# Grade 12 – Book A

**(First edition – CAPS)**

**TEACHER'S GUIDELINES**

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This book was compiled and processed by E.J. Du Toit in 2023.

Website: www.abcbooks.co.za





WITH SPECIAL THANKS AND ACKNOWLEDGMENT TO THE DEPARTMENT OF EDUCATION FOR THE USE OF EXTRACTS FROM OLD PAPERS.

ISBN 978-1-928336-68-6


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# Chapter A1
# Sequences and series

## A1.1 Arithmetic sequences and series:
### A1.1.1 Arithmetic sequences:

An arithmetic sequence (AS) has a constant difference.

General term: $T_n = a + (n-1)d$

With: a $\rightarrow$ the first term

d $\rightarrow$ constant difference: $d = T_2 - T_1 = T_3 - T_2 = T_8 - T_7$ ……….

$n$ $\rightarrow$ position of the term $[n \in \mathbb{N}_0]$

**Ex. 1 Consider the sequence: $5 ; 2 ; -1 ; -4 ; \ldots\ldots$**

**(a) Calculate the general term of the sequence.**
**(b) Calculate $T_{34}$**
**(c) Calculate $n$ if $T_n = -58$**
**(d) Solve for $x$ if $5T_x - 3T_{x+1} = 1$**

(a) $a = 5$

$d = T_2 - T_1 = 2 - 5 = -3$ or $d = T_3 - T_2 = -1 - 2 = -3$

$T_n = a + (n-1)d$

$\therefore T_n = 5 + (n-1)(-3)$

$\therefore T_n = 5 - 3n + 3$

$\therefore T_n = 8 - 3n$

(b) $T_n = 8 - 3n$

$\therefore T_{34} = 8 - 3(34)$

$\therefore T_{34} = -94$

(c) $T_n = 8 - 3n$

$\therefore -58 = 8 - 3n$

$\therefore 3n = 8 + 58$

$\therefore 3n = 66$

$\therefore n = \frac{66}{3}$

$\therefore n = 22$

(d) $5T_x - 3T_{x+1} = 1$

$\therefore 5(8 - 3x) - 3[8 - 3(x + 1)] = 1$

$\therefore 40 - 15x - 3[8 - 3x - 3] = 1$

$\therefore 40 - 15x - 24 + 9x + 9 = 1$

$\therefore -6x = 1 - 25 = -24$

$\therefore x = 4$

Exercise 1: Date:________________________

(1) Write the next three terms in each of the following sequences:
[Also indicate which of the sequences are arithmetic sequences. **AS**]

(a) $\times 2 \quad \times 2 \quad \times 2$
$3\ ;\ 6\ ;\ 12\ ;\ 24\ ;\ \mathbf{48\ ;\ 96\ ;\ 192}$

(b) $-7 \quad -7 \quad -7$
$3\ ;\ -4\ ;\ -11\ ;\ -18\ ;\ \mathbf{-25\ ;\ -32\ ;\ -39}$ **AS**

(c) $+2 \quad +2 \quad +2$
$113\ ;\ 115\ ;\ 117\ ;\ 119\ ;\ \mathbf{121\ ;\ 123\ ;\ 125}$ **AS**

(d) $+0,2 \quad +0,2 \quad +0,2$
$0{,}17\ ;\ 0{,}37\ ;\ 0{,}57\ ;\ 0{,}77\ ;\ \mathbf{0{,}97\ ;\ 1{,}17\ ;\ 1{,}37}$ **AS**

(e) $\frac{1}{2}\ ;\ \frac{2}{3}\ ;\ \frac{3}{4}\ ;\ \frac{4}{5}\ ;\ \mathbf{\frac{5}{6}\ ;\ \frac{6}{7}\ ;\ \frac{7}{8}}$ **[Numerator and denominator increase with 1.]**

(f) $+6 \quad +10 \quad +14 \quad +18 \quad +22 \quad +26$
$2\ ;\ 8\ ;\ 18\ ;\ 32\ ;\ \mathbf{50\ ;\ 72\ ;\ 98}$ **[Increases with 4 more each time.]**

(2) Write the first four terms in each of the following sequences:
[Also indicate which of the sequences are arithmetic sequences. **AS**]

(a) $T_n = -3n \rightarrow \mathbf{-3\ ;\ -6\ ;\ -9\ ;\ -12}$ **AS**

(b) $T_n = 2^n \rightarrow \mathbf{2\ ;\ 4\ ;\ 8\ ;\ 16}$

(c) $T_n = n + 10 \rightarrow \mathbf{11\ ;\ 12\ ;\ 13\ ;\ 14}$ **AS**

(d) $T_n = 4n + 1 \rightarrow \mathbf{5\ ;\ 9\ ;\ 13\ ;\ 17}$ **AS**

(e) $T_n = n^2 \rightarrow \mathbf{1\ ;\ 4\ ;\ 9\ ;\ 16}$

(f) $T_n = \frac{n}{2} \rightarrow \mathbf{\frac{1}{2}\ ;\ 1\ ;\ 1\frac{1}{2}\ ;\ 2}$ **AS**

(3) Consider the sequence: 3 ; 7 ; 11 ; 15 ; ... ....

(a) Calculate the general term of the sequence.

$\mathbf{a = 3}$

$\mathbf{d = 7 - 3 = 4}$

$\mathbf{T_n = ?}$

$\mathbf{T_n = a + (n-1)d}$

$\mathbf{= 3 + (n-1)(4)}$

$\mathbf{= 3 + 4n - 4}$

$\mathbf{\therefore\ T_n = 4n - 1}$

(b) Calculate $T_{25}$

$$T_n = 4n - 1$$

$$T_{25} = 4(25) - 1$$

$$\therefore T_{25} = 99$$

(c) Calculate $n$ if $T_n = 87$

$$T_n = 4n - 1$$

$$\therefore 87 = 4n - 1$$

$$\therefore 87 + 1 = 4n$$

$$\therefore 4n = 88$$

$$\therefore n = 22$$

(4) How many terms are there in the following sequence: $65\ ;\ 59\ ;\ 53\ ;\ 47\ ;\ \dots\dots\dots ;\ -85$?

$a = 65$

$d = 59 - 65 = -6$

$T_n = -85$

$n = ?$

$$T_n = a + (n - 1)d$$

$$-85 = 65 + (n - 1)(-6)$$

$$\therefore -85 = 65 - 6n + 6$$

$$\therefore 6n = 71 + 85$$

$$\therefore 6n = 156$$

$$\therefore n = 26$$

(5) Place 6 terms between 8 and 29 for it to form an arithmetic sequence.

$\therefore\ 8\ ;\ \_\_\ ;\ \_\_\ ;\ \_\_\ ;\ \_\_\ ;\ \_\_\ ;\ \_\_\ ;\ 29$

$a = 8$

$d = ?$

$T_8 = 29$

$n = 8$

$$T_n = a + (n - 1)d$$

$$29 = 8 + (8 - 1)d$$

$$29 = 8 + 7d$$

$$\therefore 21 = 7d$$

$$\therefore d = 3$$

$\therefore\ 8\ ;\ 11\ ;\ 14\ ;\ 17\ ;\ 20\ ;\ 23\ ;\ 26\ ;\ 29$

(6) The first three terms of an AS are: $\underset{\mathbf{T_1}}{x - 1}$ ; $\underset{\mathbf{T_2}}{2x + 1}$ ; $\underset{\mathbf{T_3}}{3 - x}$

(a) Calculate the value of $x$.

**For AS:** $T_2 - T_1 = T_3 - T_2$

$\therefore\ (2x + 1) - (x - 1) = (3 - x) - (2x + 1)$

$\therefore\ 2x + 1 - x + 1 = 3 - x - 2x - 1$

$\therefore\ x + 2 = 2 - 3x$

$\therefore\ x + 3x = 2 - 2$

$\therefore\ 4x = 0$

$\therefore\ x = 0$

(b) Write down the first five terms of the sequence.

$T_1 = x - 1 = 0 - 1 = -1$

$T_2 = 2x + 1 = 2(0) + 1 = 0 + 1 = 1$

$T_3 = 3 - x = 3 - (0) = 3$ $\rightarrow$ $d = 2$

$\therefore\ T_4 = 5$

and $T_5 = 7$ $\therefore$ **Sequence: $-1$ ; $1$ ; $3$ ; $5$ ; $7$**

(c) Write down the $n^{th}$ term.

$a = -1$ $T_n = a + (n - 1)d$

$d = 2$ $= -1 + (n - 1)(2)$

$T_n = ?$ $= -1 + 2n - 2$

$\therefore\ T_n = 2n - 3$

(d) Determine the 80$^{th}$ term of the sequence.

$T_n = 2n - 3$

$\therefore\ T_{80} = 2(80) - 3$

$\therefore\ T_{80} = 157$

(7) If $T_n = 5 - 2n$, calculate:

(a) the first term of the sequence.

$T_n = 5 - 2n$

$\therefore T_1 = 5 - 2(1)$

$\therefore T_1 = 3$

(b) the constant difference of the sequence.

$T_2 = 5 - 2(2) = 5 - 4 = 1$

$\therefore d = T_2 - T_1 = 1 - 3$

$\therefore d = -2$

(c) $T_{24} + 3T_{56}$

$= [5 - 2(24)] + 3[5 - 2(56)]$

$= [-43] + 3[-107]$

$= -364$

(8) The general term of an AS is $T_n = 4n + 3$.

Calculate: $T_{2x} - 2T_{x-1}$

$T_{2x} - 2T_{x-1}$

$= [4(2x) + 3] - 2[4(x - 1) + 3]$

$= [8x + 3] - 2[4x - 4 + 3]$

$= [8x + 3] - 2[4x - 1]$

$= 8x + 3 - 8x + 2$

$= 5$

(9) The third term in an AS is equal to 18 and $T_{10} = -17$.

Determine the first three terms of the sequence.

$T_3 = a + (3 - 1)d = a + 2d = 18$ and

$T_{10} = a + (10 - 1)d = a + 9d = -17$

$\therefore \quad a + 9d = -17$

$-(a + 2d = 18)$

$7d = -35$

$\therefore \quad d = -5$

$a + 2(-5) = 18 \rightarrow a + -10 = 18$

$\therefore a = 28$

$\therefore$ Sequence: 28 ; 23 ; 18 ; 13 ; … …

(10) The sum of the third term and the fourth term of an AS is 26 and the difference between the eleventh and the tenth terms of the same sequence is 4. Determine the first term and the value of term eighty-four.

$T_3 + T_4 = 26$ and $T_{11} - T_{10} = 4 \rightarrow d$

$(a + 2d) + (a + 3d) = 26$

$2a + 5d = 26$

$\therefore \quad 2a + 5(4) = 26$

$\therefore \quad 2a = 26 - 20 = 6$

$\therefore \quad a = 3 \rightarrow T_1$

$\therefore \quad T_{84} = a + 83d$

$\therefore \quad T_{84} = 3 + 83(4)$

$\therefore \quad T_{84} = 335$

## A1.1.2 Arithmetic series:

### A1.1.2.1 Formula:

**Formula for the sum ($S_n$) of an AS:**

$$S_n = a + [a + d] + \dots\dots + [a + (n-2)d] + [a + (n-1)d]$$

$$+ S_n = [a + (n-1)d] + [a + (n-2)d] + \dots\dots + [a + d] + a$$

$$\therefore 2S_n = [2a + (n-1)d] + [2a + (n-1)d] + \dots\dots + [2a + (n-1)d] + [2a + (n-1)d]$$

$$\therefore 2S_n = n[2a + (n-1)d]$$

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$

or if $\ell \rightarrow$ last term

$$S_n = a + [a + d] + \dots\dots + [\ell - d] + \ell$$

$$+ S_n = \ell + [\ell - d] + \dots\dots + [a + d] + a$$

$$\therefore 2S_n = [a + \ell] + [a + \ell] + \dots\dots + [a + \ell] + [a + \ell]$$

$$\therefore 2S_n = n[a + \ell]$$

$$\therefore S_n = \frac{n}{2}[a + \ell]$$

**Ex. 2 Calculate: $5 + 2 - 1 - 4 - \dots\dots - 94$**

$T_n = a + (n-1)d$ with $S_n = \frac{n}{2}[a + \ell]$

$\therefore T_n = 5 + (n-1)(-3)$ with $\therefore S_{34} = \frac{34}{2}[5 + (-94)]$

$\therefore -94 = 5 - 3n + 3$ $\qquad \therefore S_{34} = -1\,513$

$\therefore 3n = 8 + 94$

$\therefore 3n = 102$ $\qquad$ or $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore n = 34$ $\qquad \therefore S_{34} = \frac{34}{2}[2(5) + (34-1)(-3)]$

$\therefore T_{34} = -94 = \ell$ [last term] $\qquad \therefore S_{34} = -1\,513$

## A1.1.2.2 Sigma-notation:

Sigma notation $\rightarrow \sum_{k=2}^{8} 5k - 1$

It reads as: Calculate the sum from where $k = 2$ up to where $k = 8$ for $(5k - 1)$.

**Ex. 3 Calculate $n$ if $\sum_{k=1}^{n} 3k - 1 = 442$**

$\therefore$ Calculate: $[3(1) - 1] + [3(2) - 1] + [3(3) - 1] + \dots\dots\dots + [3(k) - 1]$

$\therefore 2 + 5 + 8 + \dots\dots + [3(n) - 1] = 442$

$\therefore a = 1$ $\qquad \therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$d = 2$ $\qquad \therefore 442 = \frac{n}{2}[2(2) + (n-1)(3)]$

$n = ?$ $\qquad \therefore 442 \times 2 = n[4 + 3n - 3]$

$S_n = 442$ $\qquad \therefore 884 = 3n^2 + 1n$

$\therefore 0 = 3n^2 + 1n - 884$

$\therefore 0 = (3n + 52)(n - 17)$

$\therefore n = \frac{-52}{3}$ or $n = 17$

N/A

$\therefore S_{17} = 442 \qquad [n \in \mathbb{N}_0]$

**Ex. 4 Write the following in sigma-notation:**

$29 + 25 + 21 + 17 + \ldots\ldots$ **(to 18 terms)**

$T_n = a + (n - 1)d$

$\therefore \quad T_n = 29 + (n - 1)(-4)$

$\therefore \quad T_n = 29 - 4n + 4$

$\therefore \quad T_n = 33 - 4n$

$\therefore$ **Sigma notation** $\rightarrow \sum_{n=1}^{18} 33 - 4n$

Exercise 2: Date:________________________

(1) Calculate:

(a) $5 + 8 + 11 + 14 + \ldots\ldots\ldots$ to 16 terms

$a = 5$ $\qquad S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 8 - 5 = 3$ $\qquad S_{16} = \frac{16}{2}[2 \times 5 + (16 - 1)(3)]$

$n = 16$ $\qquad = 8[10 + 15 \times 3]$

$S_n = ?$ $\qquad S_{16} = 440$

(b) $9 + 12 + 15 + 18 + \ldots\ldots\ldots + 264$

$a = 9$ $\qquad T_n = a + (n - 1)d$

$d = 3$ $\qquad 264 = 9 + (n - 1)(3)$

$n = ?$ $\qquad 264 = 9 + 3n - 3$

$T_n = 264$ $\qquad 258 = 3n$

$S_n = ?$ $\qquad \therefore \quad n = 86$

$\therefore \quad S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{86} = \frac{86}{2}[2 \times 9 + (86 - 1)(3)]$

$= 43[18 + 85 \times 3]$

$S_{86} = 11\,739$ or $S_{86} = \frac{86}{2}[9 + 264] = 11\,739$

(c) $36 + 31 + 26 + 21 + \ldots\ldots\ldots$ to 34 terms

$a = 36$ $\qquad S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 31 - 36 = -5$ $\qquad S_{34} = \frac{34}{2}[2 \times 36 + (34 - 1)(-5)]$

$n = 34$ $\qquad = 17[72 + 33 \times -5]$

$S_n = ?$ $\qquad S_{34} = -1\,581$

(d) $\sum_{k=1}^{7}(4k - 1)$

Sequence: $4(1) - 1$ ; $4(2) - 1$ ; $4(3) - 1$ ; …… ; $4(7) - 1$

$= 3$ ; $7$ ; $11$ ; …… ; $27$

$a = 3$ $\qquad S_n = \frac{n}{2}[2a + (n - 1)d]$ $\qquad$ or $\quad S_n = \frac{n}{2}[a + \ell]$

$d = 4$ $\qquad S_7 = \frac{7}{2}[2 \times 3 + (7 - 1)(4)]$ $\qquad S_7 = \frac{7}{2}[3 + 27]$

$n = 7$ $\qquad = 3{,}5[6 + 6 \times 4]$ $\qquad S_7 = 105$

$\ell = 27$ $\qquad S_7 = 105$

$S_n = ?$

(e) $-7 - 2 + 3 + 8 + \ldots\ldots\ldots + 123$

$a = -7$ $\qquad T_n = a + (n - 1)d$ $\qquad \therefore\ S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 5$ $\qquad 123 = -7 + (n - 1)(5)$ $\qquad S_{27} = \frac{27}{2}[2 \times -7 + (27 - 1)(5)]$

$n = ?$ $\qquad 123 = -7 + 5n - 5$ $\qquad = 13{,}5[-14 + 26 \times 5]$

$T_n = 123$ $\qquad 135 = 5n$ $\qquad S_{27} = 1\,566$

$S_n = ?$ $\qquad \therefore\ n = 27$ $\qquad$ or $\quad S_{27} = \frac{27}{2}[-7 + 123] = 1\,566$

(f) $\sum_{n=3}^{12}(3 - n)$

**Sequence:** $3-(3)\ ;\ 3-(4);\ 3-(5);\ \ldots\ldots\ ;\ 3-(12)$

$= 0\ ;\ -1\ ;\ -2\ ;\ \ldots\ldots\ ;\ -9$

$a = 0$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$ **or** $S_n = \frac{n}{2}[a + \ell]$

$d = -1$ $\quad S_{10} = \frac{10}{2}[2 \times 0 + (10-1)(-1)]$ $\quad S_{10} = \frac{10}{2}[0 + (-9)]$

$n = 10$ $\quad = 5[0 + 9 \times -1]$ $\quad S_{10} = -45$

$\ell = -9$ $\quad S_{10} = -45$

$S_n = ?$

(g) $n$ if $\sum_{k=1}^{n}(3k - 2) = 92$

**Sequence:** $3(1)-2\ ;\ 3(2)-2\ ;\ 3(3)-2\ ;\ \ldots\ldots\ ;\ 3(n)-2$

$= 1\ ;\ 4\ ;\ 7\ ;\ \ldots\ldots$

$a = 1$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$d = 3$ $\quad 92 = \frac{n}{2}[2 \times 1 + (n-1)(3)]$

$n = ?$ $\quad 184 = n[2 + 3n - 3]$

$S_n = 92$ $\quad 184 = 2n + 3n^2 - 3n$

$0 = 3n^2 - n - 184$

$0 = (3n + 23)(n - 8)$

$\therefore\ n = -\frac{23}{3}$ N/A $(n \in \mathbb{N}_0)$ **or** $n = 8$

(h) $-66 - 64 - 62 - 60 - \ldots\ldots\ldots - 22$

$a = -66$

$d = 2$

$n = ?$

$T_n = -22$

$S_n = ?$

$T_n = a + (n - 1)d$

$-22 = -66 + (n-1)(2)$

$-22 = -66 + 2n - 2$

$46 = 2n$

$\therefore \quad n = 23$

$\therefore \quad S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{23} = \frac{23}{2}[2 \times -66 + (23 - 1)(2)]$

$= 11,5[-132 + 22 \times 2]$

$S_{23} = -1\,012$

or $S_{23} = \frac{23}{2}[-66 + (-22)] = -1\,012$

(i) $\sum_{i=1}^{300} \left(\frac{i}{2}\right)$

**Sequence:** $\frac{(1)}{2}$ ; $\frac{(2)}{2}$; $\frac{(3)}{2}$…… ; $\frac{(300)}{2}$

$= \frac{1}{2}$ ; $1$ ; $1\frac{1}{2}$ ; …… ; $150$

$a = \frac{1}{2}$

$d = \frac{1}{2}$

$n = 300$

$\ell = 150$

$S_n = ?$

$S_{300} = \frac{300}{2}\left[2 \times \frac{1}{2} + (300 - 1)\left(\frac{1}{2}\right)\right]$

$S_{300} = \frac{300}{2}\left[1 + (299)\left(\frac{1}{2}\right)\right]$

$= 150\left[1 + (299)\left(\frac{1}{2}\right)\right]$

$S_{300} = 22\,575$

or $S_n = \frac{n}{2}[a + \ell]$

$S_{300} = \frac{300}{2}\left[\frac{1}{2} + 150\right]$

$S_{300} = 22\,575$

(j) $n$ if $0{,}3 + 1{,}1 + 1{,}9 + 2{,}7 + \ldots\ldots\ldots$ (to $n$ terms) $= 24{,}8$

$\mathbf{a = 0{,}3}$

$\mathbf{d = 0{,}8}$

$\boldsymbol{n = ?}$

$\mathbf{S_n = 24{,}8}$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$24{,}8 = \frac{n}{2}[2 \times 0{,}3 + (n - 1)(0{,}8)]$$

$$49{,}6 = n[0{,}6 + 0{,}8n - 0{,}8]$$

$$49{,}6 = 0{,}6n + 0{,}8n^2 - 0{,}8n$$

$$0 = 0{,}8n^2 - 0{,}2n - 49{,}6 \qquad [\times 10]$$

$$0 = 8n^2 - 2n - 496$$

$$0 = 4n^2 - n - 248$$

$$0 = (4n + 31)(n - 8)$$

$\therefore\ n = \frac{-31}{4}$ N/A ($n \in \mathbb{N}_0$) **or** $\boldsymbol{n = 8}$

(2) The $n^{\text{th}}$ term of an AS is $2n + 3$. Determine:

(a) the first three terms of the sequence.

$$T_n = 2n + 3$$

$$T_1 = 2(1) + 3 = 2 + 3 = 5$$

$$T_2 = 2(2) + 3 = 4 + 3 = 7$$

$$T_3 = 2(3) + 3 = 6 + 3 = 9$$

$\therefore$ **5 ; 7 ; 9**

(b) the 18th term of the sequence.

$$T_n = 2n + 3$$

$$T_{18} = 2(18) + 3$$

$$T_{18} = 36 + 3$$

$$\therefore T_{18} = 39$$

(c) how many terms in the sequence have a sum of 4 352.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$4\,352 = \frac{n}{2}[2(5) + (n - 1)(2)]$$

$$8\,704 = n[10 + 2n - 2]$$

$$8\,704 = 8n + 2n^2$$

$$0 = 8n + 2n^2 - 8\,704$$

$$0 = 2n^2 + 8n - 8\,704$$

$$0 = n^2 + 4n - 4\,352$$

$$0 = (n + 68)(n - 64)$$

$n + 68 = 0$ or $n - 64 = 0$

$\therefore$ $n = -68$ N/A ($n \in \mathbb{N}_0$) or $n = 64$

(3) The following is given: $\sum_{t=2}^{11}(3 - 3t)$

(a) Write down the first three terms.

$T_1 = 3 - 3(2) = 3 - 6 = -3 \rightarrow T_1 = -3$

$T_2 = 3 - 3(3) = 3 - 9 = -6 \rightarrow T_2 = -6$

$T_3 = 3 - 3(4) = 3 - 12 = -9 \rightarrow T_3 = -9$

(b) Determine the sum of the series.

$a = -3$

$d = -3$

$n = 10$

$S_n = ?$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{10} = \frac{10}{2}[2 \times -3 + (10 - 1)(-3)]$$

$$= 5[-6 + 9 \times -3]$$

$$= 5[-6 - 27]$$

$$S_{10} = -165$$

(4) If $S_n = n^2 + n$, determine $T_7$.

$$S_7 = 7^2 + 7 = 49 + 7 = 56$$

$$S_6 = 6^2 + 6 = 36 + 6 = 42$$

But $T_7 = S_7 - S_6 = 56 - 42$

$\therefore T_7 = 14$

(5) In an AS it is given that $S_6 = 123$ and $T_5 = 25$.

(a) Determine the first term and the constant difference of the sequence.

$S_6 = 123$ and $T_5 = 25$

But $S_6 = \frac{6}{2}[2a + (6-1)d]$ $T_5 = a + (5-1)d$

$\therefore 123 = 3[2a + 5d]$ $25 = a + 4d$

$\therefore 123 = 6a + 15d$ ← $25 - 4d = a$

$\therefore 123 = 6(25 - 4d) + 15d$

$\therefore 123 = 150 - 24d + 15d$

$\therefore 123 - 150 = -9d$

$\therefore -27 = -9d$

$\therefore d = \frac{-27}{-9} = 3$

$\therefore a = 25 - 4(3)$

$\therefore a = 25 - 12$

$\therefore a = 13$

(b) Determine $n$ for which $S_n < 403$.

$$S_n = \frac{n}{2}[2a + (n - 1)d] < 403$$

$$\therefore \frac{n}{2}[2(13) + (n - 1)(3)] < 403$$

$$\therefore n[26 + 3n - 3] < 806$$

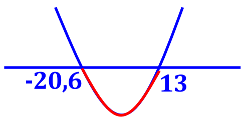
$$\therefore 26n + 3n^2 - 3n < 806$$

$$\therefore 3n^2 + 23n - 806 < 0$$

$$\therefore (3n + 62)(n - 13) < 0$$

$$\therefore 0 \leq n < 13$$

$$(n \in \mathbb{N}_0)$$




(6) Given: $-1 \ ; \ 2 \ ; \ 5 \ ; \ 8 \ ; \ \ldots\ldots\ldots$

(a) Determine the twentieth term of the sequence.

$a = -1$ $\qquad$ $T_n = a + (n - 1)d$

$d = 3$ $\qquad$ $\therefore T_{20} = -1 + (20 - 1)(3)$

$n = 20$ $\qquad$ $\therefore T_{20} = -1 + (19)(3) = -1 + 57$

$T_n = ?$ $\qquad$ $\therefore T_{20} = 56$

(b) Determine the sum of the first twenty terms.

$a = -1$ $\qquad$ $S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 3$ $\qquad$ $\therefore S_{20} = \frac{20}{2}[2(-1) + (20 - 1)(3)]$

$n = 20$ $\qquad$ $\therefore S_{20} = 10[-2 + (19)(3)] = 10[-2 + 57]$

$S_n = ?$ $\qquad$ $\therefore S_{20} = 550$

(c) Which term in the sequence is equal to 56?

$a = -1$ $\qquad$ $T_n = a + (n - 1)d$

$d = 3$ $\qquad$ $\therefore T_n = -1 + (n - 1)(3)$

$n = ?$ $\qquad$ $\therefore 56 = -1 + 3n - 3$

$T_n = 56$ $\qquad$ $\therefore 56 + 1 + 3 = 3n$

$\therefore 3n = 60$

$\therefore n = 20$

(d) How many terms in the sequence must be added to give a sum of 259?

$\mathbf{a = -1}$ $\quad$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$\mathbf{d = 3}$ $\quad$ $\therefore\ 259 = \frac{n}{2}[2(-1) + (n-1)(3)]$

$n = ?$ $\quad$ $259 = \frac{n}{2}[-2 + 3n - 3]$

$S_n = 374$ $\quad$ $\therefore\ 518 = 3n^2 - 5n$

$$0 = 3n^2 - 5n - 518$$

$$\therefore\ 0 = (3n + 37)(n - 14)$$

$\therefore\ n = \frac{-37}{3}$ N/A ($n \in \mathbb{N}_0$) $\quad$ or $\quad$ $n = 14$

(7) The first three terms of an AS are: $7x - 1$ ; $2x + 3$ and $3 - 5x$

(a) Determine the value of $x$.

**AS** $\rightarrow$ **d** $\rightarrow$ $T_2 - T_1 = T_3 - T_2$

$\therefore\ (2x + 3) - (7x - 1) = (3 - 5x) - (2x + 3)$

$\therefore\ 2x + 3 - 7x + 1 = 3 - 5x - 2x - 3$

$$-5x + 4 = -7x$$

$$-5x + 7x = -4$$

$$2x = -4$$

$$\therefore\ x = -2$$

(b) Write down the first four terms of the sequence.

$T_1 = 7x - 1 = 7(-2) - 1 = -14 - 1 = -15$

$T_2 = 2x + 3 = 2(-2) + 3 = -4 + 3 = -1$

$T_3 = 3 - 5x = 3 - 5(-2) = 3 + 10 = 13$

$\therefore$ **The sequence:** $-15\ ;\ -1\ ;\ 13\ ;\ 27\ ;\ \ldots\ldots$

(c) Calculate: $T_{18}$.

$a = -15$ $\quad$ $T_n = a + (n-1)d$

$d = 14$ $\quad$ $\therefore\ T_{18} = -15 + (18 - 1)(14)$

$n = 18$ $\quad$ $\therefore\ T_{18} = -15 + (17)(14)$

$$= -15 + 238$$

$T_n = ?$ $\quad$ $\therefore\ T_{18} = 223$

(d) How many terms should be added to the first 18 terms to have a sum of 2 360?

$a = -15$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$d = 14$ $\quad \therefore\ 2\,360 = \frac{n}{2}[2(-15) + (n-1)(14)]$

$n = ?$ $\quad 2\,360 = \frac{n}{2}[-30 + 14n - 14]$

$S_n = 2\,360$ $\quad \therefore\ 4720 = 14n^2 - 44n$

$0 = 14n^2 - 44n - 4\,720$

$0 = 7n^2 - 22n - 2\,360$

$\therefore\ 0 = (7n + 118)(n - 20)$

$\therefore\ n = \frac{-118}{7}$ N/A $(n \in \mathbb{N}_0)$ or $n = 20$

**$\therefore$ Two terms should be added to the first 18 terms to have a sum of 2 360.**

(8) The first term in an AS is 23 and the last term is $-369$.
The sum of the series is $-9\,861$.

(a) Determine the number of terms in the series.

$a = 23$ $\quad S_n = \frac{n}{2}[a + \ell]$

$\ell = -369$ $\quad \therefore\ -9\,861 = \frac{n}{2}[23 + (-369)]$

$n = ?$ $\quad -9\,861 = \frac{n}{2}[23 - 369]$

$S_n = -9\,861$ $\quad \therefore\ -9\,861 = \frac{n}{2} \times -346$

$\therefore\ -9\,861 = -173n$

$\therefore\ n = \frac{-9\,861}{-173}$

$\therefore\ n = 57$

(b) Determine the constant difference of the terms in the series.

$a = 23$ $\quad T_n = a + (n-1)d$

$d = ?$ $\quad \therefore\ -369 = 23 + (57 - 1)d$

$n = 57$ $\quad \therefore\ -369 = 23 + 56d$

$T_n = -369$ $\quad \therefore\ -369 = 23 + 56d$

$\therefore\ -56d = 23 + 369$

$\therefore\ -56d = 392$

$\therefore\ d = -7$

(c) Determine the thirteenth term.

$a = 23$ $\qquad T_n = a + (n - 1)d$

$d = -7$ $\qquad \therefore T_{13} = 23 + (13 - 1)(-7)$

$n = 13$ $\qquad \therefore T_{13} = 23 - 84$

$T_{13} = ?$ $\qquad \therefore T_{13} = -61$

(9) Given arithmetic sequence: $3k + 2$ ; $5k + 1$ ; $7k$ ; $9k - 1$ ; … … …

(a) Determine the constant difference in terms of $k$.

$d = T_2 - T_1$ or $d = T_3 - T_2$

$\therefore d = (5k + 1) - (3k + 2)$ $\qquad d = (7k) - (5k + 1)$

$= 5k + 1 - 3k - 2$ $\qquad = 7k - 5k - 1$

$\therefore d = 2k - 1$

(b) Determine the sum of the first twelve terms in terms of $k$.

$a = 3k + 2$ $\qquad S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 2k - 1$ $\qquad \therefore S_{12} = \frac{12}{2}[2(3k + 2) + (12 - 1)(2k - 1)]$

$n = 12$ $\qquad \therefore S_{12} = \frac{12}{2}[2(3k + 2) - (11)(2k - 1)]$

$S_n = ?$ $\qquad \therefore S_{12} = 6[6k + 4 + 22k - 11]$

$\therefore S_{12} = 6[28k - 7]$

$\therefore S_{12} = 168k - 42$

## A1.1.3 Applications:

**Ex. 5 Rachel is training for a marathon. She runs 20 km on the first day's preparation. She decides to run 3 km further as the previous day on each day of her preparation.**

**(a) If she follows this exercise program, calculate during which day of exercise she will run 56 km.**

**(b) Calculate Rachel's total distance she ran for the first 10 preparatory exercises.**

(a) AS: 20 ; 23 ; 26 ; 29 ; …… ; 56

$\therefore a = 20$ $\qquad T_n = a + (n - 1)d$

$d = 3$ $\qquad \therefore 56 = 20 + (n - 1)(3)$

$n = ?$ $\qquad \therefore 56 = 20 + 3n - 3$

$T_n = 56$ $\qquad \therefore -3n = 17 - 56$

$\therefore -3n = -39$

$\therefore n = 13$

**∴ Rachel will run 56 km on the thirteenth day.**

**(b)** $S_{10} = ?$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{10} = \frac{10}{2}[2(20) + (10-1)(3)]$

$\therefore S_{10} = 335$

$\therefore$ **Rachel's total distance for the first 10 days is 335 km.**

Exercise 3: Date:________________________

(1) Cory cuts sixteen pieces of string for a project. The shortest piece of string is 28 cm in length and the longest piece is 88 cm in length. If the lengths of the sixteen pieces of string are written down, it forms an AS.

(a) Calculate the length of the 13th string in the sequence.

**Sequence:** $28 ; \ldots\ldots ; 88$

$a = 28$ $\quad T_n = a + (n-1)d$

$d = ?$ $\quad \therefore T_{16} = 28 + (16-1)d$

$n = 16$ $\quad \therefore 88 = 28 + 15d$

$T_{16} = 88$ $\quad \therefore -15d = 28 - 88 = -60$

$\therefore d = \frac{-60}{-15} = 4$

$a = 28$

$d = 4$ $\quad \therefore T_{13} = 28 + (13-1)(4)$

$n = 13$ $\quad \therefore T_{13} = 28 + 48$

$T_{13} = ?$ $\quad \therefore T_{13} = 76$

$\therefore$ **The $13^{th}$ piece of string is 76 cm in length.**

(b) Calculate the total length of all 16 pieces of string.

$a = 28$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$d = 4$ $\quad \therefore S_{16} = \frac{16}{2}[2(28) + (16-1)(4)]$

$n = 16$ $\quad = 8[56 + 15 \times 4]$

$S_n = ?$ $\quad \therefore S_{16} = 928$

$\therefore$ **The total length of the 16 pieces of string is 928 cm.**