

# **Graad 12 – Boek A**

**(Eerste KABV uitgawe)**

## **INHOUD:**

### **Bladsy:**

A1. Rye en reekse	3
A2. Logaritmes en funksie inverses	103
A3. Finansiële Wiskunde	137

Hierdie boek is opgestel en verwerk deur E.J. Du Toit in 2023.

Webtuiste: [www.abcbooks.co.za](http://www.abcbooks.co.za)

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**MET SPESIALE DANK EN ERKENNING AAN DIE DEPARTEMENT VAN ONDERWYS VIR DIE GEBRUIK VAN UITTREKSELS UIT OU VRAESTELLE.**

ISBN 978-1-928336-51-8

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# Hoofstuk A1

## Rye en reekse

### A1.1 Rekenkundige rye en reekse:

#### A1.1.1 Rekenkundige rye:

'n Rekenkundige ry (RR) het 'n konstante verskil.

Algemene term:  $T_n = a + (n - 1)d$

Met:  $a \rightarrow$  die eerste term

$d \rightarrow$  konstante verskil:  $d = T_2 - T_1 = T_3 - T_2 = T_8 - T_7 \dots\dots\dots$

$n \rightarrow$  posisie van die term  $[n \in \mathbb{N}_0]$

**Vb. 1** Beskou die ry: **5 ; 2 ; -1 ; -4 ; ... ..**

**(a) Bereken die algemene term van die ry.**

**(b) Bereken  $T_{34}$**

**(c) Bereken  $n$  as  $T_n = -58$**

**(d) Los op vir  $x$  as  $5T_x - 3T_{x+1} = 1$**

**(a)  $a = 5$**

**$d = T_2 - T_1 = 2 - 5 = -3$  of  $d = T_3 - T_2 = -1 - 2 = -3$**

**$T_n = a + (n - 1)d$**

**$\therefore T_n = 5 + (n - 1)(-3)$**

**$\therefore T_n = 5 - 3n + 3$**

**$\therefore T_n = 8 - 3n$**

**(b)  $T_n = 8 - 3n$**

**$\therefore T_{34} = 8 - 3(34)$**

**$\therefore T_{34} = -94$**

**(c)  $T_n = 8 - 3n$**

**$\therefore -58 = 8 - 3n$**

**$\therefore 3n = 8 + 58$**

**$\therefore 3n = 66$**

**$\therefore n = \frac{66}{3}$**

**$\therefore n = 22$**

**(d)  $5T_x - 3T_{x+1} = 1$**

**$\therefore 5(8 - 3x) - 3[8 - 3(x + 1)] = 1$**

**$\therefore 40 - 15x - 3[8 - 3x - 3] = 1$**

**$\therefore 40 - 15x - 24 + 9x + 9 = 1$**

**$\therefore -6x = 1 - 25 = -24$**

**$\therefore x = 4$**



Oefening 1:

Datum: \_\_\_\_\_

(1) Skryf die volgende drie terme in elk van die volgende rye neer:

[Dui ook aan watter van die rye is rekenkundige rye.]

(a)  $3 ; 6 ; 12 ; 24 ;$  \_\_\_\_\_

(b)  $3 ; -4 ; -11 ; -18 ;$  \_\_\_\_\_

(c)  $113 ; 115 ; 117 ; 119 ;$  \_\_\_\_\_

(d)  $0,17 ; 0,37 ; 0,57 ; 0,77 ;$  \_\_\_\_\_

(e)  $\frac{1}{2} ; \frac{2}{3} ; \frac{3}{4} ; \frac{4}{5} ;$  \_\_\_\_\_

(f)  $2 ; 8 ; 18 ; 32 ;$  \_\_\_\_\_

(2) Skryf die eerste vier terme in elk van die volgende rye neer:

[Dui ook aan watter van die rye is rekenkundige rye.]

(a)  $T_n = -3n$  \_\_\_\_\_

(b)  $T_n = 2^n$  \_\_\_\_\_

(c)  $T_n = n + 10$  \_\_\_\_\_

(d)  $T_n = 4n + 1$  \_\_\_\_\_

(e)  $T_n = n^2$  \_\_\_\_\_

(f)  $T_n = \frac{n}{2}$  \_\_\_\_\_

(3) Beskou die ry:  $3 ; 7 ; 11 ; 15 ; \dots$ 

(a) Bereken die algemene term van die ry.

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(b) Bereken  $T_{25}$

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(c) Bereken  $n$  as  $T_n = 87$

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(4) Hoeveel terme is daar in die volgende ry: 65 ; 59 ; 53 ; 47 ; ... .. ; -85?

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(5) Plaas 6 terme tussen 8 en 29 sodat dit 'n rekenkundige ry vorm.

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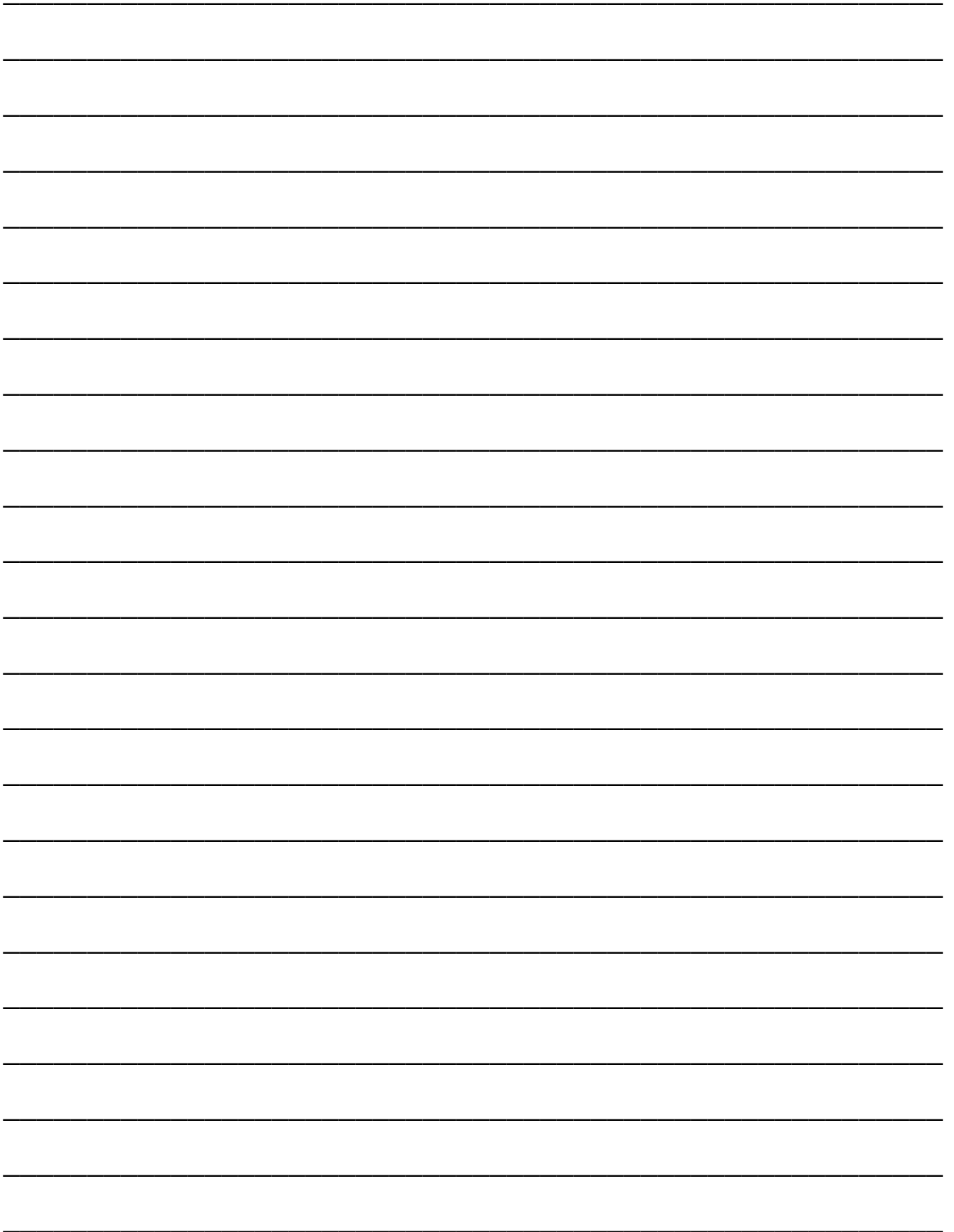
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(6) Die eerste drie terme van 'n RR is:  $x - 1$  ;  $2x + 1$  ;  $3 - x$

(a) Bereken die waarde van  $x$ .

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(b) Skryf die eerste vyf terme van die ry neer.

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(c) Skryf die  $n^{\text{de}}$  term neer.

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(d) Bepaal die  $80^{\text{ste}}$  term van die ry.

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(7) As  $T_n = 5 - 2n$ , bereken:

(a) die eerste term van die ry.

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(b) die konstante verskil van die ry.

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(c)  $T_{24} + 3 T_{56}$

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(8) Die algemene term van 'n RR is  $T_n = 4n + 3$ .

Bereken:  $T_{2x} - 2 T_{x-1}$

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(9) In 'n RR is die derde term gelyk aan 18 en  $T_{10} = -17$ .

Bepaal die eerste drie terme van die ry.

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- (10) Die som van die derde term en die vierde term van 'n RR is 26 en die verskil tussen dieselfde ry se elfde en tiende term is 4. Bepaal die eerste term en dan ook die waarde van term vier-en-tagtig.

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## **A1.1.2 Rekenkundige reekse:**

### **A1.1.2.1 Formule:**

**Formule vir die som ( $S_n$ ) van 'n RR:**

$$S_n = a + [a + d] + \dots + [a + (n-2)d] + [a + (n-1)d]$$

$$+ S_n = [a + (n-1)d] + [a + (n-2)d] + \dots + [a + d] + a$$

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$$\therefore 2 S_n = [2a + (n-1)d] + [2a + (n-1)d] + \dots + [2a + (n-1)d] + [2a + (n-1)d]$$

$$\therefore 2 S_n = n [2a + (n-1)d]$$

$$\therefore S_n = \frac{n}{2} [2a + (n-1)d]$$

of as  $l \rightarrow$  laaste term

$$S_n = a + [a + d] + \dots + [l - d] + l$$

$$+ S_n = l + [l - d] + \dots + [a + d] + a$$

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$$\therefore 2 S_n = [a + l] + [a + l] + \dots + [a + l] + [a + l]$$

$$\therefore 2 S_n = n [a + l]$$

$$\therefore S_n = \frac{n}{2} [a + l]$$

Vb. 2 Bereken:  $5 + 2 - 1 - 4 - \dots - 94$

$$\begin{aligned}
 T_n &= a + (n - 1)d & \text{met} & & S_n &= \frac{n}{2}[a + \ell] \\
 \therefore T_n &= 5 + (n - 1)(-3) & \text{met} & & \therefore S_{34} &= \frac{34}{2}[5 + (-94)] \\
 \therefore -94 &= 5 - 3n + 3 & & & \therefore S_{34} &= -1\,513 \\
 \therefore 3n &= 8 + 94 & & & & \\
 \therefore 3n &= 102 & & & \text{of } S_n &= \frac{n}{2}[2a + (n - 1)d] \\
 \therefore n &= 34 & & & \therefore S_{34} &= \frac{34}{2}[2(5) + (34 - 1)(-3)] \\
 \therefore T_{34} &= -94 = \ell \text{ [laaste term]} & & & \therefore S_{34} &= -1\,513
 \end{aligned}$$

### A1.1.2.2 Sigma-notasie:

$$\text{Sigma notasie} \rightarrow \sum_{k=2}^8 5k - 1$$

Wat lees as: Bereken die som vanaf waar  $k = 2$  tot by  $k = 8$  vir  $(5k - 1)$ .

Vb. 3 Bereken  $n$  as  $\sum_{k=1}^n 3k - 1 = 442$

$$\therefore \text{Bereken: } [3(1) - 1] + [3(2) - 1] + [3(3) - 1] + \dots + [3(k) - 1]$$

$$\therefore 2 + 5 + 8 + \dots + [3(n) - 1] = 442$$

$$\therefore a = 1$$

$$\therefore S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$d = 2$$

$$\therefore 442 = \frac{n}{2}[2(2) + (n - 1)(3)]$$

$$n = ?$$

$$\therefore 442 \times 2 = n[4 + 3n - 3]$$

$$S_n = 442$$

$$\therefore 884 = 3n^2 + 1n$$

$$\therefore 0 = 3n^2 + 1n - 884$$

$$\therefore 0 = (3n + 52)(n - 17)$$

$$\therefore n = \frac{-52}{3} \quad \text{of} \quad n = 17$$

Nvt

$$\therefore S_{17} = 442 \quad [n \in \mathbb{N}_0]$$

**Vb. 4 Skryf die volgende in sigma-notasie:**

$$29 + 25 + 21 + 17 + \dots \quad (\text{tot } 18 \text{ terme})$$

$$T_n = a + (n - 1)d$$

$$\therefore T_n = 29 + (n - 1)(-4)$$

$$\therefore T_n = 29 - 4n + 4$$

$$\therefore T_n = 33 - 4n$$

$$\therefore \text{Sigma notasie} \rightarrow \sum_{n=1}^{18} 33 - 4n$$

Oefening 2:

Datum: \_\_\_\_\_

(1) Bereken:

(a)  $5 + 8 + 11 + 14 + \dots$  tot 16 terme

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(b)  $9 + 12 + 15 + 18 + \dots + 264$ 


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(c)  $36 + 31 + 26 + 21 + \dots$  tot 34 terme

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(d)  $\sum_{k=1}^7 (4k - 1)$

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(e)  $-7 - 2 + 3 + 8 + \dots + 123$

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(f)  $\sum_{n=3}^{12} (3 - n)$

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(g)  $n$  as  $\sum_{k=1}^n (3k - 2) = 92$

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(j)  $n$  as  $0,3 + 1,1 + 1,9 + 2,7 + \dots$  (tot  $n$  terme) =  $24,8$

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(2) Die  $n^{\text{de}}$  term van 'n RR is  $2n + 3$ . Bepaal:

(a) die eerste drie terme van die ry.

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(b) die  $18^{\text{de}}$  term van die ry.

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(c) hoeveel terme in die ry het 'n som van 4 352.

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(3) Die volgende is gegee:  $\sum_{t=2}^{11} (3 - 3t)$

(a) Skryf die eerste drie terme neer.

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(b) Bepaal die som van die reeks.

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(b) Bepaal  $n$  waarvoor  $S_n < 403$ .

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(6) Gegee:  $-1 ; 2 ; 5 ; 8 ; \dots$

(a) Bepaal die twintigste term van die ry.

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(b) Bepaal die som van die eerste twintig terme.

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(c) Watter term in die ry is gelyk aan 56?

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(d) Hoeveel terme moet bymekaar getel word om 'n som van 259 te kry?

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(7) Die eerste drie terme van 'n RR is:  $7x - 1$  ;  $2x + 3$  en  $3 - 5x$

(a) Bepaal die waarde van  $x$ .

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(b) Skryf die eerste vier terme van die ry neer.

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(c) Bereken  $T_{18}$ .

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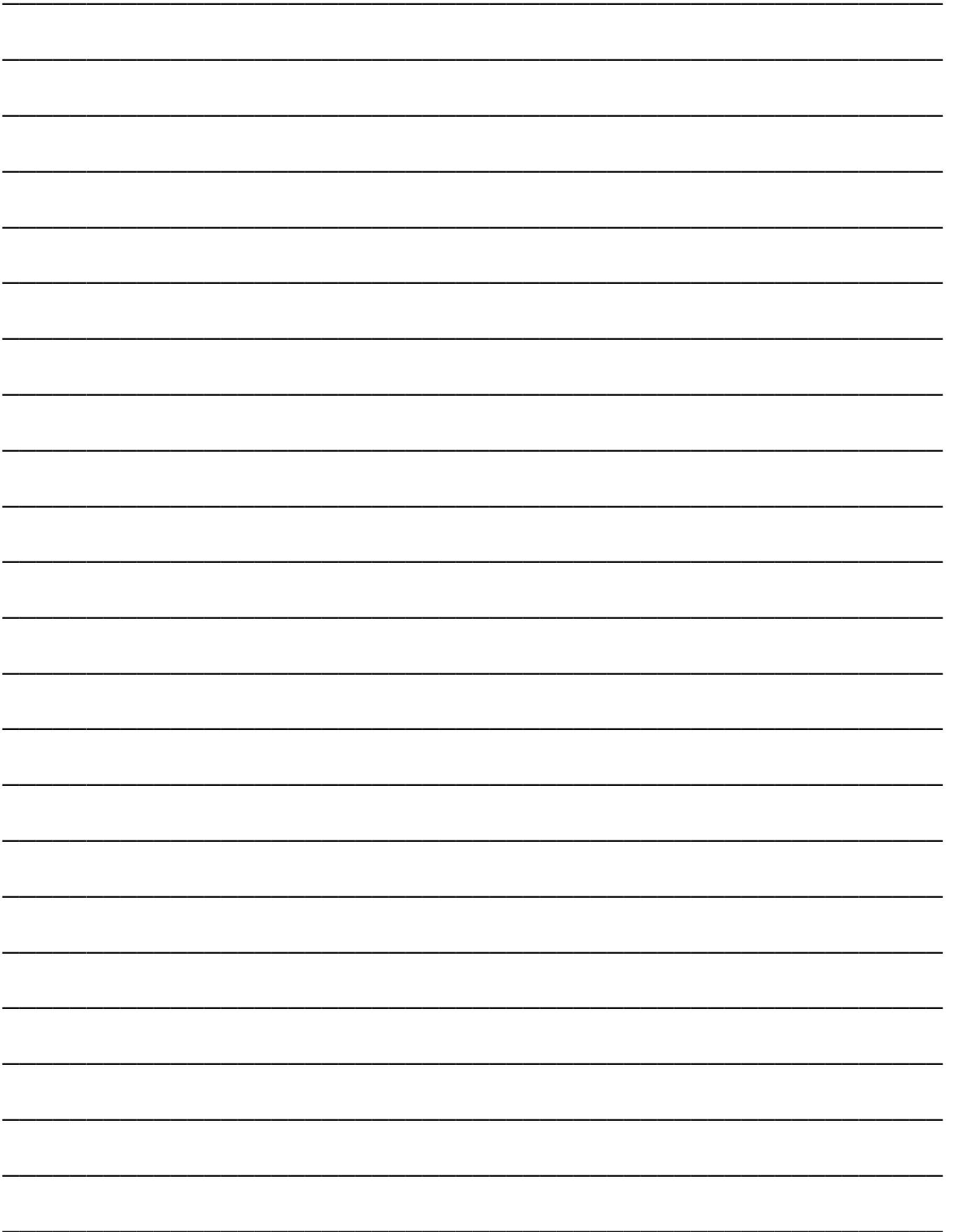
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(c) Bepaal die dertiende term.

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(9) Gegee rekenkundige ry:  $3k + 2$  ;  $5k + 1$  ;  $7k$  ;  $9k - 1$  ; ... ..

(a) Bepaal die konstante verskil in terme van  $k$ .

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(b) Bepaal die som van die eerste twaalf terme in terme van  $k$ .

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### **A1.1.3 Toepassings:**

**Vb. 5 Rachel oefen vir 'n marathon. Sy hardloop 20 km op die eerste dag se voorbereiding. Sy besluit om elke dag as sy oefen 3 km verder te hardloop as die vorige keer.**

**(a) Indien sy hierdie oefenprogram volg, bereken tydens watter dag van oefening sy 56 km sal hardloop.**

**(b) Bereken Rachel se totale afstand wat sy gehardloop het vir die eerste 10 voorbereidende oefeninge.**

**(a) RR: 20 ; 23 ; 26 ; 29 ; ..... ; 56**

$$\therefore a = 20$$

$$d = 3$$

$$n = ?$$

$$T_n = 56$$

$$T_n = a + (n - 1)d$$

$$\therefore 56 = 20 + (n - 1)(3)$$

$$\therefore 56 = 20 + 3n - 3$$

$$\therefore -3n = 17 - 56$$

$$\therefore -3n = -39$$

$$\therefore n = 13$$

**$\therefore$  Rachel sal op die dertiende dag 56 km hardloop.**

(b)  $S_{10} = ?$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\therefore S_{10} = \frac{10}{2}[2(20) + (10 - 1)(3)]$$

$$\therefore S_{10} = 335$$

**$\therefore$  Rachel se totale afstand vir die eerste 10 dae is 335 km.**

Oefening 3:

Datum: \_\_\_\_\_

- (1) Cory sny sestien stukkies tou vir 'n projek. Die kortste van die toue is 28 cm lank en die langste van die toue is 88 cm lank. Indien die lengte van die sestien toue van kort tot lank neergeskryf word, vorm dit 'n RR.
- (a) Bereken die lengte van die 13<sup>de</sup> tou in die ry.

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- (b) Bereken die totale lengte van al 16 toue.

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