Graad 12 – Boek D
(Eerste KABV uitgawe)

ONDERWYSERS HANDLEIDING

INHOUD:

Bladsy:

Hierdie boek is opgestel en verwerk deur E.J. Du Toit in 2023.

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MET SPESIALE DANK EN ERKENNING AAN DIE DEPARTEMENT VAN ONDERWYS VIR DIE GEBRUIK VAN UITTREKSELS UIT OU VRAESTELLE.

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Besoek ook www.abcmathsandscience.co.za vir ekstra oefeninge, toetse en vraestelle.

Hoofstuk D1
Analitiese Meetkunde

D1.1 Hersiening graad 10 en 11:

Gradiënt van 'n reguitlyn deur punte P en Q: $m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P}$

Toepassings van gradiënt:

* Ewewydige lyne het dieselfde gradiënt → ($m_1 = m_2$).
* Lyne wat loodreg is op mekaar se gradiënte het 'n produk van −1 → ($m_1 \times m_2 = -1$).
* Die inklinasie hoek van 'n reguitlyn word verkry deur $\tan\theta = m$.
* Lyne met positiewe gradiënte lê almal in een rigting en is stygend (inklinasie hoek is 'n skerphoek) en lyne met negatiewe gradiënte almal in 'n ander rigting en is dalend (inklinasie hoek is 'n stomphoek)!
* Kollineêre punte of punte wat saamlynig het dieselfde gradiënt.

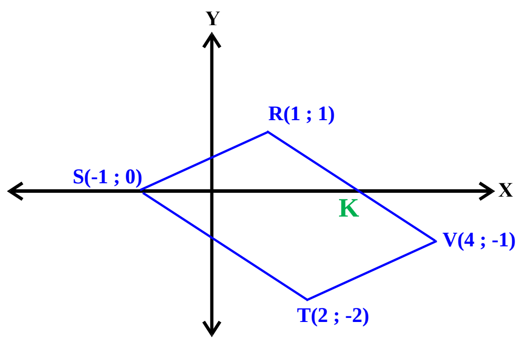
Afstand tussen enige twee punte: $d(PQ) = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$

Middelpunt van 'n lynstuk PQ: $M(x_M ; y_M) = \left(\frac{x_P + x_Q}{2} ; \frac{y_P + y_Q}{2}\right)$

Vergelyking van 'n reguitlyn: $y = mx + c$ **of** $y - y_1 = m(x - x_1)$

Oefening 1: Datum: ____________________

(1) Gegee: R(1 ; 1), S(−1 ; 0), T(2 ; −2) en V(4 ; −1)



(a) Bewys dat RSTV 'n parallelogram is deur gebruik te maak van die gradiënt formule.

$m_{RS} = \frac{y_S - y_R}{x_S - x_R} = \frac{0 - 1}{-1 - 1} = \frac{-1}{-2} = \frac{1}{2}$

$m_{TV} = \frac{y_V - y_T}{x_V - x_T} = \frac{-1 - (-2)}{4 - 2} = \frac{-1 + 2}{2} = \frac{1}{2}$

$m_{RV} = \frac{y_V - y_R}{x_V - x_R} = \frac{-1 - 1}{4 - 1} = \frac{-2}{3} = -\frac{2}{3}$

$m_{ST} = \frac{y_T - y_S}{x_T - x_S} = \frac{-2 - 0}{2 - (-1)} = \frac{-2}{2 + 1} = -\frac{2}{3}$

∴ **RS // TV → Gradiënte is gelyk**

∴ **RV // ST → Gradiënte is gelyk**

∴ **RSTV is 'n parallelogram, want die pare teenoorstaande sye is ewewydig.**

(b) Bereken die koördinate van die snypunt van die diagonale van die parallelogram.

Die koördinate van die snypunt van die diagonale → middelpunt van VS of TR

$$M_{VS} = \left(\frac{x_V + x_S}{2} ; \frac{y_V + y_S}{2}\right)$$

$$\therefore\ M_{VS} = \left(\frac{4 + (-1)}{2} ; \frac{-1 + 0}{2}\right)$$

$$\therefore\ M_{VS} = \left(\frac{3}{2} ; -\frac{1}{2}\right)$$

(c) Bepaal die verhouding tussen die sylengtes van die parallelogram.

$$d(VT) = \sqrt{(x_V - x_T)^2 + (y_V - y_T)^2}$$

$$\therefore\ d(VT) = \sqrt{(4-2)^2 + (-1-(-2))^2}$$

$$\therefore\ d(VT) = \sqrt{(2)^2 + (1)^2}$$

$$\therefore\ d(VT) = \sqrt{4+1} = \sqrt{5}$$

$$d(VR) = \sqrt{(x_V - x_R)^2 + (y_V - y_R)^2}$$

$$\therefore\ d(VR) = \sqrt{(4-1)^2 + (-1-1)^2}$$

$$\therefore\ d(VR) = \sqrt{(3)^2 + (-2)^2}$$

$$\therefore\ d(VR) = \sqrt{9+4} = \sqrt{13}$$

VT = RS → Teenoostaande sye van parm ← VR = TS

$$\therefore\ \frac{VT}{VR} = \frac{\sqrt{5}}{\sqrt{13}} = \frac{5}{13} \quad \text{of} \quad \frac{VR}{VT} = \frac{13}{5}$$

(d) Bereken die grootte van $S\hat{R}V$, afgerond tot een desimaal.

$$\tan R\hat{K}X = m_{RV} = -\frac{2}{3} \quad \rightarrow \quad \text{Sien (a) en K op skets}$$

$$\therefore\ R\hat{K}X = 180° - 33,69\ldots°$$

$$\therefore\ R\hat{K}X = 146,309\ldots°$$

$$\tan K\hat{S}R = m_{RS} = \frac{1}{2} \quad \rightarrow \quad \text{Sien (a)}$$

$$\therefore\ K\hat{S}R = 26,565\ldots°$$

$$\therefore\ S\hat{R}V = R\hat{K}X - K\hat{S}R \quad \rightarrow \quad \text{Buite } \angle \text{ van } \Delta$$

$$\therefore\ S\hat{R}V = 146,309\ldots° - 26,565\ldots°$$

$$\therefore\ S\hat{R}V \approx 119,74°$$

(2) A(−1 ; 3), B(3 ; 5) en C(7 ; 7)

(a) Is A, B en C kollineêr? Toon alle bewerkings.

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{5-3}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$$

$$m_{CB} = \frac{y_B - y_C}{x_B - x_C} = \frac{5-7}{3-7} = \frac{-2}{-4} = \frac{1}{2}$$

$\therefore$ **A, B en C is kolineêr want** $m_{AB} = m_{CB}$

(b) Toon aan dat AB = BC. $\rightarrow$ **A(−1 ; 3), B(3 ; 5) en C(7 ; 7)**

$$d(AB) = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$
$$\therefore\ d(AB) = \sqrt{(3-(-1))^2 + (5-3)^2}$$
$$\therefore\ d(AB) = \sqrt{(4)^2 + (2)^2}$$
$$\therefore\ d(AB) = \sqrt{16+4} = \sqrt{20}$$

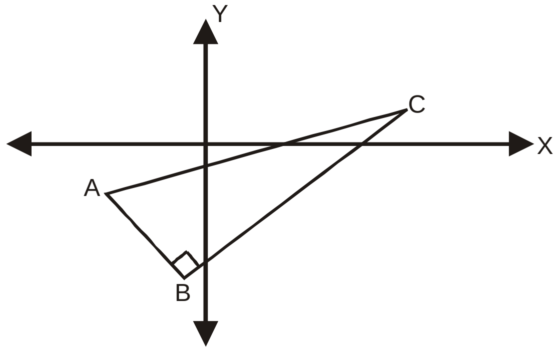
$$d(CB) = \sqrt{(x_B - x_C)^2 + (y_B - y_C)^2}$$
$$\therefore\ d(CB) = \sqrt{(3-7)^2 + (5-7)^2}$$
$$\therefore\ d(CB) = \sqrt{(-4)^2 + (-2)^2}$$
$$\therefore\ d(CB) = \sqrt{16+4} = \sqrt{20}$$

$\therefore$ **AB = BC**

(c) Is B die middelpunt van lynstuk AC? Motiveer jou antwoord.

Ja, want A, B en C lê op 'n reguit lyn (is kollineêr) en AB = BC.

(3) A(−4 ; −2), B(−1 ; −5) en C(x ; 2)



(a) Bereken die gradiënt van AB.

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{-5-(-2)}{-1-(-4)} = \frac{-5+2}{-1+4} = \frac{-3}{3}$$

$\therefore\ m_{AB} = -1$

(b) Skryf die gradiënt van BC neer.

$m_{BC} = 1$ $\rightarrow$ **AB ⊥ BC**

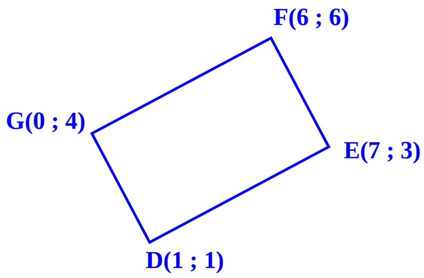
(c) Bereken nou die waarde van x.

$$m_{BC} = \frac{-5-2}{-1-x} = 1 \quad \rightarrow \quad \therefore \quad -7 = -1 - x$$
$$\therefore \quad x = -1 + 7 \quad \rightarrow \quad \therefore \quad x = 6$$

(d) Indien BC = $\sqrt{98}$, bereken die omtrek en oppervlakte van Δ ABC. Laat jou antwoord in die eenvoudigste wortelvorm.

$$d(AB) = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} \quad \rightarrow \quad A(-4\,;\,-2) \text{ en } B(-1\,;\,-5)$$
$$\therefore\ d(AB) = \sqrt{(-1-(-4))^2 + (-5-(-2))^2}$$
$$\therefore\ d(AB) = \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = 8$$
$$\therefore\ AC^2 = AB^2 + BC^2 \quad \rightarrow \quad \text{Pythagoras}$$
$$\therefore\ AC^2 = \left(\sqrt{18}\right)^2 + \left(\sqrt{98}\right)^2 = 18 + 98 = 116$$
$$\therefore\ AC = \sqrt{116}$$
$$\text{Omtrek} = \sqrt{18} + \sqrt{98} + \sqrt{116} = 3\sqrt{2} + 7\sqrt{2} + 2\sqrt{29}$$
$$\therefore\ \text{Omtrek} = 10\sqrt{2} + 2\sqrt{29}$$
$$\text{Opp} = \frac{1}{2} \times \text{ Basis} \times \perp \text{Hoogte} = \frac{1}{2} \times \sqrt{18} \times \sqrt{98} = \frac{1}{2} \times 3\sqrt{2} \times 7\sqrt{2}$$
$$\therefore\ \text{Opp} = \frac{1}{2} \times 21 \times 2$$
$$\therefore\ \text{Opp} = 21$$

(4) D(1 ; 1), E(7 ; 3), F(6 ; 6) en G(0 ; 4)



(a) Toon aan dat DEFG 'n reghoek is.

$m_{DE} = \frac{y_E - y_D}{x_E - x_D} = \frac{3-1}{7-1} = \frac{2}{6} = \frac{1}{3}$

$m_{EF} = \frac{y_F - y_E}{x_F - x_E} = \frac{6-3}{6-7} = \frac{3}{-1} = -3$

$m_{FG} = \frac{y_G - y_F}{x_G - F} = \frac{4-6}{0-6} = \frac{-2}{-6} = \frac{1}{3}$

$m_{GD} = \frac{y_D - y_G}{x_D - x_G} = \frac{1-4}{1-0} = \frac{-3}{1} = -3$

∴ DE // FG en EF // GD → Gradiënte is gelyk

Maar $m_{DE} \times m_{EF} = \frac{1}{3} \times 3 = -1$ → DE ⊥ EF

∴ DEFG is 'n reghoek, want beide pare teenoorstaande sye is ewewydig en die aangrensende sye is loodreg op mekaar → alle hoeke is gelyk aan 90°

(b) Bepaal die vergelyking van EG.

$m_{EG} = \frac{y_G - y_E}{x_G - x_E} = \frac{4-3}{0-7} = \frac{1}{-7}$ deur die punt G(0 ; 4) → y-afsnit want $x = 0$

∴ $y = mx + c$

∴ $y = -\frac{1}{7}x + 4$

(c) Bepaal die vergelyking van EF.

$m_{EF} = -3$ → Sien (a)

∴ $y - y_1 = m(x - x_1)$ deur die punte E(7 ; 3) en F(6 ; 6)

∴ $y - 3 = -3(x - 7)$ of $y - 6 = -3(x - 6)$

∴ $y = -3x + 21 + 3$ $y = -3x + 18 + 6$

∴ $y = -3x + 24$ **$y = -3x + 24$**

(d) Bereken die oppervlakte van reghoek DEFG.

$d(DE) = \sqrt{(x_E - x_D)^2 + (y_E - y_D)^2}$ of $d(FG) = \sqrt{(x_G - x_F)^2 + (y_G - y_F)^2}$

∴ $d(DE) = \sqrt{(7-1)^2 + (3-1)^2}$ $d(FG) = \sqrt{(0-6)^2 + (4-6)^2}$

∴ $d(DE) = \sqrt{(6)^2 + (2)^2}$ $d(FG) = \sqrt{(-6)^2 + (-2)^2}$

∴ $d(DE) = \sqrt{36+4} = \sqrt{40}$ $d(FG) = \sqrt{36+4} = \sqrt{40}$

$d(DG) = \sqrt{(x_G - x_D)^2 + (y_G - y_D)^2}$ of $d(EF) = \sqrt{(x_E - x_F)^2 + (y_E - y_F)^2}$

∴ $d(DG) = \sqrt{(0-1)^2 + (4-1)^2}$ $d(EF) = \sqrt{(7-6)^2 + (3-6)^2}$

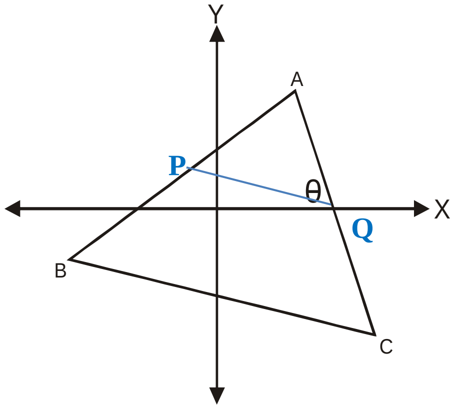
∴ $d(DG) = \sqrt{(-1)^2 + (3)^2}$ $d(EF) = \sqrt{(1)^2 + (-3)^2}$

∴ $d(DG) = \sqrt{1+9} = \sqrt{10}$ $d(EF) = \sqrt{1+9} = \sqrt{10}$

∴ Opp reghoek DEFG = L × B = $\sqrt{40} \times \sqrt{10} = \sqrt{400}$

∴ Opp reghoek DEFG = 20 vierkante eenhede

(5) A(2 ; 4), B(−4 ; −2) en C(4 ; −4) is die hoekpunte van Δ ABC.



(a) Bepaal die koördinate van P en Q as P en Q die middelpunte is van onderskeidelik AB en AC.

$$\mathbf{P = M_{AB} = \left(\frac{x_A + x_B}{2} ; \frac{y_A + y_B}{2}\right)}$$

$$\therefore\ \mathbf{P = M_{AB} = \left(\frac{2-4}{2} ; \frac{4-2}{2}\right) = \left(\frac{-2}{2} ; \frac{2}{2}\right)}$$

$$\therefore\ \mathbf{P = (-1 ; 1)}$$

$$\mathbf{Q = M_{AC} = \left(\frac{x_A + x_C}{2} ; \frac{y_A + y_C}{2}\right)}$$

$$\therefore\ \mathbf{Q = M_{AC} = \left(\frac{2+4}{2} ; \frac{4-4}{2}\right) = \left(\frac{6}{2} ; \frac{0}{2}\right)}$$

$$\therefore\ \mathbf{Q = (3 ; 0)}$$

(b) Bewys dat PQ // BC.

$$m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P} = \frac{0-1}{3-(-1)} = \frac{-1}{4}$$

$$m_{BC} = \frac{y_C - y_B}{x_C - x_B} = \frac{-4-(-2)}{4-(-4)} = \frac{-4+2}{4+4} = \frac{-2}{8} = \frac{-1}{4}$$

∴ PQ // BC → Gradiënte gelyk

(c) Bewys dat PQ $= \frac{1}{2}$ BC.

$$\mathbf{d(PQ)} = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2} \quad \textbf{en} \quad \mathbf{d(BC)} = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2}$$

$$\therefore\ \mathbf{d(PQ)} = \sqrt{(3-(-1))^2 + (0-1)^2} \qquad \mathbf{d(BC)} = \sqrt{(4-(-4))^2 + (-4-(-2))^2}$$

$$\therefore\ \mathbf{d(PQ)} = \sqrt{(4)^2 + (-1)^2} \qquad \mathbf{d(BC)} = \sqrt{(8)^2 + (-2)^2}$$

$$\therefore\ \mathbf{d(PQ)} = \sqrt{16+1} = \sqrt{17} \qquad \mathbf{d(BC)} = \sqrt{64+4} = \sqrt{68}$$

Maar $\sqrt{68} = \sqrt{4} \times \sqrt{17} = 2\sqrt{17}$

$$\therefore\ \mathbf{BC = 2\ PQ} \ \rightarrow\ \mathbf{PQ = \tfrac{1}{2}BC}$$

(d) Bereken die grootte van θ tot die naaste graad.

$$\tan A\hat{Q}X = m_{AC}$$

$$\therefore\ \tan A\hat{Q}X = \frac{-4-4}{4-2} = \frac{-8}{2} = -4$$

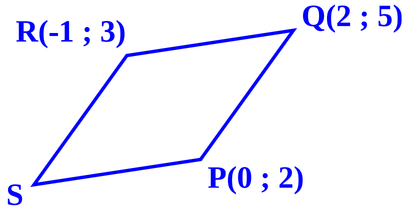
$$\therefore\ A\hat{Q}X = 180° - 76° \approx 104°$$

Maar $\theta = 180° - A\hat{Q}X$

$$\therefore\ \theta = 180° - 104°$$

$$\therefore\ \theta = 76°$$

(6) P(0 ; 2), Q(2 ; 5), R(−1 ; 3) en S is die hoekpunte van parallelogram PQRS.



(a) Bereken die gradiënt van lyn QR.

$$m_{QR} = \frac{y_R - y_Q}{x_R - x_Q} = \frac{3-5}{-1-2} = \frac{-2}{-3}$$

$\therefore\ m_{QR} = \frac{2}{3}$

(b) Bepaal die vergelyking van lyn PS.

$y = mx + c$ → Deur punt $P(0\,;2)$ → y-afsnit want $x = 0$

$\therefore\ y = \frac{2}{3}x + 2$ → $m_{PS} = m_{QR}$ Lyne ewewydig (Oorstaande sye van parm)

(c) Toon aan dat PQRS 'n ruit is.

$d(QR) = \sqrt{(x_R - x_Q)^2 + (y_R - y_Q)^2}$ en $d(PQ) = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$

$\therefore\ d(QR) = \sqrt{(-1-2)^2 + (3-5)^2}$ $d(PQ) = \sqrt{(2-0)^2 + (5-2)^2}$

$\therefore\ d(QR) = \sqrt{(-3)^2 + (-2)^2}$ $d(PQ) = \sqrt{(2)^2 + (3)^2}$

$\therefore\ d(QR) = \sqrt{9+4}$ $d(PQ) = \sqrt{4+9}$

$\therefore\ d(QR) = \sqrt{13}$ $d(PQ) = \sqrt{13}$

$\therefore\ QR = PQ$

∴ PQRS is 'n ruit want die parallelogram PQRS se aangrensende sye is ewe lank

(d) Bereken die koördinate van die punt waar die diagonale van PQRS mekaar sny.

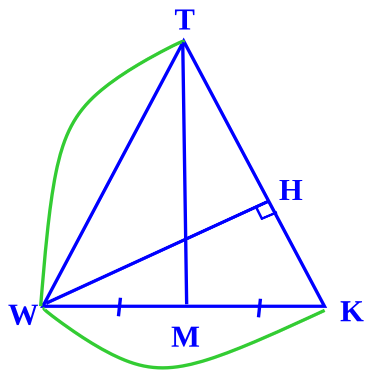
Diagonale sny by middelpunt van PR.

$\therefore\ M_{PR} = \left(\frac{x_P + x_R}{2}\,;\frac{y_P + y_R}{2}\right)$

$\therefore\ M_{PR} = \left(\frac{0-1}{2}\,;\frac{2+3}{2}\right)$

$\therefore\ M_{PR} = \left(-\frac{1}{2}\,;\frac{5}{2}\right)$

(7) TWK is 'n gelykbenige driehoek met TW = WK. T(7 ; 8), W(1 ; 6) en K(−1 ; y).



(a) Bereken die lengte van TW.

$d(TW) = \sqrt{(x_W - x_T)^2 + (y_W - y_T)^2}$

$\therefore\ d(TW) = \sqrt{(1-7)^2 + (6-8)^2}$

$\therefore\ d(TW) = \sqrt{(-6)^2 + (-2)^2}$

$\therefore\ d(TW) = \sqrt{36+4}$

$\therefore\ d(TW) = \sqrt{40}$

(b) Bereken die waarde(s) van y.

$$d(KW) = \sqrt{(x_W - x_K)^2 + (y_W - y_K)^2}$$

$\therefore$ $d(KW) = \sqrt{(1-(-1))^2 + (6-y)^2}$

Maar KW = TW $\rightarrow$ Gegee

$\therefore$ $\sqrt{40} = \sqrt{(1+1)^2 + (6-y)^2}$

$\therefore$ $40 = 4 + 36 - 12y + y^2$

$\therefore$ $0 = y^2 - 12y$

$\therefore$ $y(y-12) = 0$

$\therefore$ $y = 0$ **of** $y = 12$

(c) Bepaal die vergelyking van die:

(i) swaartelyn deur T vir $y > 0$. $\rightarrow$ **TM met M die middelpunt van WK**

$M_{WK} = \left(\frac{x_W + x_K}{2} ; \frac{y_W + y_K}{2}\right) = \left(\frac{1-1}{2} ; \frac{6+12}{2}\right) \rightarrow y > 0 \rightarrow y = 12$

$\therefore$ $M = (0 ; 9)$

$m_{TM} = \frac{y_M - y_T}{x_M - x_T} = \frac{9-8}{0-7} = \frac{1}{-7} = -\frac{1}{7}$ **deur** $M = (0 ; 9)$ $\rightarrow$ y**-afsnit want** $x = 0$

$\therefore$ $y = -\frac{1}{7}x + 9$

(ii) hoogtelyn deur W vir $y \leq 0$. $\rightarrow$ **WH met WH ⊥ TK**

$m_{TK} = \frac{y_K - y_T}{x_K - x_T} = \frac{0-8}{-1-7} = \frac{-8}{-8} = 1 \rightarrow y \leq 0 \rightarrow y = 0 \rightarrow K(-1 ; 0)$

$\therefore$ $m_{WH} = -1 \rightarrow m_1 \times m_2 = -1$ **want WH ⊥ TK**

$\therefore$ $y - y_1 = m(x - x_1)$ **deur** $W(1 ; 6)$

$\therefore$ $y - 6 = -1(x - 1)$

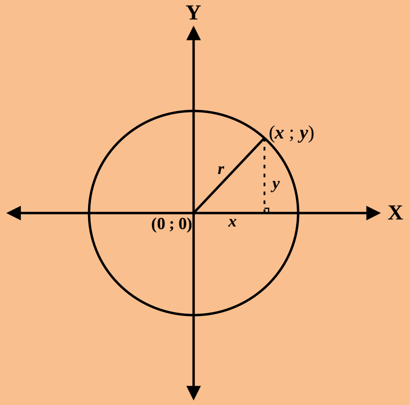
$\therefore$ $y = -1x + 1 + 6 \rightarrow$ $\therefore$ $y = -x + 7$

D1.2 <u>Sirkels</u>:
D1.2.1 <u>Sirkels met oorsprong as middelpunt</u>:

Die vergelyking van 'n sirkel met middelpunt (0 ; 0):

$$x^2 + y^2 = r^2$$

met $(x ; y)$ enige punt op die omtrek van die sirkel en r die radius van die sirkel.



D1.2.2 Ander sirkels:

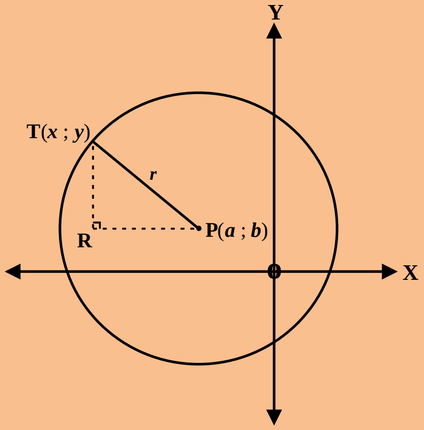
Die vergelyking van 'n sirkel met middelpunt $(a\,;\,b)$:

$(x-a)^2+(y-b)^2=r^2$

met $(x\,;\,y)$ enige punt op die omtrek van die sirkel en r die radius van die sirkel.

Die radius kan bereken word met die afstandsformule:

$\mathrm{d(PT)}=\sqrt{(x_P-x_T)^2+(y_P-y_T)^2}$



Vb. 1 Bepaal die koördinate van die middelpunt en die lengte van die radius van die volgende sirkel:

$x^2+6x+y^2-4y=12$

Gebruik vierkantsvoltooiing om die vergelyking van die sirkel in standaardvorm $[(x-a)^2+(y-b)^2=r^2]$ te skryf:

$$x^2+6x+y^2-4y=12$$

$$x^2+6x+\left(\frac{6}{2}\right)^2+y^2-4y+\left(\frac{-4}{2}\right)^2=12+\left(\frac{6}{2}\right)^2+\left(\frac{-4}{2}\right)^2$$

$$x^2+6x+(3)^2+y^2-4y+(-2)^2=12+9+4$$

$$(x+3)^2+(y-2)^2=25$$

$\therefore$ **MP** $=(-3\,;\,2)$ **en** $r^2=25$

$\therefore r=5$

Vb. 2 Bepaal die vergelyking van die sirkel met middelpunt $(-2\,;\,5)$ deur die punt $(1\,;\,-1)$.

$(x-a)^2+(y-b)^2=r^2$ **met MP** $=(\overset{a}{-2}\,;\,\overset{b}{5})$

$\therefore (x-(-2))^2+(y-5)^2=r^2$

$\therefore (1+2)^2+(-1-5)^2=r^2$ **deur** $(\overset{x}{1}\,;\,\overset{y}{-1})$

$\therefore (3)^2+(-6)^2=r^2$

$\therefore r^2=9+36=45$

$\therefore (x+2)^2+(y-5)^2=45$

Oefening 2: Datum: ______________________

(1) Bepaal die vergelykings van die volgende sirkels:

(a) met middelpunt (5 ; 2) en radius 6.

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\therefore (x - 5)^2 + (y - 2)^2 = 6^2$$

$$\therefore (x - 5)^2 + (y - 2)^2 = 36$$

(b) met middelpunt (−1 ; 3) en radius $\sqrt{12}$.

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\therefore (x - (-1))^2 + (y - 3)^2 = (\sqrt{12})^2$$

$$\therefore (x + 1)^2 + (y - 3)^2 = 12$$

(c) met middelpunt (4 ; −2) en deur die punt (−2 ; 0).

$$(x - a)^2 + (y - b)^2 = r^2 \quad \rightarrow \quad (-2 - 4)^2 + (0 + 2)^2 = r^2$$

$$\therefore (x - 4)^2 + (y - (-2))^2 = r^2 \qquad \therefore r^2 = (-6)^2 + (2)^2 = 36 + 4 = 40$$

$$\therefore (x - 4)^2 + (y + 2)^2 = 40$$

(d) met middelpunt (−2 ; −3) en deur die punt (2 ; −1).

$$(x - a)^2 + (y - b)^2 = r^2 \quad \rightarrow \quad (2 + 2)^2 + (-1 + 3)^2 = r^2$$

$$\therefore (x - (-2))^2 + (y - (-3))^2 = r^2 \qquad \therefore r^2 = (4)^2 + (2)^2 = 16 + 4 = 20$$

$$\therefore (x + 2)^2 + (y + 3)^2 = 20$$

(2) Bepaal die koördinate van die volgende sirkels se middelpunte en die lengte van die radius:

(a) $(x - 4)^2 + (y - 2)^2 = 36 \rightarrow$ **MP(4 ; 2) met** $r^2 = 36 \rightarrow r = 6$

(b) $x^2 + y^2 - 10y = 6$

$$\therefore x^2 + y^2 - 10y + 25 = 6 + 25$$

$$\therefore x^2 + (y - 5)^2 = 31 \rightarrow$$ **MP(0 ; 5) met** $r^2 = 31 \rightarrow r = \sqrt{31}$

(c) $(x + 3)^2 + (y + 6)^2 = 20 \rightarrow$ **MP(−3 ; −6) met** $r^2 = 20 \rightarrow r = \sqrt{20}$

(d) $(x + 5)^2 + (y + 5)^2 = 280 \rightarrow$ **MP(−5 ; −5) met** $r^2 = 280 \rightarrow r = \sqrt{280}$

(e) $(x - 1)^2 + (y + 2)^2 = 9 \rightarrow$ **MP(1 ; −2) met** $r^2 = 9 \rightarrow r = 3$

(f) $x^2 + (y - 4)^2 = 100$ → **MP$(0\,;4)$** **met** $\boldsymbol{r^2 = 100}$ → $\boldsymbol{r = 10}$

(g) $x^2 - 8x + y^2 - 6y = 12$

$\therefore\ \boldsymbol{x^2 - 8x + 16 + y^2 - 6y + 9 = 12 + 6 + 9}$

$\therefore\ \boldsymbol{(x-4)^2 + (y-3)^2 = 27}$ → **MP$(4\,;3)$** **met** $\boldsymbol{r^2 = 36}$ → $\boldsymbol{r = 6}$

(h) $\left(x + \frac{1}{2}\right)^2 + (y + 2)^2 = 48$ → **MP$\left(-\frac{1}{2};-2\right)$** **met** $\boldsymbol{r^2 = 48}$ → $\boldsymbol{r = \sqrt{48}}$

(i) $(x-6)^2 + y^2 = 1$ → **MP$(6\,;0)$** **met** $\boldsymbol{r^2 = 1}$ → $\boldsymbol{r = 1}$

(j) $2x^2 + 2y^2 - 4x - y = 2$

$\boldsymbol{x^2 + y^2 - 2x - \frac{1}{2}y = 1}$ → **÷ deur 2**

$\therefore\ \boldsymbol{x^2 - 2x + 1 + y^2 - \frac{1}{2}y + \frac{1}{4} = 1 + 1 + \frac{1}{4}}$

$\therefore\ \boldsymbol{(x-1)^2 + \left(y - \frac{1}{2}\right)^2 = 2\frac{1}{4} = \frac{9}{4}}$ → **MP$\left(1\,;\frac{1}{2}\right)$** **met** $\boldsymbol{r^2 = \frac{9}{4}}$ → $\boldsymbol{r = \frac{3}{2}}$

(3) Bepaal of die punt $(3\,;\,-2)$ op die sirkel met middelpunt $(-1\,;\,5)$ sal lê.
Die radius van die sirkel is 8.

$\boldsymbol{(x - a)^2 + (y - b)^2 = r^2}$

$\therefore\ \boldsymbol{(x - (-1))^2 + (y - 5)^2 = 8^2}$

$\therefore\ \boldsymbol{(x - (-1))^2 + (y - 5)^2 = 8^2}$

$\therefore\ \boldsymbol{(x+1)^2 + (y - 5)^2 = 64}$

$\therefore$ **LK** $\boldsymbol{= (3+1)^2 + (-2 - 5)^2 = (4)^2 + (-7)^2 = 16 + 49 = 65}$

$\therefore$ **Nee, die punt $(3\,;-2)$ lê nie op die sirkel nie maar buite die sirkel want $65 > r^2 = 64$**

(4) Die vergelyking van die sirkel deur die punt $(-3\,;\,-1)$ is $x^2 + 10x + y^2 - 2y + p = 0$.

(a) Bepaal die koördinate van die middelpunt van die sirkel.

$\therefore\ \boldsymbol{x^2 + 10x + 25 + y^2 - 2y + 1 = -p + 25 + 1}$

$\therefore\ \boldsymbol{(x+5)^2 + (y-1)^2 = -p + 26}$ → **MP$(-5\,;1)$**

(b) Bereken die waarde van p.

$\boldsymbol{(x+5)^2 + (y-1)^2 = -p + 26}$

$\therefore\ \boldsymbol{(-3+5)^2 + (-1-1)^2 = -p + 26}$ → **deur punt $(-3\,;-1)$**

$\therefore\ \boldsymbol{(2)^2 + (-2)^2 = -p + 26}$

$\therefore\ \boldsymbol{4 + 4 = -p + 26}$

$\therefore\ \boldsymbol{p = 26 - 4 - 4}$

$\therefore\ \boldsymbol{p = 18}$

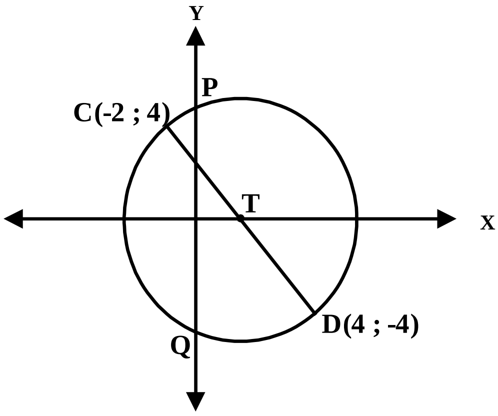
(5) Bepaal die vergelyking van die sirkel met middelpunt $(-4\,;\,-3)$ en deursnee 18.

Deursnee 18 → Radius = 9

$\therefore\ (x-(-4))^2 + (y-(-3))^2 = 9^2$

$\therefore\ (x+4)^2 + (y+3)^2 = 81$

(6) CD is die middellyn van 'n sirkel met T die middelpunt van CD. Bereken:



(a) die koördinate van die sirkel se middelpunt.

$M_{CD} = \left(\frac{x_C + x_D}{2}\,;\,\frac{y_C + y_D}{2}\right)$

$\therefore\ M_{CD} = \left(\frac{-2+4}{2}\,;\,\frac{4-4}{2}\right)$

$\therefore\ T = (1\,;\,0)$

(b) die vergelyking van die sirkel.

$(x-a)^2 + (y-b)^2 = r^2$ → **Met middelpunt T$(1\,;\,0)$**

$\therefore\ (x-1)^2 + (y-0)^2 = r^2$

$\therefore\ (-2-1)^2 + (4-0)^2 = r^2$ → **Deur C$(-2\,;\,4)$ of D$(4\,;\,-4)$**

$\therefore\ r^2 = (-3)^2 + (4)^2 = 9 + 16 = 25$

$\therefore\ (x-1)^2 + y^2 = 25$

(c) die lengte van PQ as P en Q die y-afsnitte van die sirkel is.

$(0-1)^2 + y^2 = 25$ → **$x = 0$ vir y-afsnitte**

$\therefore\ y^2 = 25 - 1 = 24$

$\therefore\ y = \pm\sqrt{24}$

$\therefore\ PQ = 2\sqrt{24}$

(d) die vergelyking van middellyn CD.

$m_{CD} = \frac{y_D - y_C}{x_D - x_C} = \frac{-4-4}{4-(-2)} = \frac{-8}{6} = -\frac{4}{3}$

$y - y_1 = m(x - x_1)$

$\therefore\ y - 4 = -\frac{4}{3}(x-(-2))$ → **Deur C$(-2\,;\,4)$ of D$(4\,;\,-4)$**

$\therefore\ y - 4 = -\frac{4}{3}(x+2)$

$\therefore\ y = -\frac{4}{3}x - \frac{8}{3} + 4$

$\therefore\ y = -\frac{4}{3}x + \frac{4}{3}$

of

$\therefore\ y - (-4) = -\frac{4}{3}(x-4)$

$\therefore\ y + 4 = -\frac{4}{3}x + \frac{16}{3}$

$\therefore\ y = -\frac{4}{3}x + \frac{16}{3} - 4$

$\therefore\ y = -\frac{4}{3}x + \frac{4}{3}$