# Grade 12 – Textbook Answers
**(First edition – CAPS)**

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# Chapter A2
# Logarithms and function inverses

**See grade 11 Functions and exponents for revision and background!**

## A2.1 Logarithms:
### A2.1.1 Definition of a logarithm:

Logarithms are the inverses of exponents.

Ex. If $2^5 = 32$ then $\log_2 32 = 5$

$\therefore$ Per definition if $y = \log_a x \Leftrightarrow x = a^y$ with $a > 0$ ; $a \neq 1$ $x > 0$

Remember: * $\log_a 1 = 0$ because $a^0 = 1$

* The natural logarithm is $\log x \Leftrightarrow \log_{10} x$

* $\log_a a = 1$ because $a^1 = a$

### A2.1.2 Laws of logarithms:

For $a > 0$ ; $a \neq 1$ ; $b > 0$ ; $b \neq 1$ ; $x > 0$ and $y > 0$

- $\log_a x + \log_a y = \log_a xy$
- $\log_a x - \log_a y = \log_a \frac{x}{y}$
- $n \log_a x = \log_a x^n$
- $\log_a x = \frac{\log_b x}{\log_b a}$

**Ex. 1 Simplify: (Without using a calculator.)**

(a) $\log_4 2 + \log_4 32$

$= \log_4(2 \times 32)$

$= \log_4(64)$

$= \log_4(4^3)$

$= 3\log_4(4)$

$= 3(1)$

$= 3$

(b) $\log 200 - \log 2$

$= \log(200 \div 2)$

$= \log 100$

$= \log_{10} 10^2$

$= 2\log_{10} 10$

$= 2(1)$

$= 2$

(c) $\log_3 36 \times \log_6 9$

$= \frac{\log 36}{\log 3} \times \frac{\log 9}{\log 6}$

$= \frac{\log 6^2}{\log 3} \times \frac{\log 3^2}{\log 6}$

$= \frac{2\log 6}{\log 3} \times \frac{2\ \log 3}{\log 6}$

$= \frac{2\cancel{\log 6}}{\cancel{\log 3}} \times \frac{2\ \cancel{\log 3}}{\cancel{\log 6}}$

$= 2 \times 2$

$= 4$

(d) $\log_4 16 + \log_3 \frac{1}{3} - \log_7 1$

$= \log_4 4^2 + \log_3 3^{-1} - 0$

$= 2\log_4 4 + (-1)\log_3 3$

$= 2(1) - 1(1)$

$= 2 - 1$

$= 1$

Ex. 2 If $\log 3 = 0{,}477$ and $\log 5 = 0{,}699$, calculate:
(Without using a calculator.)

(a) $\log 45$

$= \log(9 \times 5)$

$= \log(3^2 \times 5)$

$= \log 3^2 + \log 5$

$= 2\ \log 3 + \log 5$

$= 2 \times 0{,}477 + 0{,}699$

$= 0{,}954 + 0{,}699$

$= 1{,}653$

(b) $\log 30$

$= \log(3 \times 10)$

$= \log 3 + \log 10$

$= \log 3 + \log 10$

$= 0{,}477 + 1$

$= 1{,}477$

Ex. 3 Solve for $x$ : (Without using a calculator.)

(a) $\log x + \log(x + 3) = 1$

$\therefore \log_{10} x(x + 3) = 1$

$\therefore 10^1 = x^2 + 3x$

$\therefore 0 = x^2 + 3x - 10$

$\therefore 0 = (x + 5)(x - 2)$

$\therefore x = -5$ or $x = 2$

but $x \neq -5$, because $x > 0$

(b) $\log_3(x + 4) - \log_3 x = \log_3 5$

$\therefore \log_3 \frac{(x+4)}{x} = \log_3 5$

$\therefore \log_3 \frac{(x+4)}{x} = \log_3 5$

$\therefore \frac{(x+4)}{x} = 5$ [Per definition]

$\therefore x + 4 = 5x$

$\therefore x - 5x = -4$

$\therefore -4x = -4$

$\therefore x = 1$

Ex. 4 Solve for $x$ : (Use a calculator and give your answer correct to 2 decimals.)

(a) $3^x = 7$

$\therefore \log_3 7 = x$

$\therefore x = \frac{\log 7}{\log 3}$

$\therefore x \approx 1{,}77$

(b) $1{,}3 = 2^{x-3}$

$\therefore \log_2 1{,}3 = x - 3$

$\therefore x - 3 = \frac{\log 1{,}3}{\log 2}$

$\therefore x - 3 = 0{,}3785 \ldots$

$\therefore x \approx 3{,}38$

Exercise 1:

(1) Write the following in logarithmic form:

(a) $7^3 = 343$

$\rightarrow \quad \mathbf{\log_7 343 = 3}$

(b) $x = \left(\frac{1}{2}\right)^2$

$\rightarrow \quad \mathbf{\log_{\frac{1}{2}} x = 2}$

(c) $y = 2^{x+1}$

$\rightarrow \quad \mathbf{\log_2 y = x + 1}$

(d) $2^{\log x} = 5$

$\rightarrow \quad \mathbf{\log_2 5 = \log x}$

(2) Write the following in exponential form:

(a) $\log_2 32 = 5$

$\rightarrow \quad \mathbf{2^5 = 32}$

(b) $\log y = k$

$\rightarrow \quad \mathbf{10^k = y}$

(c) $m = \log_3 k$

$\rightarrow \quad \mathbf{3^m = k}$

(d) $\log_3 \frac{1}{27} = -3$

$\rightarrow \quad \mathbf{3^{-3} = \frac{1}{27}}$

(3) Write the following as separate logarithms with base 10 if $\{x ; y ; t ; p\} > 0$:

(a) $\log \frac{xy}{p}$

$= \mathbf{\log xy - \log p}$

$= \mathbf{\log x + \log y - \log p}$

(b) $\log_t p^2 t$

$= \mathbf{2 \log_t p + \log_t t}$

$= \mathbf{2 \frac{\log p}{\log t} + 1}$

(4) Write the following as a single logarithm if $\{x ; y ; t ; p\} > 0$:

(a) $\log t - \log y + 2 \log p$

$= \mathbf{\log t - \log y + \log p^2}$

$= \mathbf{\log \frac{tp^2}{y}}$

(b) $\log_2(x-2) - \log_2(x+1) - \log_2 x$

$= \mathbf{\log_2 \frac{(x-2)}{x(x+1)}}$

(5) Simplify without using a calculator:

(a) $\log 25 + \log 8 - \log 2$

$= \log(25 \times 8 \div 2)$

$= \log 100$

$= \log 10^2$

$= 2 \log_{10} 10 = 2 \times 1$

$= 2$

(b) $\log_2 16 + 3 \log_3 \left(\frac{1}{9}\right) - \log_{15} 1$

$= \log_2 2^4 + 3 \log_3 3^{-2} - \log_{15} 15^0$

$= 4 \log_2 2 + (3 \times -2) \log_3 3 - 0 \log_{15} 15$

$= 4 \times 1 + (-6) \times 1 - 0 \times 1 = 4 - 6 - 0$

$= -2$

(c) $\dfrac{\log 32 - \log 243}{\log 3 - \log 2}$

$= \dfrac{\log 2^5 - \log 3^5}{\log 3 - \log 2}$

$= \dfrac{5 \log 2 - 5 \log 3}{\log 3 - \log 2}$

$= \dfrac{5(\log 2 - \log 3)}{-1(\log 2 - \log 3)}$

$= \dfrac{5\cancel{(\log 2 - \log 3)}}{-1\cancel{(\log 2 - \log 3)}}$

$= -5$

(d) $$\frac{\log_5 27 + \log_5 9}{\log_5 \sqrt{3}}$$

$$= \frac{\log_5 27 \times 9}{\log_5 3^{\frac{1}{2}}} = \frac{\log_5 243}{\frac{1}{2}\log_5 3}$$

$$= 2 \times \log_3 243$$

$$= 2 \times \log_3 3^5$$

$$= 2 \times 5 \log_3 3 = 2 \times 5$$

$$= 10$$

or

$$= \frac{\log_5 3^5}{\frac{1}{2}\log_5 3}$$

$$= \frac{5 \log_5 3}{\frac{1}{2}\log_5 3}$$

$$= \frac{5 \cancel{\log_5 3}}{\frac{1}{2}\cancel{\log_5 3}}$$

$$= 5 \div \frac{1}{2}$$

$$= 10$$

(e) $$\log 8\,000 - \log 8$$

$$= \log \frac{8\,000}{8}$$

$$= \log 1\,000$$

$$= \log 10^3$$

$$= 3 \log 10$$

$$= 3 \times 1$$

$$= 3$$

(f) $$\frac{1}{2}\log_4 16 + \log_{0,2} 0{,}04 - \log_3 \sqrt{27} - \log 25 \times \log_5 1$$

$$= \frac{1}{2}\log_4 4^2 + \log_{0,2}(0,2)^2 - \log_3 \sqrt{3^3} - \log 25 \times 0$$

$$= \frac{1}{2}\log_4 4^2 + \log_{0,2}(0,2)^2 - \log_3 \sqrt{3^3} - \log 25 \times 0$$

$$= \frac{1}{2} \times 2 \times \log_4 4 + 2 \times \log_{0,2} 0,2 - \log_3 3^{\frac{3}{2}} - 0$$

$$= \frac{1}{2} \times 2 \times 1 + 2 \times 1 - \frac{3}{2} \times \log_3 3$$

$$= 1 + 2 - \frac{3}{2} \times 1$$

$$= 3 - \frac{3}{2}$$

$$= \frac{3}{2}$$

(6) Solve for $x$: [Where necessary, round off correct to 2 decimals.]

(a) $\log_4 2x = 3$

$\therefore \quad 4^3 = 2x$

$\therefore \quad 64 = 2x$

$\therefore \quad x = 32$

(b) $\log_3(x + 2) + \log_3 x = 1$

$\therefore \quad \log_3 x(x+2) = 1$ or $\log_3 x(x+2) = \log_3 3$

$\therefore \quad 3^1 = x^2 + 2x$ $\leftarrow$ EB $\Leftrightarrow$ EE

$\therefore \quad 0 = x^2 + 2x - 3$

$\therefore \quad 0 = (x+3)(x-1)$

$\therefore \quad x = -3$ or $x = 1$

N/A $\rightarrow$ Ex. $\log_3 -3$ is not admissible by definition

(c) $\log_2(2x + 12) - 2 = \log_2 x$

$\therefore \quad \log_2(2x+12) - 2\log_2 2 = \log_2 x$

$\therefore \quad \log_2(2x+12) - \log_2 2^2 = \log_2 x$

$\therefore \quad \log_2 \frac{(2x+12)}{4} = \log_2 x$

$\therefore \quad \frac{(2x+12)}{4} = x$ $\leftarrow$ EB $\Leftrightarrow$ EE

$\therefore \quad 2x + 12 = 4x$

$\therefore \quad 12 = 4x - 2x$

$\therefore \quad 2x = 12$

$\therefore \quad x = 6$

(d) $7^{3x} = 14$

$\therefore \quad \log_7 14 = 3x$

$\therefore \quad \frac{\log 14}{\log 7} = 3x$

$\therefore \quad 3x = 1,356 \ldots$ $\rightarrow$ **Use a calculator**

$\therefore \quad 3x = 1,356 \ldots$

$\therefore \quad x \approx 0,45$

(7) Write the following in terms of $m$ and/or $n$ if $\log 6 = m$ and $\log 3 = n$:

(a) $\log 18$

$= \log 3 \times 6$

$= \log 3 + \log 6$

$= m + n$

(b) $\log_{27} 36$

$= \frac{\log 36}{\log 27} = \frac{\log 6^2}{\log 3^3}$

$= \frac{2 \log 6}{3 \log 3}$

$= \frac{2m}{3n}$

(c) $\log 300$

$= \log 3 \times 100$

$= \log 3 + \log 100$

$= \log 3 + \log 10^2 = \log 3 + 2 \log 10$

$= n + 2$

(d) $\log 20$

$= \log \frac{60}{3}$

$= \log 6 \times 10 - \log 3$

$= \log 6 + \log 10 - \log 3$

$= m + 1 - n$

## A2.2 Inverses:

The rule for the reflection in the line $x = y$ is: $(x\,;\,y) \;\Leftrightarrow\; (y\,;\,x)$

This reflection in the line $y = x$ is referred to as the inverse $\Leftrightarrow$ it means that the $x$ and $y$ swap places!

The inverse of $f(x)$ is written as $f^{-1}(x)$.

**Ex. 5 Determine $f^{-1}(x)$ in each of the following in the form $f^{-1}(x) = \ldots$ :**

**(a)** $f(x) = 5x^2$

$\therefore$ **For $f$:** $y = 5x^2$

$\therefore$ **For $f^{-1}$:** $x = 5y^2$

$\therefore \quad \frac{x}{5} = y^2$

$\therefore \quad y = \pm\sqrt{\frac{x}{5}}$

$\therefore \quad f^{-1}(x) = \pm\sqrt{\frac{x}{5}}$

**(b)** $f: x \to \frac{3}{x+2}$

$\therefore$ **For $f$:** $y = \frac{3}{x+2}$

$\therefore$ **For $f^{-1}$:** $x = \frac{3}{y+2}$

$\therefore \quad y + 2 = \frac{3}{x}$

$\therefore \quad y = \frac{3}{x} - 2$

$\therefore \quad f^{-1}(x) = \frac{3}{x} - 2$

Exercise 2:

(1) Determine $f^{-1}(x)$ in each of the following and write it in the form $f^{-1}(x) = \ldots\ldots$

(a) $f(x) = 3x - 4 \;\to\; y = 3x - 4$

$f^{-1}$: $x = 3y - 4$

$\therefore \quad 3y = x + 4$

$\therefore \quad y = \frac{x+4}{3}$

$\therefore \quad f^{-1}(x) = \frac{x+4}{3}$

(b) $f(x) = 5^x \;\to\; y = 5^x$

$f^{-1}$: $x = 5^y$

$\therefore \quad y = \log_5 x$

$\therefore \quad f^{-1}(x) = \log_5 x$

(c) $f(x) = -2x^2 \;\to\; y = -2x^2$

$f^{-1}$: $x = -2y^2$

$\therefore \quad y^2 = \frac{x}{-2}$

$\therefore \quad y = \pm\sqrt{-\frac{x}{2}}$

$\therefore \quad f^{-1}(x) = y = \pm\sqrt{-\frac{x}{2}}$

(d) $f(x) = \log_{0,5} x \;\to\; y = \log_{0,5} x$

$f^{-1}$: $x = \log_{0,5} y$

$\therefore \quad y = 0,5^x$

$\therefore \quad f^{-1}(x) = 0,5^x$

(2) Determine $g^{-1}(x)$ in each of the following and write it in the form $g^{-1}: x \to$ …….

(a) $g : x \to \frac{x}{4} \quad \to \quad y = \frac{x}{4}$

$g^{-1}$: $x = \frac{y}{4}$

$\therefore \quad y = 4x$

$\therefore \quad g^{-1}: x \to 4x$

(b) $g : x \to \log_3 x \quad \to \quad y = \log_3 x$

$g^{-1}$: $x = \log_3 y$

$\therefore \quad y = 3^x$

$\therefore \quad g^{-1}: x \to 3^x$

(c) $g : x \to 3^{x+1} \quad \to \quad y = 3^{x+1}$

$g^{-1}$: $x = 3^{y+1}$

$\therefore \quad y + 1 = \log_3 x$

$\therefore \quad y = \log_3 x - 1$

$\therefore \quad g^{-1}: x \to \log_3 x - 1$

(d) $g : x \to -0{,}5x \quad \to \quad y = -0{,}5x$

$g^{-1}$: $x = -0{,}5y$

$\therefore \quad y = \frac{x}{-0{,}5}$

$\therefore \quad g^{-1}: x \to -\frac{x}{0{,}5}$

(3) Determine $h$ in each of the following and write it in the form $h(x) =$ …….

(a) $h^{-1}(x) = \log_7 x \quad \to \quad y = \log_7 x$

$h$: $x = \log_7 y$

$\therefore \quad y = 7^x$

$\therefore \quad h(x) = 7^x$

(b) $h^{-1}(x) = \frac{x-2}{3} \quad \to \quad y = \frac{x-2}{3}$

$h$: $x = \frac{y-2}{3}$

$\therefore \quad 3x = y - 2$

$\therefore \quad y = 3x + 2$

$\therefore \quad h(x) = 3x + 2$

(c) $h^{-1}(x) = \frac{1}{4}x^2 \quad \to \quad y = \frac{1}{4}x^2$

$h$: $x = \frac{1}{4}y^2$

$\therefore \quad 4x = y^2$

$\therefore \quad y = \pm\sqrt{4x}$

$\therefore \quad h(x) = \pm\sqrt{4x}$

(d) $h^{-1}(x) = \log x \quad \to \quad y = \log x$

$h$: $x = \log y$

$\therefore \quad y = 10^x$

$\therefore \quad h(x) = 10^x$

(4) Consider the following: $p(x) = \{(1;7)\,;(2;8)\,;(3:9)\,;(4;10)\}$

(a) Is $p$ a function? Motivate your answer.

**Yes, $p$ is a function, because none of the $x$-coordinates are repeated.**

(b) Write down the range of $p^{-1}(x)$.

$\mathbf{R_{p^{-1}} = \{1;2;3;4\}}$ $\rightarrow$ $\mathbf{R_{p^{-1}} = D_p}$

(5) Explain the difference between $f^{-1}(x)$ and $(f(x))^{-1}$.

**$f^{-1}(x)$ is the inverse of a function and $\left(f(x)\right)^{-1}$ is the reciprocal of the function.**

$\therefore$ **If $f(x) = 2x$, then $f^{-1}(x) = \frac{x}{2}$ and $\left(f(x)\right)^{-1} = \frac{1}{2x}$.**

$\rightarrow$ $y = 2x$ $\rightarrow$ **For $f^{-1}$: $x = 2y$**

# A2.3 Graphs of inverses:
## A2.3.1 Graphs of inverses of the straight line:

**See grade 11 Linear Functions for revision and background!**

If the function $f(x) = mx + c$ is given, the inverse will obtained as follow:

$f(x) = mx + c \iff y = mx + c$

$\therefore$ For the inverse the $x$ and $y$ swap place: $x = my + c$

$\Longrightarrow y = \frac{x - c}{m}$ [Make $y$ the subject!]

$\therefore f^{-1}(x) = \frac{x - c}{m}$ $\Longrightarrow$ Inverse function

**Ex. 6 Given: $g(x) = 2x - 4$**

**(a) Determine $g^{-1}(x) = \ldots..$**

**(b) Sketch $g(x)$ and $g^{-1}(x)$ on the same system of axes.**

**(a)** $g(x)$: $y = 2x - 4 \iff$ $\therefore g^{-1}(x)$: $x = 2y - 4$

$\therefore 2y = x + 4$

$\therefore y = \frac{1}{2}x + 2$

$\therefore g^{-1}(x) = \frac{1}{2}x + 2$

**(b) For $g(x)$:** $x$**-intercept** $(y = 0)$ $y$**-intercept** $(x = 0)$

$2x - 4 = 0$ $\quad\quad$ $y = 2(0) - 4$

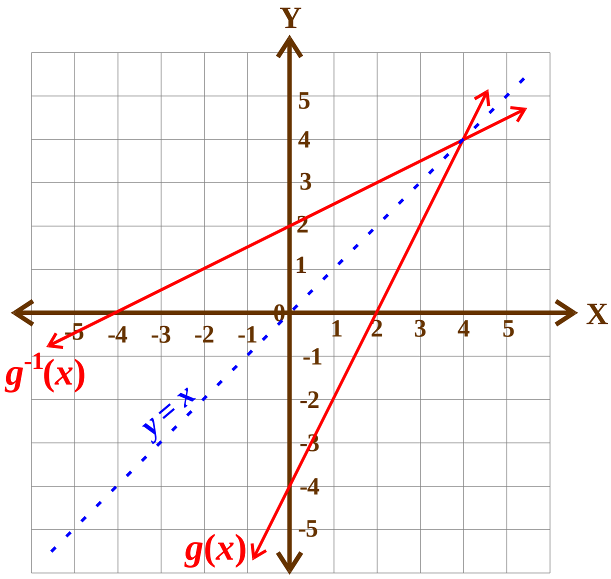
$\therefore \quad x = 2$ $\quad\quad$ $\therefore\ y = -4$

$\therefore \quad (2\ ;\ 0)$ **and** $(0\ ;\ -4)$

**For $g^{-1}(x)$:** $x$ **and** $y$ **of** $g(x)$ **swap place:**

$y$**-intercept** $(x = 0)$ $\quad\quad$ $x$**-intercept** $(y = 0)$

$\therefore\ (0\ ;\ 2)$ **and** $(-4\ ;\ 0)$



## A2.3.2 Graphs of inverses of the parabola:

**See grade 11 Quadratic Functions for revision and background!**

Ex. 7 Given: $g(x) = 2x^2$ with $x \geq 0$

(a) Determine $g^{-1}(x) = \ldots\ldots$

(b) Sketch $g(x)$ and $g^{-1}(x)$ on the same system of axes.

(c) Write down the domain of $g^{-1}(x)$.

(a) $g(x)$: $y = 2x^2$ with $x \geq 0$ $\Leftrightarrow$ $\therefore$ $g^{-1}(x)$: $x = 2y^2$ with $y \geq 0$

$\therefore$ $y^2 = \frac{x}{2}$

$\therefore$ $y = \pm\sqrt{\frac{x}{2}}$

but $y \geq 0$

$\therefore$ $g^{-1}(x) = +\sqrt{\frac{x}{2}}$

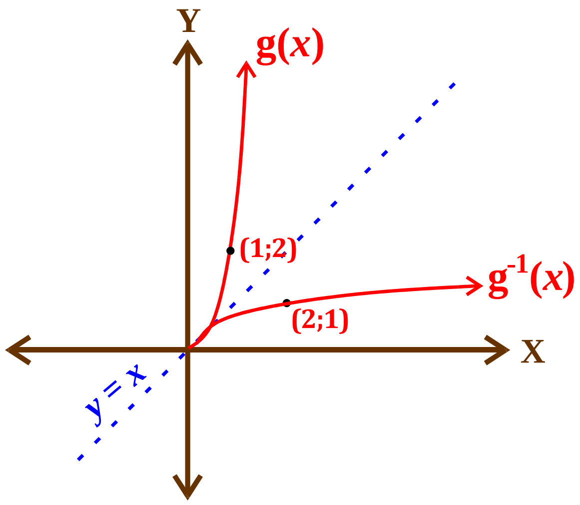
(b) For $g(x)$: Use a table, because the $x$-and-$y$ intercepts and the turning point is $(0\ ;\ 0)$.

| | | | |
|---|---|---|---|
| $x$ | 0 | 1 | 2 |
| $y$ | 0 | 2 | 8 |

$x \geq 0$

For $g^{-1}(x)$: $x$ and $y$ of $g(x)$ swap place:

| | | | |
|---|---|---|---|
| $x$ | 0 | 2 | 8 |
| $y$ | 0 | 1 | 2 |




(c) $D_{g^{-1}}$: $x \geq 0$

## A2.3.3 Graphs of inverses of the exponential function:

**See grade 11 Exponential Functions for revision and background!**

If the function $f(x) = a^x$ is given, the inverse will obtained as follow:

$f(x) = a^x \iff y = a^x$

$\therefore$ For the inverse, the $x$ and $y$ swap places: $x = a^y$

$\Longrightarrow y = \log_a x$ [Make $y$ the subject!]

$\therefore$ The inverse of an exponential function is a logarithmic function.

**Ex. 8 Given: $g(x) = 2^x$**

**(a) Determine $g^{-1}(x) = \ldots\ldots$**

**(b) Sketch $g(x)$ and $g^{-1}(x)$ on the same system of axes.**

**(c) Write down the equation of the asymptote of $g^{-1}(x)$.**

**(a)** $g(x)$: $y = 2^x \iff \therefore g^{-1}(x)$: $x = 2^y$

$\therefore y = \log_2 x$

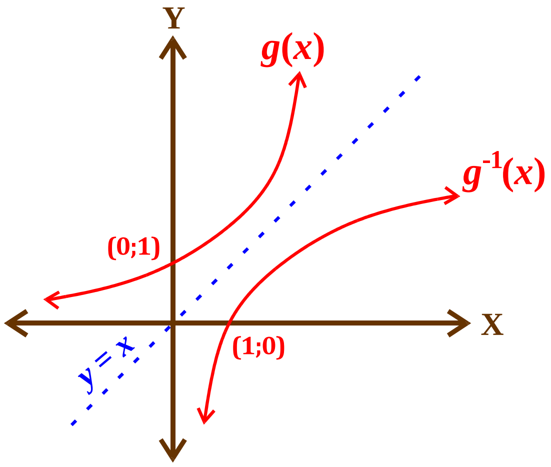
$\therefore g^{-1}(x) = \log_2 x$

**(b) For $g(x)$:**

| $x$ | $-1$ | $0$ | $1$ |
|---|---|---|---|
| $y$ | $\frac{1}{2}$ | $1$ | $2$ |

**For $g^{-1}(x)$: $x$ and $y$ of $g(x)$ swap places:**

| $x$ | $\frac{1}{2}$ | $1$ | $2$ |
|---|---|---|---|
| $y$ | $-1$ | $0$ | $1$ |




**(c) Asymptote of $g^{-1}(x)$:**

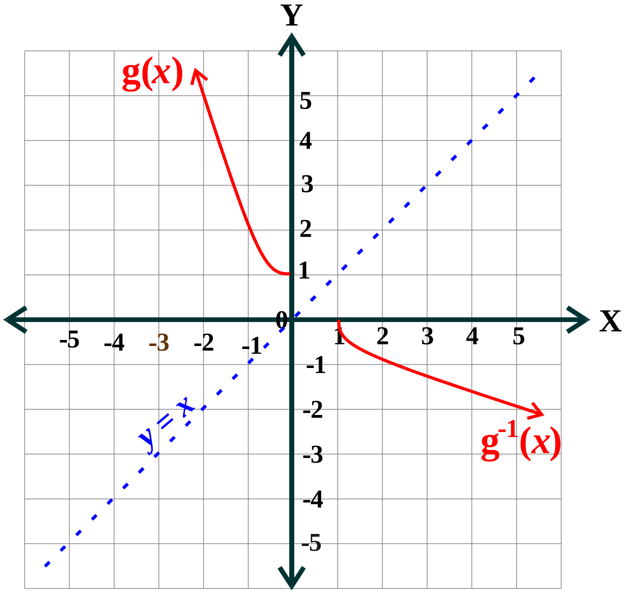
$x = 0$

Exercise 3:

(1) (a) Sketch: $g(x) = x^2 + 1$ for $x \leq 0$.

| $x$ | $0$ | $-1$ | $-2$ |
|---|---|---|---|
| $y$ | $1$ | $2$ | $5$ |

$x \leq 0$




(b) Determine $g^{-1}$ and write it in the form $g^{-1}(x) = \ldots\ldots\ldots$ $g(x) = x^2 + 1 = y$

$g^{-1}$: $y^2 + 1 = x$ for $y \leq 0$

$\therefore$ $y^2 = x - 1$

$\therefore$ $y = \pm\sqrt{x-1}$ $\rightarrow$ $g^{-1}(x) = -\sqrt{x-1}$

(c) Sketch $g^{-1}(x)$ on the same system of axes as $g(x)$.

| $x$ | $1$ | $2$ | $5$ |
|---|---|---|---|
| $y$ | $0$ | $-1$ | $-2$ |

$y \leq 0$

(d) Write down the range of $g^{-1}(x)$.

$R_{g^{-1}}$: $y \leq 0$

(2) Given: $h(x) = 2^{-x}$ $\rightarrow$ $h(x) = 2^{-x} = (2^{-1})^x = \left(\frac{1}{2}\right)^x = y$

(a) Determine $h^{-1}$ and write it in the form $h^{-1}(x) = \ldots\ldots\ldots$

$h^{-1}$: $\left(\frac{1}{2}\right)^y = x$

$\therefore$ $y = \log_{\frac{1}{2}} x$

$\therefore$ $h^{-1} = \log_{\frac{1}{2}} x$

(b) Sketch $h$ and $h^{-1}$ on the same system of axes.

**$h$:**

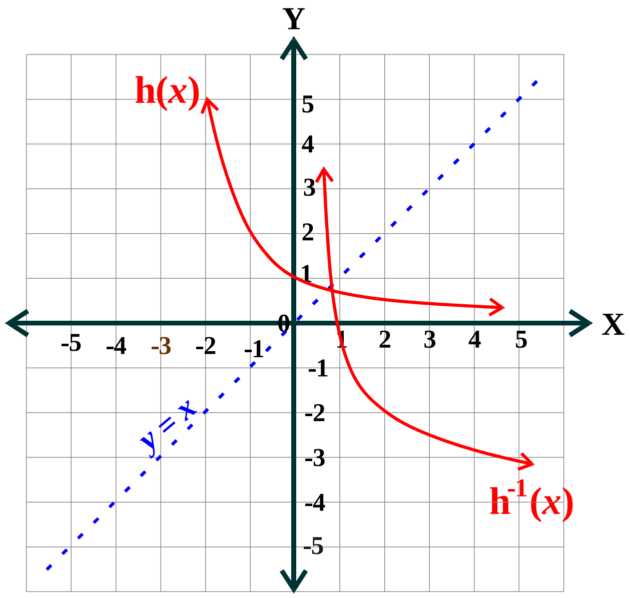
| $x$ | $-1$ | $0$ | $1$ |
|---|---|---|---|
| $y$ | $2$ | $1$ | $\frac{1}{2}$ |

**Asymptote: $y = 0$**

**$h^{-1}$:**

| $x$ | $2$ | $1$ | $\frac{1}{2}$ |
|---|---|---|---|
| $y$ | $-1$ | $0$ | $1$ |

**Asymptote: $x = 0$**




(c) Write down the domain of $h^{-1}(x)$. $\mathbf{D_{h^{-1}}: x > 0}$

(d) If $p$ is the reflection of $h$ in the $y$-axis, determine the equation of $p$ and write it in the form $p(x) = \ldots\ldots.$

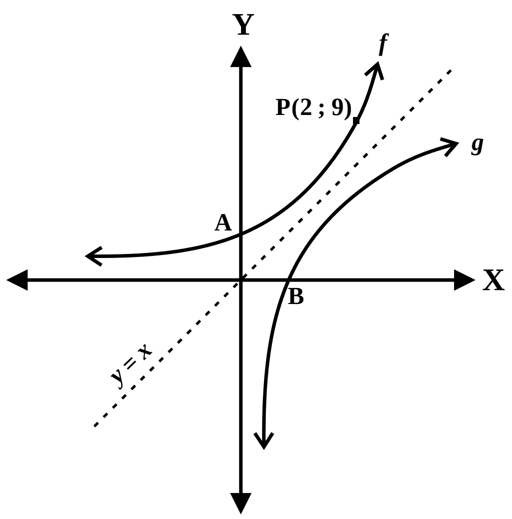
$$p(x) = \left(\frac{1}{2}\right)^{-x}$$

$\therefore \quad p(x) = 2^x$

(e) Determine $p^{-1}$ and write it in the form $p^{-1}(x) = \ldots\ldots.$

$\therefore \quad p^{-1}(x) = \log_2 x$

(3) Given: $f(x) = a^x$ and $g(x)$ with P(2 ; 9).




(a) Determine the value of $a$.

$y = f(x) = a^x$ through P(2 ; 9) ($x$ $y$)

$\therefore \quad 9 = a^2$

$\therefore \quad a = 3 \quad \rightarrow \quad a > 0$

(b) Give the coordinates of A.

**A(0; 1)**

(c) Determine the equation of $g(x)$, if $g(x)$ is the mirror image of $f(x)$ in the line $y = x$.

$g(x)$ **is the inverse of** $f(x)$ **with** $f(x) = y = 3^x$

$\therefore$ **for** $g$: $x = 3^y$

$\therefore$ $y = g(x) = \log_3 x$

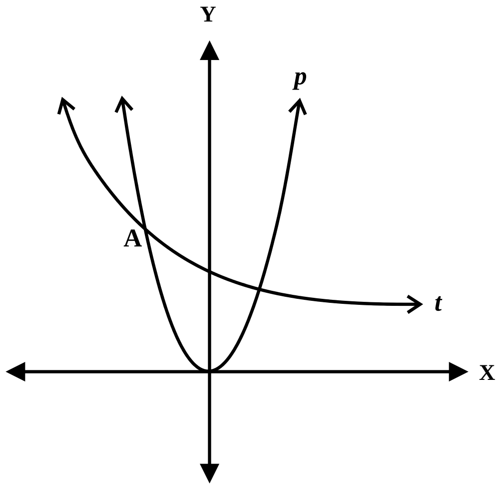
(d) Give the coordinates of B. $\mathbf{B(1;0)}$

(e) For which values of $x$ will $g(x)$ be defined?

**Defined → Domain** $\therefore$ $D_g$: $x > 0$

(f) Write down the equation of the asymptote of $g(x)$. $y$**-axis** → $x = 0$

(4) Given: $t(x) = a^x$ and $p(x) = bx^2$ met $A(-2\,;\,4)$.




(a) Determine the values of $a$ and $b$.

$t(x) = y = a^x$ **through** $\overset{x}{\phantom{}}\overset{y}{\phantom{}}$ $A(-2\,;4)$

$\therefore$ $4 = a^{-2}$

$\therefore$ $4 = \frac{1}{a^2}$

$\therefore$ $a^2 = \frac{1}{4}$

$\therefore$ $a = \frac{1}{2}$ → $a > 0$

$p(x) = y = bx^2$ **through** $A(-2\,;4)$

$\therefore$ $4 = b(-2)^2$

$\therefore$ $4 = b(-2)^2$

$\therefore$ $4 = b(4)$

$\therefore$ $b = 1$

(b) Write down the following: $t^{-1}(x) = \ldots\ldots$

$t(x) = y = \left(\frac{1}{2}\right)^x$

**For** $t^{-1}(x)$: $x = \left(\frac{1}{2}\right)^y$

$\therefore$ $y = \log_{\frac{1}{2}} x$

$\therefore$ $t^{-1}(x) = \log_{\frac{1}{2}} x$

(c) Write down the following: $p^{-1}(x) = \ldots\ldots$

$p(x) = 1x^2$ → $y = x^2$

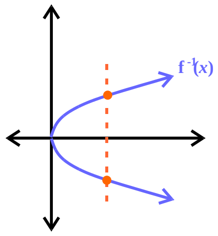
**For** $p^{-1}(x)$: $x = y^2$

$\therefore$ $y = \pm\sqrt{x}$

$\therefore$ $p^{-1}(x) = \pm\sqrt{x}$

(d) Explain why $p^{-1}(x)$ is not a function.

**Because $p^{-1}(x)$ each input does not have a unique output.**
**See sketch.**

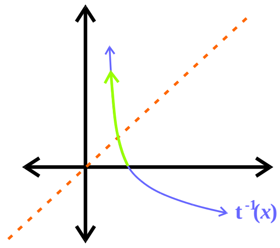



(e) Determine $x$ for which $t^{-1}(x) \geq 0$.

$\therefore \quad y \geq 0$

$\therefore \quad 0 < x \leq 1$

**See the solution in green.**




(f) Calculate: $t^{-1}(0{,}25) + p(3)$

$= \log_{\frac{1}{2}} 0{,}25 + (3)^2$

$= 2 + 9$

$= 11$

(5) The graph of $f(x) = a^x$ is sketched alongside.

The point $B(3; 8)$ lies on the graph of $f$.

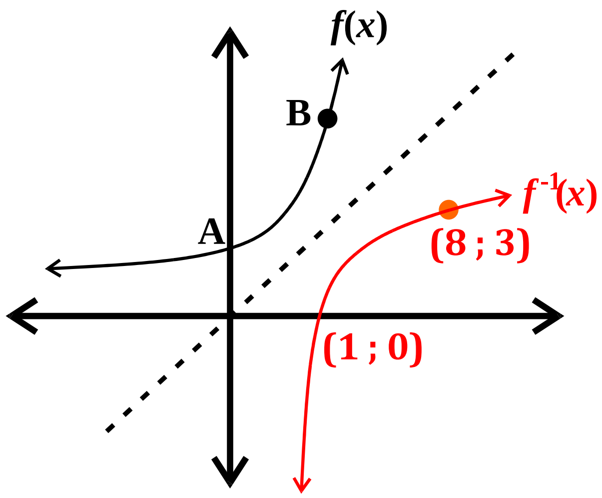
(a) Show that $a = 2$.

$f(x) = y = a^x$ **through** $B(\overset{x}{3} \,;\, \overset{y}{8})$

$\therefore \quad 8 = a^3$

$\therefore \quad 2^3 = a^3$

$\therefore \quad a = 2$ **EB ⟺ EE**




(b) Write down the coordinates of A.

**A(0; 1)** **because** $y = 2^0 = 1$

(c) Write down the equation of $f^{-1}(x)$ in the form $f^{-1}(x) = \cdots$

$f(x) = y = 2^x$

**For $f^{-1}(x)$:** $x = 2^y$

$\therefore \quad f^{-1}(x) = y = \log_2 x$

(d) Sketch the graph of $f^{-1}$.

Show the $x$-intercept and ONE other point. **See sketch.**

(e) For which values of $x$ will $f^{-1}(x) = f(x)$?

**None** $\rightarrow$ **$f^{-1}(x)$ and $f(x)$ don't intersect.**

(f) Write down the equation of $g$ if $g$ is the reflection of $f$ in the $y$-axis.

$\boldsymbol{g(x) = \left(\frac{1}{2}\right)^x}$ **or** $\boldsymbol{g(x) = 2^{-x}}$

(g) Write down the equation of $h$ if $h$ is the reflection of $f^{-1}$ in the $x$-axis.

$\boldsymbol{h(x) = -\log_2 x}$

(h) Are $g$ and $h$ one another's inverse? Motivate your answer.

**Yes,** **because** $\boldsymbol{g(x) = y = 2^{-x}}$ $\rightarrow$ $\boldsymbol{g^{-1}}$: $\boldsymbol{x = 2^{-y}}$

$\therefore \quad \boldsymbol{-y = \log_2 x}$

$\therefore \quad \boldsymbol{y = -\log_2 x = h(x)}$

(i) For which values of $x$ will $f^{-1}(x) \geq 0$? $\rightarrow$ $\boldsymbol{y \geq 0}$ **for** $\boldsymbol{f^{-1}}$

$\therefore \ \boldsymbol{x \geq 1}$

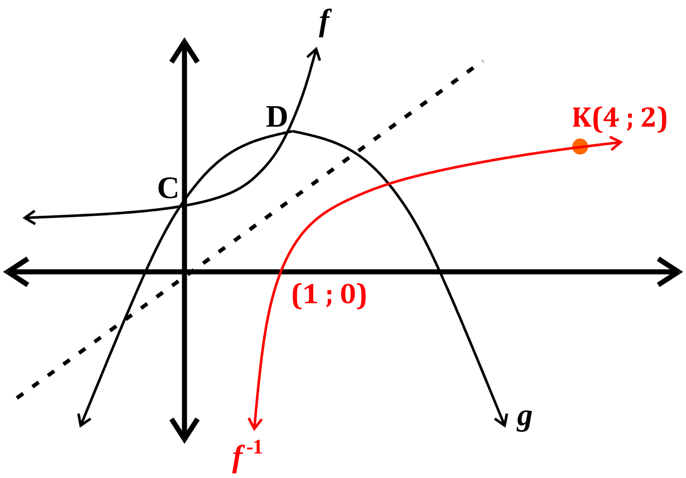
(j) Calculate: $f^{-1}(2) + f(-2)$

$= \boldsymbol{\log_2 2 + 2^{-2}}$

$= \boldsymbol{1 + \frac{1}{2^2}}$

$= \boldsymbol{1 + \frac{1}{4} = 1\frac{1}{4}}$

(6) On the right is the graphs of $f(x) = 2^x$ and $g(x) = -(x-1)^2 + b$, with $b$ as a constant value. The graphs of $f$ and $g$ intersects on the $y$-axis at C. D is the turning point of $g$.




(a) Show that $b = 2$.

**C is the $y$-intercept of both $f$ and $g$.**

**For $f$:** $\boldsymbol{y = 2^x}$ **with** $\boldsymbol{x = 0}$ $\rightarrow$ **for $y$-intercept**

$\therefore \quad \boldsymbol{y = 2^0 = 1} \quad \rightarrow \quad \boldsymbol{C(0; 1)}$

**For $g$:** $\boldsymbol{y = -(x-1)^2 + b}$ **through** $\boldsymbol{C(\overset{x}{0}\,;\overset{y}{1})}$

$\therefore \quad \boldsymbol{1 = -(0-1)^2 + b}$

$\therefore \quad \boldsymbol{1 = -(-1)^2 + b}$

$\therefore \quad \boldsymbol{1 = -1 + b}$

$\therefore \quad \boldsymbol{b = 2}$

(b) Write down the coordinates of the turning point of $g$. **TP: $(1; 2)$**

(c) Write down the equation of $f^{-1}(x)$ in the form $y = \ldots\ldots$.

**$f^{-1}(x) = y = \log_2 x$**

(d) Sketch the graph of $f^{-1}$ on the same graph as given above.

Show on your graph the $x$-intercept and the coordinates of one other point.

**See graph!**

**Other point: If $x = 4$ $\rightarrow$ $y = \log_2 4 = 2$**

**$\therefore$ $K(4; 2)$**

(e) Write down the equation of $h$ if $h(x) = g(x + 1) - 2$.

$$h(x) = \left[-\left((x+1)-1\right)^2 + 2\right] - 2$$

$$h(x) = [-(x+1-1)^2 + 2] - 2$$

$$h(x) = [-(x)^2 + 2] - 2$$

$$h(x) = -x^2 + 2 - 2$$

$$h(x) = -x^2$$

(f) How can the domain of $h$ be restricted so that $h^{-1}$ will be a function?

**The domain of $h$ should be limited as below for $h^{-1}$ to be a function:**

$$x \leq 0 \quad \text{or} \quad x \geq 0$$

**$\rightarrow$ The range of $h^{-1}$ will be as follow:**

$$y \leq 0 \quad \text{or} \quad y \geq 0$$

(g) Determine the maximum value of $2^{2-(x-1)^2}$.

**The maximum value of $2^{2-(x-1)^2}$ will be at the maximum value of $[2 - (x-1)^2]$**

$\therefore$ $2 - (x-1)^2 = -(x-1)^2 + 2$

**$\rightarrow$ maximum value is 2 $\Longrightarrow$ read from the turning point $(1; 2)$**

**$\therefore$ maximum value of $2^{2-(x-1)^2} = 2^2$**

**$\therefore$ maximum value of $2^{2-(x-1)^2} = 4$**

# REVISION FROM PAST PAPERS:

Exercise A:

Consider the function $f(x) = y = \left(\frac{1}{3}\right)^x \rightarrow y = 3^{-x}$

(1) Is $f$ an increasing or decreasing function? Give a reason for your answer. (2)

**Decreasing function** ✓ **Reason: $0 < a < 1$** ✓ **or** **As $x$ increases, $f(x)$ will decrease.**

(2) Calculate $f^{-1}(x)$ in the form $y = \ldots\ldots$ (2)

$f^{-1}(x)$: $x = \left(\frac{1}{3}\right)^y$ ✓ or $x = 3^{-y}$

$\therefore \quad y = \log_{\frac{1}{3}} x$ ✓ $\therefore \quad y = -\log_3 x$

(3) Write down the equation of the asymptote of $f(x) - 5$. (1)

$y = -5$ ✓

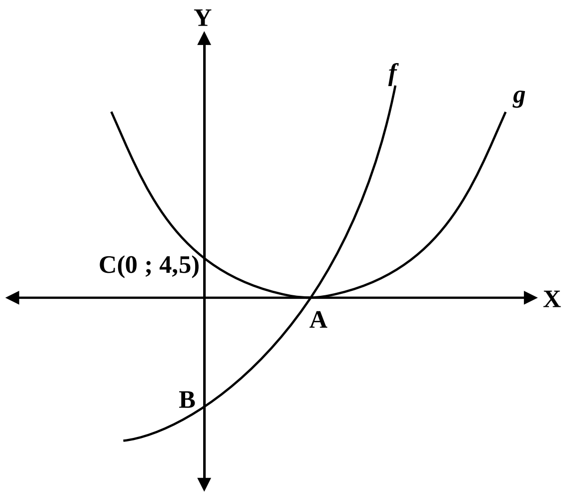
(4) Describe the transformation of $f$ to $g$ if $g(x) = \log_3 x$. (2)

**A reflection in the line $y = x$ followed by a reflection in the $x$-axis.** ✓✓

**or** **A reflection in the $y$-axis followed by a reflection in the line $y = x$.**

Exercise B:

The graphs of $f(x) = 2^x - 8$ and $g(x) = ax^2 + bx + c$ sketched below. B and C$(0\ ;\ 4{,}5)$ are the $y$-intercepts of the graphs of $f$ and $g$ respectively. The two graphs intersect at A, which is the turning point of the graph of $g$ and the $x$-intercept of the graphs of $f$ and $g$.




(1) Determine the coordinates of A and B. (4)

**A: $x$-intercept of $g$ $\rightarrow$ $y = 0$**

$\rightarrow\ \mathbf{0 = 2^x - 8}$ ✓

$\rightarrow\ \mathbf{8 = 2^3 = 2^x}$

$\rightarrow\ \therefore\ \mathbf{x = 3}$

$\therefore$ **A(3 ; 0)** ✓

**B: $y$-intercept of $g$ $\rightarrow$ $x = 0$**

$\rightarrow\ \mathbf{y = 2^0 - 8}$ ✓

$\rightarrow\ \mathbf{y = 1 - 8}$

$\rightarrow\ \mathbf{y = -7}$

$\therefore$ **B(0 ; −7)** ✓

(2) Write down the equation of the asymptote of graph $f$. (1)

$\mathbf{y = -8}$ ✓

(3) Determine the equation of $h$ if $h(x) = f(2x) + 8$. (2)

$\mathbf{h(x) = f(2x) + 8 = 2^{2x} - 8 + 8}$ $\rightarrow$ $\mathbf{h(x) = 2^{2x} = 4^x}$ ✓✓

(4) Determine the equation of $h^{-1}$ in the form $y = \ldots..$ (2)

$\mathbf{h^{-1}:}$ $\mathbf{x = 4^y}$ $\rightarrow$ $\mathbf{y = \log_4 x}$ ✓✓

(5) Write down the equation of $p$, if $p$ is the reflection of $h^{-1}$ in the $x$-axis. (1)

$\mathbf{p(x) = -\log_4 x}$ **or** $\mathbf{p(x) = \log_{\frac{1}{4}} x}$ ✓

(6) Calculate $\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$. Show ALL calculations. (4)

$= [g(0) + g(1) + g(2) + g(3)] - [g(4) + g(5)]$ ✓

$= g(0) + g(1) + g(2) + g(3) - g(4) - g(5)$ **with $x = 3$ the line of symmetry**

$= g(0) + g(3)$ ✓ **because** $g(1) = g(5)$ **and** $g(2) = g(4)$

$= 4,5 - 0$ ✓ → **Given C$(0\,;4,5)$ and calculated in (a) A$(3\,;0)$**

$= 4,5$ ✓

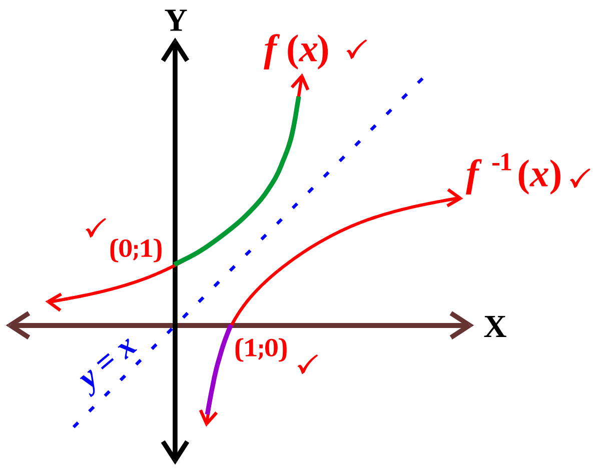
Exercise C:

Given: $f(x) = 3^x$

(1) Determine an equation for $f^{-1}$ in the form $f^{-1}(x) = \ldots$ (1)

$f^{-1}(x) = \log_3 x$ ✓

(2) Sketch the graphs of $f$ and $f^{-1}$, clearly showing ALL intercepts with the axes. (4)




(3) Write down the domain of $f^{-1}$. (2)

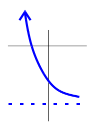
$D_{f^{-1}}: x > 0$ **or** $x \in (0\,;\infty)$ ✓✓

(4) For which values of $x$ will $f(x).f^{-1}(x) \leq 0$? (2)

$0 < x < 1$ ✓✓ **where $f(x) > 0$ and $f^{-1}(x) < 0$**

(5) Write down the range of $h(x) = 3^{-x} - 4$. (2)

$R_h: y > -4$ **or** $y \in (-4\,;\infty)$ ✓✓

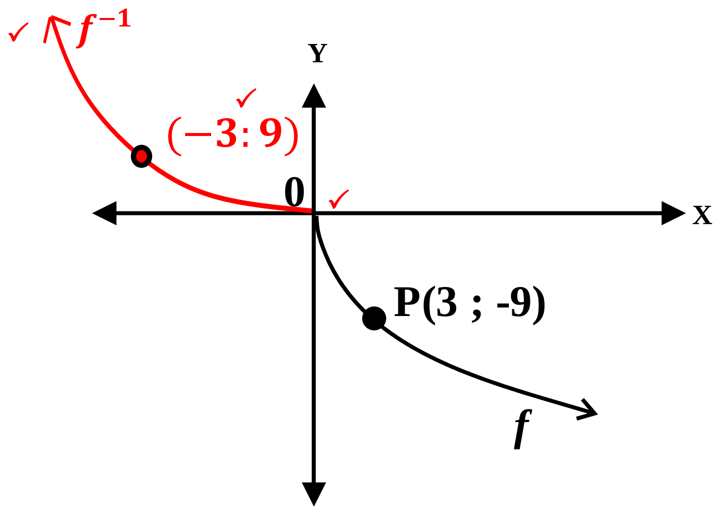


(6) Write down an equation for $g$ is the graph of $g$ is the image of the graph of $f$ after $f$ has been translated two units to the right and reflected about the $x$-axis. (2)

$g(x) = -3^{x-2}$ ✓✓

Exercise D:

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched below.
The point P$(3\,;\,-9)$ lies on the graph of $f$.




(1) Use the graph to determine the values of $x$ for which $f(x) \geq -9$. (2)

$0 \leq x \leq 3$ ✓✓ **For $0 < x < 3$ only 1 mark**

(2) Write down the equation of $f^{-1}$ in the form $y = \ldots$.. Indicate ALL restrictions. (3)

$f^{-1}:$ $x = -\sqrt{27y}$ → $(x)^2 = \left(-\sqrt{27y}\right)^2$ ✓

∴ $x^2 = 27y$

∴ $y = \frac{x^2}{27} = \frac{1}{27}x^2$ ✓ **for** $x \leq 0$ **or** $x \in (-\infty\,;0)$ ✓

(3) Sketch $f^{-1}$, the inverse of $f$. Indicate the intercept(s) with the axes and the coordinates of ONE other point. (3)

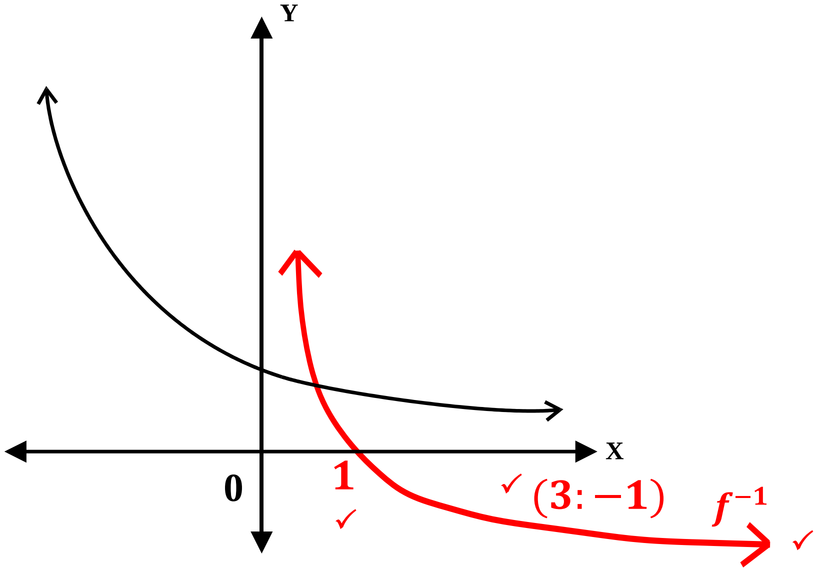
**See sketch above!**

(4) Describe the transformation of $f$ to $g$ if $g(x) = \sqrt{27x}$ for $x \geq 0$. (1)

**Reflection about the $x$-axis** **or** $(x\,;y) \to (x:-y)$ **for** $x \geq 0$ ✓

Exercise E:

The graph of $f(x) = \left(\frac{1}{3}\right)^x$ is sketched below.




(1) Write down the domain of $f$. (1)

$\mathbb{R}$ **or** $x \in (-\infty\,;\infty)$ ✓

(2) Write down the equation of the asymptote of $f$. (1)

$y = 0$ ✓

(3) Write down the equation of $f^{-1}$ in the form $y = \ldots\ldots$ (2)

$f^{-1}$: $x = \left(\frac{1}{3}\right)^y = 3^{-y}$ ✓ $\rightarrow$ $y = \log_{\frac{1}{3}} x = -\log_3 x$ ✓

(4) Sketch the graph of $f^{-1}$. Indicate the $x$- intercept and the coordinates of ONE other point. **See sketch above!** (3)

(5) Write down the equation of the asymptote of $f^{-1}(x+2)$. (2)

$x = -2$ ✓✓ **Graph moves two units to the left**

(6) Prove that: $[f(x)]^2 - [f(-x)]^2 = f(2x) - f(-2x)$ for all values of $x$. (3)

**LHS** $= [f(x)]^2 - [f(-x)]^2 = \left[\left(\frac{1}{3}\right)^x\right]^2 - \left[\left(\frac{1}{3}\right)^{-x}\right]^2$ ✓

$\therefore$ **LHS** $= 3^{-2x} - 3^{2x}$ ✓

**RHS** $= f(2x) - f(-2x) = \left(\frac{1}{3}\right)^{2x} - \left(\frac{1}{3}\right)^{-2x}$ ✓

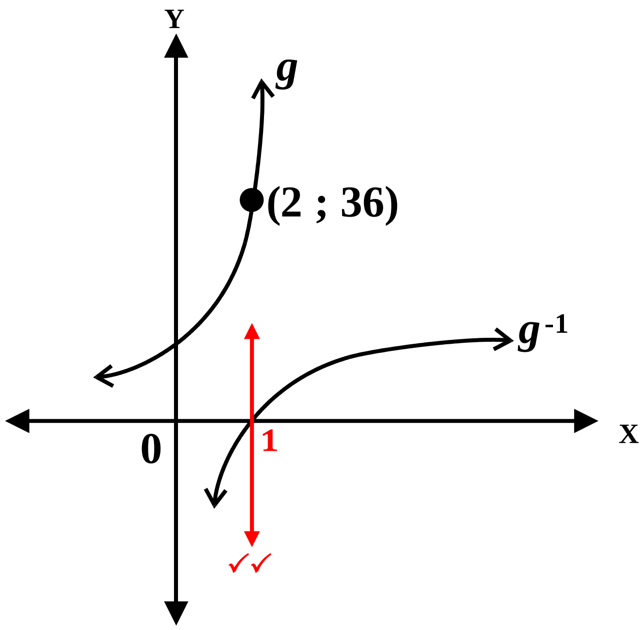
$\therefore$ **RHS** $= 3^{-2x} - 3^{2x}$

$\therefore$ **LHS = RHS**

$\therefore$ $[f(x)]^2 - [f(-x)]^2 = f(2x) - f(-2x)$

Exercise F:

The graphs of $g(x) = k^x$, with $k > 0$ and $y = g^{-1}(x)$ is sketched below. The point $(2\,;\,36)$ is a point on $g$.




(1) Determine the value of $k$. (2)

$g(x) = k^x \quad \rightarrow \quad y = k^x \quad \rightarrow \quad 36 = k^2$ ✓ $\quad \therefore \quad k = 6$ ✓ $(k > 0)$

(2) Write down the equation of $g^{-1}$ in the form $y = \ldots\ldots$ (2)

$g^{-1}: \quad x = 6^y$ ✓ $\quad \rightarrow \quad y = \log_6 x$ ✓

(3) For which value(s) of $x$ will $g^{-1}(x) \leq 0$? (2)

$0 < x \leq 1$ or $x \in (0\,;1]$ ✓

(4) Write down the domain of $h$, for $h(x) = g^{-1}(x - 3)$. (1)

$x > 3$ or $x \in (3\,;\infty)$ ✓

(5) Sketch the graph of the inverse of $y = 1$. **See sketch above!** (2)

**Inverse of $y = 1 \quad \rightarrow \quad x = 1$**

(6) Is the inverse of $y = 1$ a function? Motivate your answer. (2)

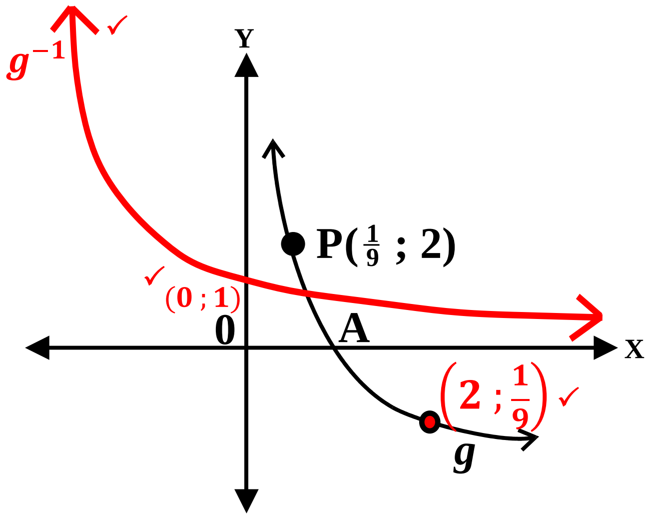
**No, $x = 1$ is not a function, because the line is not a one-to-one fuction.** ✓✓

Exercise G:

(1) Given the graph of $g(x) = \log_{\frac{1}{3}} x$

A is the $x$-intercept of $g$.

$P\left(\frac{1}{9}\,;2\right)$ is a point on $g$.




(1) Write down the coordinates of A. (1)

**A(1 ; 0)** ✓

(2) Sketch the graph of $g^{-1}$ and indicate intercepts as well the coordinates of ONE other that will lie on the graph. **See graph above!** (3)

(3) Write down the domain of $g^{-1}$. (1)

$\mathbb{R}$ **or** $x \in (-\infty\,;\infty)$ ✓