$

$$= 3\frac{2}{3}$$

(2) $0,1\dot{3}$

$$= \frac{13}{99}$$

(3) $22,3\dot{9}$

$$= 22 + 0,3 + 0,0\dot{9}$$

$$= 22 + \frac{3}{10} + 0,9 \div 10$$

$$= 22 + \frac{3}{10} + \frac{9}{9} \times \frac{1}{10}$$

$$= 22 + \frac{3}{10} + 1 \times \frac{1}{10}$$

$$= 22 + \frac{3}{10} + \frac{1}{10}$$

$$= 22 + \frac{3+1}{10} = 22\frac{4}{10}$$

$$= 22\frac{2}{5}$$

(4) $-1,1\dot{3}\dot{5}$ of $-1,\overline{135}$

$$= -1\frac{135}{999}$$

$$= -1\frac{135}{999} \div \frac{27}{27}$$

$$= -1\frac{5}{37}$$

(5) $0,7\dot{7}$

$$= \frac{7}{9}$$

(6) $0,00\dot{3}$

$$= 0,3 \div 100$$

$$= \frac{3}{9} \times \frac{1}{100} = \frac{3}{900}$$

$$= \frac{1}{300}$$

(7) $1,\overline{214}$

$$= 1\frac{214}{999}$$

(8) $3,2\dot{5}\dot{8}$

$$= 3 + 0,2 + 0,0\dot{5}\dot{8}$$

$$= 3 + \frac{2}{10} + 0,5\dot{8} \div 10$$

$$= 3 + \frac{2}{10} + \frac{58}{99} \times \frac{1}{10}$$

$$= 3 + \frac{2}{10} \times \frac{99}{99} + \frac{58}{990}$$

$$= 3 + \frac{198}{990} + \frac{58}{990} = 3\frac{256}{990}$$

$$= 3\frac{128}{495}$$

☺ Bereken die volgende sonder die gebruik van 'n sakrekenaar: $0,4 + \frac{2}{3}$

$$\begin{aligned} 0,4 + \frac{2}{3} &= \frac{4}{9} + \frac{2}{3} \\ &= \frac{4}{9} + \frac{2}{3} \times \frac{3}{3} \\ &= \frac{4}{9} + \frac{6}{9} = \frac{10}{9} = 1\frac{1}{9} \end{aligned}$$

A1.3 Irrasionale en Reële getalle:

Irrasionale getalle kan nie uitgedruk word as die verhouding tussen twee heelgetalle nie. Hierdie getalle is oneindige, nie-repeterende desimale breuke.

Vb. 7 Irrasionale getalle:

- $\sqrt{2}$ of $\sqrt{7}$ of $\sqrt{\frac{3}{4}}$ ens. want 2 ; 7 en 3 is nie volkome vierkante nie!
- $\sqrt[3]{12}$ of $\sqrt[3]{100}$ ens. want 12 en 100 is nie volkome derdemagte nie!

Waar die volgende rasionale getalle is:

- $\sqrt{4}$ of $\sqrt{0,01}$ of $\sqrt{\frac{25}{9}}$ ens. want 4 ; 0,01 ; 25 en 9 is volkome vierkante!
- $\sqrt[3]{27}$ of $\sqrt[3]{125}$ ens. want 27 en 125 is volkome derdemagte!

Die reële getalle, \mathbb{R} bestaan uit die rasionale getalle, \mathbb{Q} en die irrasionale getalle, \mathbb{Q}' . Onthou dat alle eindige en repeterende desimale breuke rasionale getalle is.

Vb.8 Bepaal die twee heelgetalle waar tussen die irrasionale getal $\sqrt{7}$ lê.

Kies die twee volkome vierkante weerskante van 7:

$$\begin{aligned} \sqrt{4} &< \sqrt{7} < \sqrt{9} \\ \therefore 2 &< \sqrt{7} < 3 \end{aligned}$$

Oefening 8:

Datum: _____

(1) Watter van die volgende getalle is Rasionale getalle (\mathbb{Q}) en watter is Irrasionale getalle (\mathbb{Q}')?

- | | | |
|---|--|--|
| (a) 14 : \mathbb{Q} | (b) $\frac{1}{5}$: \mathbb{Q} | (c) $\sqrt{81}$: \mathbb{Q}
= 9 |
| (d) 0,12 : \mathbb{Q}
= $-\sqrt{4} = -2$ | (e) $\sqrt{18}$: \mathbb{Q}' | (f) $12,2\dot{3}$: \mathbb{Q} |
| (g) $-\sqrt{\frac{12}{3}}$: \mathbb{Q} | (h) 0,2945 ... : \mathbb{Q}' | (i) $\sqrt[3]{64}$: \mathbb{Q}
= 4 |
| (j) π : \mathbb{Q}' | (k) $\sqrt[5]{32}$: \mathbb{Q}
= 2 | (l) $\frac{11}{7}$: \mathbb{Q} |

(2) Tussen watter twee heelgetalle lê die volgende irrasionale getalle:

(a) $-\sqrt{12}$
 $-\sqrt{16} < -\sqrt{12} < -\sqrt{9}$
 $-4 < -\sqrt{12} < -3$

(b) $\sqrt{66}$
 $\sqrt{64} < \sqrt{66} < \sqrt{81}$
 $8 < \sqrt{66} < 9$

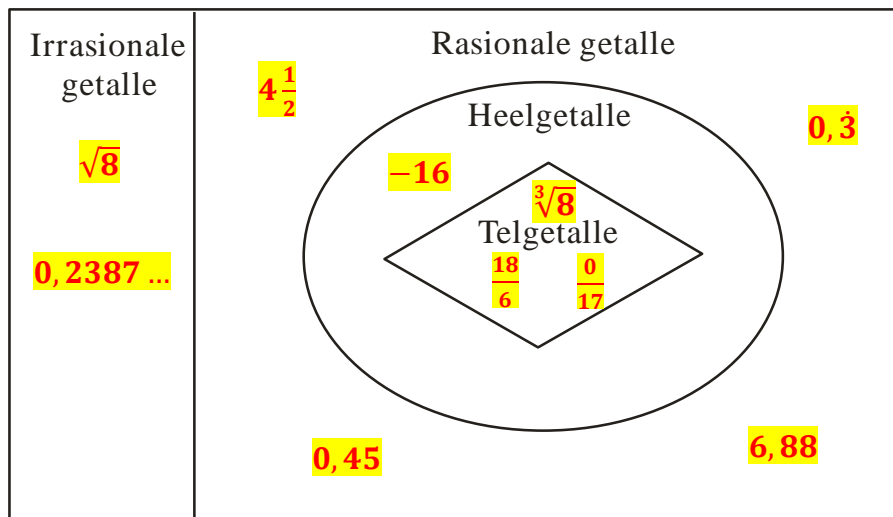
(c) $\sqrt[3]{5}$
 $\sqrt[3]{1} < \sqrt[3]{5} < \sqrt[3]{8}$
 $1 < \sqrt[3]{5} < 2$

(d) $\sqrt[5]{2}$
 $\sqrt[5]{1} < \sqrt[5]{2} < \sqrt[5]{32}$
 $1 < \sqrt[5]{2} < 2$

(3) Die onderstaande diagram is 'n opsomming van al die getalle wat op skoolvlak gebruik word. Plaas die volgende getalle op die regte plek in die tabel; waar nodig, vereenvoudig die getal:

$$4\frac{1}{2} ; \sqrt[3]{8} = 2 ; \sqrt{8} ; -16 ; 0,45 ; 0,3 ; \frac{18}{6} = 3 ; 0,2387... ; \frac{0}{17} = 0 ; 6,88$$

Reële getalle:



☺ (1) Buiten die reële getalle kry ons ook nog die nie-reële getalle.

Gee 'n voorbeeld van 'n nie-reële getal. $\sqrt{-1} ; \sqrt[3]{-2} ; \dots$

(2) Wat word die versameling genoem wat alle reële en nie-reële getalle bevat?

Komplekse getalle

A1.4 Voostelling van versamelings van getalle:

Versamelings van getalle kan op die volgende maniere geskryf of voorgestel word:

A1.4.1 Versamelingskeurdernotasie:

Vb. 9 Skryf die volgende versamelings van getalle in versamelingskeurdernotasie:

(a) Alle natuurlike getalle groter as 6: $\{x / x > 6 ; x \in \mathbb{N}\}$

(b) Alle reële getalle tussen -2 en 5 : $\{m : -2 < m < 5 ; m \in \mathbb{R}\}$

A1.4.2 Intervalnotasie:

Slegs versamelings wat deel van die reële getalle uitmaak, kan in intervalnotasie geskryf word!

Vb. 10 Skryf die volgende in intervalnotasie:

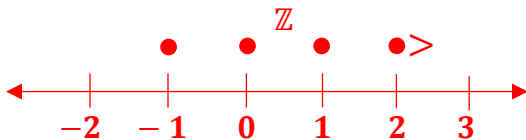
(a) Die reële getalle tussen -2 en 4 met 4 ingesluit: $x \in (-2 ; 4]$ Oop, geslote interval!

(b) $\{m / m > 7 ; m \in \mathbb{R}\}$: $m \in (7 ; \infty)$ Oop interval!

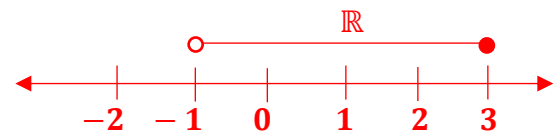
A1.4.3 Getallelyne:

Vb. 11 Stel die volgende op 'n getallelyn voor :

(a) $\{-1 ; 0 ; 1 ; 2 ; \dots \dots \dots\}$



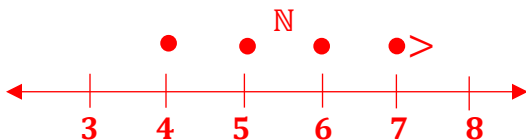
(b) $\{x : -1 < x \leq 3 ; x \in \mathbb{R}\}$



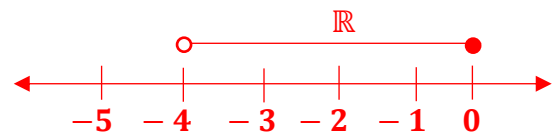
A1.4.4 Oplos van lineêre ongelykhede:

Vb. 12 Bereken x in elk van die volgende en stel jou antwoord op 'n getallelyn voor:

(a) $x - 2 \geq 2$ as $x \in \mathbb{N}$
 $x \geq 2 + 2$
 $x \geq 4$



(b) $-3 < x + 1 \leq 1$ as $x \in \mathbb{R}$
 $-3 - 1 < x \leq 1 - 1$
 $-4 < x \leq 0$



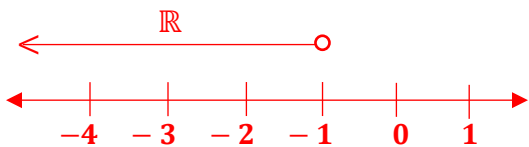
Oefening 9:

Datum: _____

(1) Skryf die volgende in interval notasie (waar van toepassing) en stel dit op 'n getallelyn voor:

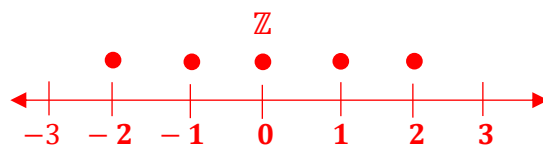
(a) $\{x : x < -1; x \in \mathbb{R}\}$

$x \in (-\infty; -1)$



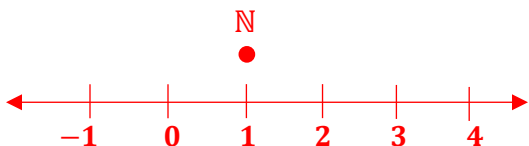
(b) $\{x : -3 < x < 3; x \in \mathbb{Z}\}$

Geen interval notasie



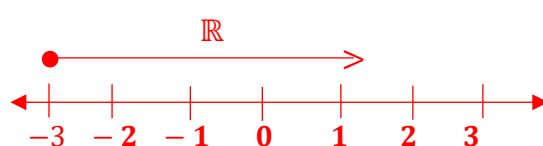
(c) $\{y : y < 2; y \in \mathbb{N}\}$

Geen interval notasie



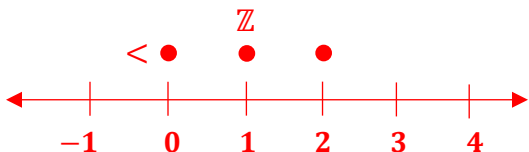
(d) $\{x : x \geq -3; x \in \mathbb{R}\}$

$x \in [-3; \infty)$



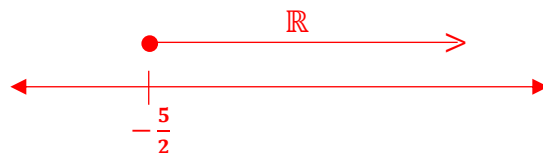
(e) $\{x/x < 3; x \in \mathbb{Z}\}$

Geen interval notasie



(f) $\{p/p \geq -\frac{5}{2}; p \in \mathbb{R}\}$

$p \in [-\frac{5}{2}; \infty)$



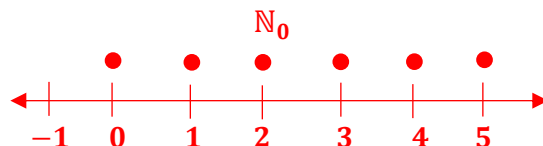
(g) $\{m : -2 \leq m < 5; m \in \mathbb{R}\}$

$m \in [-2; 5)$



(h) $\{x : x \leq 5; x \in \mathbb{N}_0\}$

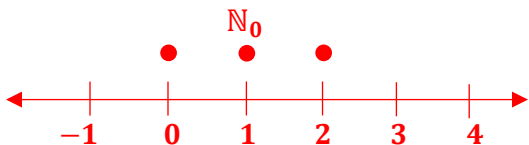
Geen interval notasie

(2) Los op vir x en stel jou antwoord op 'n getallelyn voor:

(a) $x + 1 \leq 3; x \in \mathbb{N}_0$

$x \leq 3 - 1$

$x \leq 2$



(b) $2x \geq -8; x \in \mathbb{R}$

$x \geq -\frac{8}{2}$

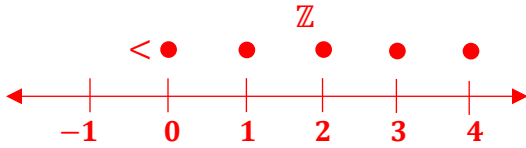
$x \geq -4$



(c) $x - 4 \leq 0; x \in \mathbb{Z}$

$$x \leq 0 + 4$$

$$x \leq 4$$

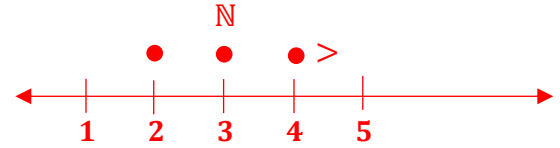


(d) $2x + 3 > 7; x \in \mathbb{N}$

$$2x > 7 - 3$$

$$x > \frac{4}{2}$$

$$x > 2$$



(e) $-6 < x - 1 \leq 6; x \in \mathbb{R}$

$$-6 + 1 < x \leq 6 + 1$$

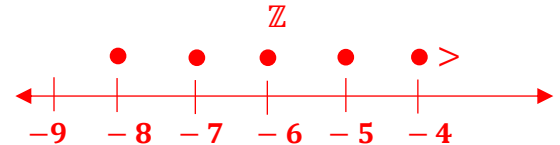
$$-5 < x \leq 7$$



(f) $x + 7 \geq -1; x \in \mathbb{Z}$

$$x \geq -1 - 7$$

$$x \geq -8$$

**A1.5 HERSIENINGSOEFENING:**

Datum: _____

(1) Herlei die volgende na gewone breuke in eenvoudigste vorm: (Sonder 'n sakrekenaar.)

(a) $14,1\dot{7}$

$$= 14 + 0,1 + 0,0\dot{7}$$

$$= 14 + \frac{1}{10} + 0,7 \div 10$$

$$= 14 + \frac{1}{10} + \frac{7}{9} \times \frac{1}{10}$$

$$= 14 + \frac{1}{10} + \frac{7}{90}$$

$$= 14 + \frac{1 \times 9}{10 \times 9} + \frac{7}{90}$$

$$= 14 + \frac{9}{90} + \frac{7}{90} = 14 \frac{16}{90}$$

$$= 14 \frac{8}{45}$$

(b) $0,\overline{1234}$

$$= \frac{1234}{9999}$$

(c) $4,68$

$$= 4 \frac{68}{100}$$

$$= 4 \frac{68}{100} \div \frac{4}{4}$$

$$= 4 \frac{17}{25}$$

(d) $5,1$

$$= 5 \frac{1}{9}$$

(2) Merk die rationale getalle tussen 0 en 10 met 'n ✓:

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
$\sqrt{9} = 3$	-1	$\sqrt{8}$	$\frac{6}{3} = 2$	$\sqrt[3]{16}$	π	$\frac{0}{3} = 0$	$\sqrt{144} = 12$	4,124....	$\sqrt{\frac{24}{6}} = \sqrt{4} = 2$
✓			✓						✓

(3) Rond die volgende breuke af tot die aantal desimale plekke aangedui in hakies:

(a) 7,199 (tot 1 des)

$$\approx 7,2$$

(b) 0,048561 (tot 4 des)

$$\approx 0,0486$$

(c) 234,34 (tot 1 des)

$$\approx 234,3$$

(d) 1 001,1989 (tot die naaste heelgetal)

$$\approx 1\ 001$$

(e) 3,997 (tot 2 des)

$$\approx 4,00$$

(f) 23,712 (tot die naaste heelgetal)

$$\approx 24$$

(4) Plaas enige twee irrasionale getalle tussen 2 en 3.

$$2 = \sqrt{4} \quad \text{en} \quad 3 = \sqrt{9}$$

$$\therefore 2 < \sqrt{5} < \sqrt{6} < \sqrt{7} < \sqrt{8} < 3$$

Enige 2!

(5) Tussen watter twee rationale getalle lê die volgende irrasionale getalle:

(a) $\sqrt{\frac{1}{2}}$

$$\sqrt{\frac{1}{1}} > \sqrt{\frac{1}{2}} > \sqrt{\frac{1}{4}}$$

$$1 > \sqrt{\frac{1}{2}} > \frac{1}{2}$$

$$\frac{1}{2} < \sqrt{\frac{1}{2}} < 1$$

(b) $\sqrt[3]{57}$

$$\sqrt[3]{27} < \sqrt[3]{57} < \sqrt[3]{64}$$

$$3 < \sqrt[3]{57} < 4$$

$$4 > \sqrt[3]{57} > 3$$

of

(6) Voltooi die ontbrekende voorstellings in die onderstaande tabel:

	Versamelingskeurdernotasie:	Intervalnotasie:	Getallelyn:
(1)	$\{x / -1 < x \leq 2; x \in \mathbb{R}\}$	$x \in (-1; 2]$	
(2)	$\{x : -2 \leq x \leq 5; x \in \mathbb{R}\}$	$x \in [-2; 5]$	
(3)	$\{y / y \leq 3; y \in \mathbb{R}\}$	$y \in (-\infty; 3]$	
(4)	$\{x / x \leq 3; x \in \mathbb{N}_0\}$	Nvt	
(5)	$\{y / y \geq 3; y \in \mathbb{N}\}$	Nvt	
(6)	$\{m : 0 < m \leq 4; m \in \mathbb{R}\}$	$m \in (0; 4]$	
(7)	$\{t : t \geq -5; t \in \mathbb{R}\}$	$t \in [-5; \infty)$	
(8)	$\{m : m \leq 6; m \in \mathbb{R}\}$	$m \in (-\infty; 6]$	
(9)	$\{x / -1 < x < 2; x \in \mathbb{Z}\}$	Nvt	
(10)	$\{x / x > -1; x \in \mathbb{R}\}$	$x \in (-1; \infty)$	

Hoofstuk A2

Algebraïese uitdrukkings

A2.1 Produkte:

A2.1.1 Distributiewe wet:

Vb. 1 Bepaal die volgende produkte deur gebruik te maak van die distributiewe wet:

$$(a) \quad (x - 2)(x + 2)$$

$$(b) \quad (3m + n)(2m + 5n)$$

$$(a) \quad x(x + 2) - 2(x + 2)$$

$$= x^2 + 2x - 2x - 4$$

$$= x^2 - 4$$

$$(b) \quad (3m + 1n)(2m + 5n)$$

$$= 6m^2 + 15mn + 2mn + 5n^2$$

$$= 6m^2 + 17mn + 5n^2$$

Oefening 1:

Datum: _____

Bepaal die volgende produkte:

$$(1) \quad (y - 4)(y + 3)$$

$$= y^2 + 3y - 4y - 12$$

$$= y^2 - y - 12$$

$$(2) \quad (p - 2)(p - 7)$$

$$= p^2 - 7p - 2p + 14$$

$$= p^2 - 9p + 14$$

$$(3) \quad (2x + 1)(x - 5)$$

$$= 2x^2 - 10x + 1x - 5$$

$$= 2x^2 - 9x - 5$$

$$(4) \quad (x - 2y)(2x - y)$$

$$= 2x^2 - 1xy - 4xy + 2y^2$$

$$= 2x^2 - 5xy + 2y^2$$

$$(5) \quad (4ab + 1)(2ab - 3)$$

$$= 8a^2b^2 - 12ab + 2ab - 3$$

$$= 8a^2b^2 - 10ab - 3$$

$$(6) \quad (5 - 7m)(2 - 3m)$$

$$= 10 - 15m - 14m + 21m^2$$

$$= 10 - 29m + 21m^2$$

$$(7) \quad (2a - 4b)(3a + 2b)$$

$$= 6a^2 + 4ab - 12ab - 8b^2$$

$$= 6a^2 - 8ab - 8b^2$$

$$(8) \quad (m + n)(2m - 1)$$

$$= 2m^2 - 1m + 2mn - n$$

$$(9) \quad (d - 12)(12 + d)$$

$$= 12d + d^2 - 144 - 12d$$

$$= d^2 - 144$$

$$(10) \quad (a^2 + 4)(a^2 + 2)$$

$$= a^4 + 2a^2 + 4a^2 + 8$$

$$= a^4 + 6a^2 + 8$$

$$\begin{aligned}
 (11) \quad & \left(\frac{1}{2}m - 6\right)(8m - 3) \\
 & = 4m^2 - \frac{3}{2}m - 48m + 18 \\
 & = 4m^2 - 49\frac{1}{2}m + 18
 \end{aligned}$$

$$\begin{aligned}
 (12) \quad & (-2k - 5)(5 + 3k) \\
 & = -10k - 6k^2 - 25 - 15k \\
 & = -6k^2 - 25k - 25
 \end{aligned}$$

$$\begin{aligned}
 (13) \quad & \left(p + \frac{1}{p}\right)\left(8p - \frac{4}{p}\right) \\
 & = 8p^2 - \frac{4p}{p} + \frac{8p}{p} - \frac{4}{p^2} \\
 & = 8p^2 - 4 + 8 - \frac{4}{p^2} \\
 & = 8p^2 + 4 - \frac{4}{p^2}
 \end{aligned}$$

$$\begin{aligned}
 (14) \quad & (abc - 2ac)(abc + 3bc) \\
 & = a^2b^2c^2 + 3ab^2c^2 - 2a^2bc^2 - 6abc^2
 \end{aligned}$$

$$\begin{aligned}
 (15) \quad & (3r^3 + 2)(2r^2 - 5) \\
 & = 6r^5 - 15r^3 + 4r^2 - 10
 \end{aligned}$$

$$\begin{aligned}
 (16) \quad & 2x(x - 5y)(3x + 2y) \\
 & = 2x(3x^2 + 2xy - 15xy - 10y^2) \\
 & = 2x(3x^2 - 13xy - 10y^2) \\
 & = 6x^3 - 26x^2y - 20xy^2
 \end{aligned}$$

$$\begin{aligned}
 (17) \quad & \left(\frac{1}{p^3q^2} - \frac{2}{p^2q}\right)\left(\frac{1}{p} + \frac{2}{q}\right) \\
 & = \frac{1}{p^4q^2} + \frac{2}{p^3q^3} - \frac{2}{p^3q} - \frac{4}{p^2q^2}
 \end{aligned}$$

$$\begin{aligned}
 (18) \quad & \left(\frac{m^2n}{3} - \frac{6}{mn}\right)\left(\frac{mn}{2} - \frac{3}{mn^2}\right) \\
 & = \frac{m^3n^2}{6} - \frac{3m^2n}{3mn^2} - \frac{6mn}{2mn} + \frac{18}{m^2n^3} \\
 & = \frac{m^3n^2}{6} - \frac{m}{n} - 3 + \frac{18}{m^2n^3}
 \end{aligned}$$

Vb. 2 Vereenvoudig:

$$(a) \quad (2a + 1)(2a - 1) = 4a^2 - 2a + 2a - 1 = 4a^2 - 1$$

$$(b) \quad (m^2 - 5n)(m^2 + 5n) = m^4 + 5m^2n - 5m^2n - 25n^2 = m^4 - 25n^2$$

Of korter:

$$(c) \quad (xy + 3)(xy - 3) = x^2y^2 - 9$$

$$(d) \quad \left(\frac{ab}{4} - \frac{1}{7}\right)\left(\frac{ab}{4} + \frac{1}{7}\right) = \frac{a^2b^2}{16} - \frac{1}{49}$$

Oefening 2:

Datum: _____

Vereenvoudig:

$$(1) \quad (abc - 2)(abc + 2)$$

$$= a^2b^2c^2 - 4$$

$$(2) \quad \left(\frac{1}{3} + 5t\right)\left(\frac{1}{3} - 5t\right)$$

$$= \frac{1}{9} - 25t^2$$

$$(3) \quad (p - 9q)(9q + p)$$

$$= (p - 9q)(p + 9q)$$

$$= p^2 - 81q^2$$

$$(4) \quad (n + 7k)(7n - k)$$

$$= 7n^2 - 1kn + 49kn - 7k^2$$

$$= 7n^2 + 48kn - 7k^2$$

$$(5) \quad (-a + 4b)(-a - 4b)$$

$$= a^2 + 4ab - 4ab - 16b^2$$

$$= a^2 - 16b^2$$

$$(6) \quad -x\left(\frac{1}{x} - x\right)\left(\frac{1}{x} + x\right)$$

$$= -x\left(\frac{1}{x^2} - x^2\right)$$

$$= \frac{-x}{x^2} + x\left(\frac{1}{x^2} - x^2\right)$$

$$= \frac{-1}{x} + x^3$$

$$(7) \quad (x^{2m} - 8)(x^{2m} + 8)$$

$$= x^{2m+2m} - 64$$

$$= x^{4m} - 64$$

$$(8) \quad (0,3 + 3q)(0,3 - 3q)$$

$$= 0,09 - 9q^2$$

$$(9) \quad (b^6c^3 + 6)(b^6c^3 + 6)$$

$$= b^{12}c^6 + 6b^6c^3 + 6b^6c^3 + 36$$

$$= b^{12}c^6 + 12b^6c^3 + 36$$

$$(10) \quad (4xk^5 - 7)(7 + 4xk^5)$$

$$= (4xk^5 - 7)(4xk^5 + 7)$$

$$= 16x^2k^{10} - 49$$

$$(11) \quad (m - 2n)^2(m + 2n)^2$$

$$= [(m - 2n)(m + 2n)]^2$$

$$= [m^2 - 4n^2]^2$$

$$= m^4 - 8m^2n^2 + 16n^4$$

$$(12) \quad \left(\frac{m}{n} + 2\right)\left(\frac{m^2}{n^2} + 4\right)\left(\frac{m}{n} - 2\right)$$

$$= \left(\frac{m}{n} + 2\right)\left(\frac{m}{n} - 2\right)\left(\frac{m^2}{n^2} + 4\right)$$

$$= \left(\frac{m^2}{n^2} - 4\right)\left(\frac{m^2}{n^2} + 4\right)$$

$$= \frac{m^4}{n^4} - 16$$

A2.1.2 Kwadrering van 'n tweeterm:**Vb. 3 Bepaal die volgende produkte:**

(a) $(2x + 1)^2$

$$= (2x + 1)(2x + 1)$$

$$= 4x^2 + 2x + 2x + 1$$

$$= 4x^2 + 4x + 1$$

(b) $\left(m - \frac{1}{m}\right)^2$

$$= \left(m - \frac{1}{m}\right)\left(m - \frac{1}{m}\right)$$

$$= m^2 - \frac{m}{m} - \frac{m}{m} + \frac{1}{m^2}$$

$$= m^2 - 2 + \frac{1}{m^2}$$

Oefening 3:

Datum: _____

Bepaal die volgende kwadrate:

(1) $(y - 11)^2$

$$= (y - 11)(y - 11)$$

$$= y^2 - 11y - 11y + 121$$

$$= y^2 - 22y + 121$$

(2) $(3p + 2q)^2$

$$= (3p + 2q)(3p + 2q)$$

$$= 9p^2 + 6pq + 6pq + 4q^2$$

$$= 9p^2 + 12pq + 4q^2$$

(3) $(-4 + 5c)^2$

$$= (-4 + 5c)(-4 + 5c)$$

$$= 16 - 20c - 20c + 25c^2$$

$$= 16 - 40c + 25c^2$$

(4) $(mn + 3)^2$

$$= (mn + 3)(mn + 3)$$

$$= m^2n^2 + 3mn + 3mn + 9$$

$$= m^2n^2 + 6mn + 9$$

(5) $(k^2 + 1)^2$

$$= (k^2 + 1)(k^2 + 1)$$

$$= k^4 + 1k^2 + 1k^2 + 1$$

$$= k^4 + 2k^2 + 1$$

(6) $(8 - 3b)^2$

$$= (8 - 3b)(8 - 3b)$$

$$= 64 - 24b - 24b + 9b^2$$

$$= 64 - 48b + 9b^2$$

(7) $\left(x - \frac{1}{2}\right)^2$

$$= \left(x - \frac{1}{2}\right)\left(x - \frac{1}{2}\right)$$

$$= x^2 - \frac{1}{2}x - \frac{1}{2}x + \frac{1}{4}$$

$$= x^2 - 1x + \frac{1}{4}$$

(8) $\left(\frac{y}{5} - 3\right)^2$

$$= \left(\frac{y}{5} - 3\right)\left(\frac{y}{5} - 3\right)$$

$$= \frac{y^2}{25} - \frac{3y}{5} - \frac{3y}{5} + 9$$

$$= \frac{y^2}{25} - \frac{6y}{5} + 9$$

$$\begin{aligned}
 (9) \quad & (5p - 2p^2)^2 \\
 &= (5p - 2p^2)(5p - 2p^2) \\
 &= 25p^2 - 10p^3 - 10p^3 + 4p^4 \\
 &= 25p^2 - 20p^3 + 4p^4
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad & \left(4 + \frac{3}{n}\right)^2 \\
 &= \left(4 + \frac{3}{n}\right)\left(4 + \frac{3}{n}\right) \\
 &= 16 + \frac{12}{n} + \frac{12}{n} + \frac{9}{n^2} \\
 &= 16 + \frac{24}{n} + \frac{9}{n^2}
 \end{aligned}$$

$$\begin{aligned}
 (11) \quad & (0,2 + 6y)^2 \\
 &= (0,2 + 6y)(0,2 + 6y) \\
 &= 0,04 + 1,2y + 1,2y + 36y^2 \\
 &= 0,04 + 2,4y + 36y^2
 \end{aligned}$$

$$\begin{aligned}
 (12) \quad & \left(\frac{2m}{p} + \frac{p^2}{3m}\right)^2 \\
 &= \left(\frac{2m}{p} + \frac{p^2}{3m}\right)\left(\frac{2m}{p} + \frac{p^2}{3m}\right) \\
 &= \frac{4m^2}{p^2} + \frac{2mp^2}{3mp} + \frac{2mp^2}{3mp} + \frac{p^4}{9m^2} \\
 &= \frac{4m^2}{p^2} + \frac{4p}{3} + \frac{p^4}{9m^2}
 \end{aligned}$$

Vb. 4 Vereenvoudig die volgende: (Korter metode!)

$$(a) \quad (m + 5n)^2 = (m)^2 + 2(m)(5n) + (5n)^2 = m^2 + 10mn + 25n^2$$

$$(b) \quad (pq - 2)^2 = p^2q^2 - 4pq + 4$$

$$(c) \quad \left(\frac{1}{3} + 3x\right)^2 = \frac{1}{9} + 2x + 9x^2$$

Oefening 4:

Datum: _____

Vereenvoudig(Gebruik die korter metode!)

$$\begin{aligned}
 (1) \quad & (x - 3)^2 \\
 &= x^2 - 6x + 9
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & (6m - 1)^2 \\
 &= 36m^2 - 12m + 1
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad & (3y + 7)^2 \\
 &= 9y^2 + 42y + 49
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad & (3 + pq)^2 \\
 &= 9 + 6pq + p^2q^2
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad & (5t^2 + 8)^2 \\
 &= 25t^4 + 80t^2 + 64
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad & \left(\frac{2}{3} - 6y\right)^2 \\
 &= \frac{4}{9} - 8y + 36y^2
 \end{aligned}$$

$$\begin{aligned}
 (7) \quad & (-2k - 5)^2 \\
 &= 4k^2 + 20k + 25
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad & \left(\frac{3p - 2q}{5m}\right)^2 \\
 &= \frac{9p^2 - 12pq + 4q^2}{25m^2}
 \end{aligned}$$

$$(9) \quad (4x^2 + 10y^2)^2$$

$$= 16x^4 + 80x^2y^2 + 100y^4$$

$$(11) \quad (8 - 3y)(8 + 3y)$$

$$= 64 - 9y^2$$

$$(10) \quad (2mn + 7)(7 + 2mn)$$

$$= (2mn + 7)(2mn + 7)$$

$$= 4m^2n^2 + 28mn + 49$$

$$(12) \quad -2(abc - 11)^2$$

$$= -2(a^2b^2c^2 - 22abc + 121)$$

$$= -2a^2b^2c^2 + 44abc - 242$$

A2.1.3 Tweeterme en drieterme:

Vb. 5 Vereenvoudig die volgende produkten:

$$(4y + 1)(y^2 - y + 5)$$

$$= 4y^3 - 4y^2 + 20y + 1y^2 - 1y + 5$$

$$= 4y^3 - 3y^2 + 19y + 5$$

Oefening 5:

Datum: _____

Vereenvoudig die volgende:

$$(1) \quad (2a - 3)(a^2 + 5a - 4)$$

$$= 2a^3 + 10a^2 - 8a - 3a^2 - 15a + 12$$

$$= 2a^3 + 7a^2 - 23a + 12$$

$$(3) \quad (1 + x)(1 - x + x^2)$$

$$= 1 - 1x + 1x^2 + 1x - 1x^2 + x^3$$

$$= 1 + x^3$$

$$(5) \quad \left(2m + \frac{1}{2}\right)\left(\frac{m^2}{4} + 4 - 4m\right)$$

$$= \frac{2m^3}{4} + 8m - 8m^2 + \frac{m^2}{8} + 2 - \frac{4m}{2}$$

$$= \frac{m^3}{2} + 8m - \frac{8m^2}{1} + \frac{1m^2}{8} + 2 - 2m$$

$$= \frac{m^3}{2} + 6m - 7\frac{7}{8}m^2 + 2$$

$$(2) \quad (m + 7)(2m^2 + 3m + 3)$$

$$= 2m^3 + 3m^2 + 3m + 14m^2 + 21m + 21$$

$$= 2m^3 + 17m^2 + 24m + 21$$

$$(4) \quad (3y - 2)(9y^2 + 6y + 4)$$

$$= 27y^3 + 18y^2 + 12y - 18y^2 - 12y - 8$$

$$= 27y^3 - 8$$

$$(6) \quad (m^2n^2 - 5)(25 + 5m^2n^2 + m^4n^4)$$

$$= 25m^2n^2 + 5m^4n^4 + m^6n^6 - 125$$

$$\quad - 25m^2n^2 - 5m^2n^2$$

$$= m^6n^6 - 125$$

A2.1.4 Die som en verskil van twee derdemagte:

Vb. 6 Beskou die volgende:

Produk:

$$(a) (x - 3)(x^2 + 3x \oplus 9) = x^3 + 3x^2 + 9x - 3x^2 - 9x - 27 = x^3 - 27$$

$$(b) (y + 5)(y^2 - 5y \oplus 25) = y^3 - 5y^2 + 25y + 5y^2 - 25y + 125 = y^3 + 125$$

$$(c) (4m - 1)(16m^2 + 4m \oplus 1) = \underline{(4m - 1)}[\underline{(4m)^2} + \underline{(4m)(1)} + \underline{(1)^2}] = 64m^3 - 1$$

$$(d) (n^2 + 2)(n^4 - 2n^2 \oplus 4) = \underline{(n^2 + 2)}[\underline{(n^2)^2} - \underline{(2)(n^2)} + \underline{(2)^2}] = n^6 + 8$$

Oefening 6:

Datum: _____

Skryf die volgende produkte direk neer, indien moontlik:

$$(1) (a + 3)(a^2 - 3a + 9)$$

$$= a^3 + 27$$

$$(2) (2y^3 + 4)(4y^6 - 8y^3 + 16)$$

$$= 8y^9 + 64$$

$$(3) \left(\frac{x}{3} - 1\right)\left(\frac{1}{9}x^2 + \frac{1}{3}x + 1\right)$$

$$= \frac{x^3}{27} - 1$$

$$(4) \left(6a^2 - \frac{1}{2}\right)\left(36a^4 + 3a^2 + \frac{1}{4}\right)$$

$$= 216a^6 - \frac{1}{8}$$

$$(5) (5q + 7)(25q^2 - 35q + 49)$$

$$= 125q^3 + 343$$

$$(6) (8 - 3m)(9m^2 + 24m + 64)$$

$$= (8 - 3m)(64 + 24m + 9m^2)$$

$$= 512 - 27m^3$$

$$(7) (x - 5)(x^2 - 5x + 25)$$

$$= x^3 - 5x^2 + 25x - 5x^2 + 25x - 125$$

$$= x^3 - 10x^2 + 50x - 125$$

$$(8) (0,1 + 0,2y)(0,01 - 0,02y + 0,04y^2)$$

$$= 0,001 + 0,008y^3$$

$$(9) (9a^4 + 6a^2b + 4b^2)(3a^2 - 2b)$$

$$= (3a^2 - 2b)(9a^4 + 6a^2b + 4b^2)$$

$$= 27a^6 - 8b^3$$

$$(10) 2(-1 + 5m)(25m^2 + 5m + 1)$$

$$= 2(5m - 1)(25m^2 + 5m + 1)$$

$$= 2(125m^3 - 1)$$

$$= 250m^3 - 2$$