

# **Grade 8 – Textbook**

**(Revised CAPS edition)**

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# Chapter A1

## Integers

### A1.1 Number systems and properties of integers:

#### Exercise 1:

Complete: \* Natural numbers:  $\mathbb{N} = \{ \underline{\hspace{15em}} \}$   
 \* Whole numbers  $\mathbb{N}_0 = \{ \underline{\hspace{15em}} \}$   
 \* Integers:  $\mathbb{Z} = \{ \underline{\hspace{15em}} \}$

The integers are expanded to include the fractions:

Rational numbers ( $\mathbb{Q}$ ): Include all fractions which can be written as  $\frac{a}{b}$ , with a and b as integers and  $b \neq 0$ . This includes all finite and recurring decimal fractions.

E.g.  $\frac{1}{3}$ ;  $0.\dot{7}$ ;  $-3\frac{2}{3}$ ; 2,34;  $\sqrt{25}$ ; 9;  $\sqrt[3]{27}$  etc.

Irrational numbers ( $\mathbb{Q}'$ ): Include all infinite and non-recurring decimal fractions.

E.g. 3,68463.....;  $\pi$ ;  $\sqrt{10}$ ;  $\sqrt[3]{4}$  etc.

Real numbers ( $\mathbb{R}$ ) consist of all rational and irrational numbers in union:  $\mathbb{Q} \cup \mathbb{Q}'$

Non-real numbers for example are:  $\sqrt{-4}$ ;  $\sqrt{-12}$  etc.

$\sqrt[3]{-8}$  and  $\sqrt[3]{-243}$  however, are real numbers, because  $\sqrt[3]{-8} = -2$  and  $\sqrt[3]{-243} = -3$ .

#### Properties of 1 and 0:

- |                                 |                    |
|---------------------------------|--------------------|
| * $m \times 0 = 0$              | * $m \times 1 = m$ |
| * $0 \div m = 0$                | * $m \div 1 = m$   |
| * $m \div 0 = \text{undefined}$ |                    |

#### Identity elements:

- \* 0 is the identity element of addition, because  $m + 0 = m$
- \* 1 is the identity element of multiplication, because  $m \times 1 = m$

#### Inverses:

- \* The sum of a number and its additive inverse is 0.  
E.g. 3 is the additive inverse of -3, because  $3 + (-3) = 3 - 3 = 0$
- \* The multiplicative inverse (reciprocal) is the number multiplied with a certain number with a result of 1. E.g. the multiplicative inverse of 3 is  $\frac{1}{3}$ , because  $3 \times \frac{1}{3} = \frac{3}{1} \times \frac{1}{3} = 1$

#### Other properties:

- \* Commutative operation:  $m \times n = n \times m$  or  $m + n = n + m$
- \* Associative operation:  $(m \times n) \times p = m \times (n \times p)$  or  $(m + n) + p = n + (m + p)$
- \* Distributive operation:  $p \times (m + n) = p \times m + p \times n$  or  $p \times (m - n) = p \times m - p \times n$

## A1.2 Rules for divisibility:

Divisor:	Rules for divisibility:
2	Last digit must be an even number or a 0.
3	Sum of all the digits must be divisible by 3.
4	Two last digits must be divisible by 4.
5	Last digit must be 5 or 0.
6	Rules for divisibility for 2 and 3 must apply.
8	Last three digits must be divisible by 8.
9	Sum of all the digits must be divisible by 9.
10	Last digit must be 0.
11	Calculate the sums of the alternate digits. The difference between these sums must be 0, or it must be divisible by 11.

*E.g. 1 Determine whether 10 527 is divisible by the numbers in the above table:*

2: NO, because the number (10 527) does not end on an even number.

3: YES, because the sum of the digits,  $1+0+5+2+7=15$  is divisible by 3.

4: NO, because 27(10 527) is not divisible by 4.

5: NO, because the number does not end on a 5 or a 0.

6: NO, because the rule of divisibility for 2 does not apply.

8: NO, because the last three digits, 527, are not divisible by 8.

9: NO, because the sum of the digits viz.  $1+0+5+2+7=15$  is not divisible by 9.

10: NO, because the last digit is not 0.

11: YES, because the difference between  $1+5+7=13$  and  $0+2=2$  with  $13 - 2 = 11$ .

### Exercise 2:

Determine whether the following numbers are divisible by the numbers in the above table:

(1) 1 275

(2) 2 772

(3) 7 920

☺ A certain number is divisible by 2, 3, 5 and 11. This number is not divisible by 8 and 9, but it is divisible by 4. Determine the smallest number that meets these conditions.

## A1.3 Factors:

*E.g. 2 The factors of 10 are:  $F_{10} = \{1 ; 2 ; 5 ; 10\}$*

### Exercise 3:

Complete:

(1)  $F_{20}$

(2)  $F_{16}$

(3)  $F_5$

(4)  $F_{32}$

(5)  $F_{15}$

(6)  $F_{28}$

(7)  $F_{12}$

(8)  $F_7$

(9)  $F_{36}$

(10)  $F_{11}$

**A1.4 Multiples:**

E.g. 3 The multiples of 10 are:  $M_{10} = \{10 ; 20 ; 30 ; \dots\dots\dots\}$

**Exercise 4:**

Complete:

- |              |              |              |              |
|--------------|--------------|--------------|--------------|
| (1) $M_6$    | (2) $M_{20}$ | (3) $M_7$    | (4) $M_{12}$ |
| (5) $M_{36}$ | (6) $M_9$    | (7) $M_{35}$ | (8) $M_{16}$ |
| (9) $M_{11}$ | (10) $M_3$   |              |              |

© Determine the multiples of 6 which are also factors of 120.

**A1.5 Prime numbers and compound numbers:****Exercise 5:**

Complete:

- (1) The definition of a prime number is ...
- (2) The smallest prime number is ...
- (3) The only even prime number is ...
- (4) The definition of a compound number is ...
- (5) Which natural number is **neither** a prime number nor a compound number?
- (6) Which natural numbers smaller than 50, are prime numbers?  
(Do the following: Encircle 2 ; 3 ; 5 and 7 and cross out all the multiples of 2 ; 3 ; 5 ; and 7.  
The numbers which are left will be the prime numbers. Remember to cross out 1 as well!)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

The prime numbers smaller than 50 are: {\_\_\_\_\_}

**A1.6 Prime factors:**

E.g. 4 The factors of 6 are:  $F_6 = \{1 ; 2 ; 3 ; 6\}$

$\therefore$  The prime factors of 6 are: **2 and 3**. (In other words they are factors which are prime numbers)

E.g. 5 The factors of 20 are:  $F_{20} = \{1 ; 2 ; 4 ; 5 ; 10 ; 20\}$

$\therefore$  The prime factors of 20 are: **2 and 5**.

E.g. 6 Determine the prime factors of 60:

2	60
2	30
3	15
5	5
	1

$$\begin{aligned} \therefore 60 &= 2 \times 2 \times 3 \times 5 \\ &= \underline{2^2 \times 3 \times 5} \end{aligned}$$

Exercise 6:

Determine the prime factors of:

- |        |          |          |          |
|--------|----------|----------|----------|
| (1) 12 | (2) 35   | (3) 32   | (4) 44   |
| (5) 48 | (6) 27   | (7) 56   | (8) 100  |
| (9) 18 | (10) 168 | (11) 588 | (12) 450 |

**A1.7 LCM and HCF:****LCM = Lowest common multiple.****HCF = Highest common factor.**

*E.g.7 Determine the LCM of 8 ; 12 and 20  
[First determine the prime factors!]*

$$\begin{aligned}
 8 &= \boxed{2 \times 2} \times 2 \\
 12 &= \boxed{2 \times 2} \times 3 \\
 20 &= \boxed{2 \times 2} \times 5
 \end{aligned}
 \quad \therefore \text{LCM} = \boxed{2 \times 2} \times 2 \times 3 \times 5 = \underline{120}$$

*E.g.8 Determine the HCF of 36 and 60.  
[First determine the prime factors!]*

$$\begin{aligned}
 36 &= \boxed{2 \times 2 \times 3} \times 3 \\
 60 &= \boxed{2 \times 2 \times 3} \times 5
 \end{aligned}
 \quad \therefore \text{HCF} = \boxed{2 \times 2 \times 3} = \underline{12}$$

Exercise 7:

(1) Determine the HCF of the following by finding the prime factors first:

- (a) 14; 21 and 35
- (b) 27; 45 and 72
- (c) 12 and 168
- (d) 38; 57 and 95
- (e) 10; 15 and 105

(2) Determine the LCM of the following by finding the prime factors first:

- (a) 6 ; 12 and 18
- (b) 8 and 20
- (c) 2 ; 6 and 11
- (d) 21 and 49
- (e) 3 ; 9 ; 12 and 60
- (f) 15 ; 45 and 270

(3) Determine the LCM and the HCF:

- (a) 16 ; 48 and 56
- (b) 5 and 24

☺ Mounting boards of (a) 24 cm<sup>2</sup>, (b) 36 cm<sup>2</sup> and (c) 18 cm<sup>2</sup> have to be cut. Determine the area of the smallest mounting board panel that should be used so that any combination of (a), (b) and/or (c) can be cut from it, without wasting any board. [Make use of prime factors.]

### A1.8 Square roots and cube roots:

E.g.9 Determine the following by using prime factors:

(a)  $\sqrt{784}$

(b)  $\sqrt[3]{3\,375}$

\*\*\*\*\*

$$\begin{array}{r|l} 2 & 784 \\ 2 & 392 \\ 2 & 196 \\ 2 & 98 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

$$\begin{aligned} \therefore 784 &= 2 \times 2 \times 2 \times 2 \times 7 \times 7 \\ &= 2^2 \times 2^2 \times 7^2 \end{aligned}$$

$$\begin{aligned} \therefore \sqrt{784} &= 2 \times 2 \times 7 \\ &= \underline{28} \end{aligned}$$

$$\begin{array}{r|l} 3 & 3\,375 \\ 3 & 1\,125 \\ 3 & 375 \\ 5 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$$\begin{aligned} \therefore 3\,375 &= 3 \times 3 \times 3 \times 5 \times 5 \times 5 \\ &= 3^3 \times 5^3 \end{aligned}$$

$$\begin{aligned} \therefore \sqrt[3]{3\,375} &= 3 \times 5 \\ &= \underline{15} \end{aligned}$$

#### Exercise 8:

Calculate: (by using prime factors)

(1)  $\sqrt{576}$

(2)  $\sqrt[3]{343}$

(3)  $\sqrt{225}$

(4)  $\sqrt{1\,024}$

(5)  $\sqrt{1\,000}$

(6)  $\sqrt[3]{4\,096}$

(7)  $\sqrt[3]{729}$

(8)  $\sqrt{5\,184}$

### A1.9 Squares and cubes:

**Remember the order of operations:**

- (1) Brackets
- (2) Powers and roots
- (3) Of  $\rightarrow \times$
- (4) Multiplication and division
- (5) Addition and subtraction

E.g.10 Calculate, without using a calculator:

(a)  $2^3 = 2 \times 2 \times 2 = \underline{8}$

(d)  $\sqrt{25 - 9} = \sqrt{16} = \underline{4}$

(b)  $5^2 = 5 \times 5 = \underline{25}$

(e)  $\sqrt[3]{12^3} = \underline{12}$

(c)  $(3 + 4)^2 = (7)^2 = \underline{49}$

Exercise 9:

Calculate, without using a calculator:

(1)  $\sqrt{144}$

(2)  $(6 - 2)^3$

(3)  $2^3 + 1^3$

(4)  $\sqrt{36} - \sqrt{4}$

(5)  $(\sqrt{26})^2$

(6)  $5^2 + 5^3$

(7)  $\sqrt{100 - 64} - \sqrt{1}$

(8)  $\sqrt[3]{27} + \sqrt{121}$

(9)  $(2 + 6)^3 + (11 - 7)^3$

(10)  $\sqrt[3]{216} + \sqrt{1024}$

(11)  $\sqrt{\sqrt{81}}$

(12)  $\sqrt{\frac{144}{4}}$

(13)  $(2 \times 3)^2$

(14)  $2^2 \times 3^2$

(15)  $5^3 - 3^3$

(16)  $(5 - 3)^3$

(17)  $2 \times 9^2$

(18)  $(\sqrt{9})^3$

(19)  $\sqrt[3]{1} - 1^3$

(20)  $3 \times \sqrt{25}$

☺ Calculate: $\sqrt[5]{25^5} + (2)^5 - (\sqrt[3]{8})^8 - (5 - 4)^{100}$
---

**A1.10 Order of integers:**Exercise 10:

(1) What will the reading on a thermometer be if the temperature:

- (a) rises from  $5^\circ\text{C}$  by  $3^\circ\text{C}$ ?
- (b) rises from  $-2^\circ\text{C}$  by  $7^\circ\text{C}$ ?
- (c) drops from  $4^\circ\text{C}$  by  $6^\circ\text{C}$ ?
- (d) drops from  $-3^\circ\text{C}$  by  $4^\circ\text{C}$ ?
- (e) drops from  $8^\circ\text{C}$  by  $3^\circ\text{C}$  and then rises by  $5^\circ\text{C}$ ?
- (f) rises from  $2^\circ\text{C}$  by  $6^\circ\text{C}$  and then drops by  $11^\circ\text{C}$ ?

(2) What will your bank balance be if you have R100 in your account and then:

(Use the original balance to answer each question!)

- (a) withdraw R70?
- (b) deposit R40 and then withdraw R60?
- (c) withdraw R140?
- (d) withdraw R20 and then later withdraw another R80?



(3) State whether the following is false or true:

- (a) 2 is a positive number.
- (b) -6 is a whole number.
- (c) 0 is a natural number.
- (d) 4 is an even, negative number.
- (e)  $\frac{2}{3}$  is an integer.
- (f) 0 is an integer.
- (g)  $\{1; 3; 5; \dots\}$  is the set of uneven natural numbers.
- (h)  $\{\dots; -2; -1; 0; 1\}$  is the set of integers less than 2.

(4) Arrange the following integers in ascending order:

- (a) 4 ; -1 ; 5 ; 0 ; 3 ; -7
- (b) -5 ; -6 ; -1 ; 2 ; 8 ; -11
- (c) -200 ; -202 ; -201 ; -205

(5) Arrange the following integers in descending order:

- (a) 10 ; -20 ; 30 ; -40 ; 50
- (b) 3 ; 4 ; 5 ; -3 ; -4 ; -5
- (c) 36 ; -3 ; -27 ; 18 ; -15

### A1.11 Properties of integers:

Exercise 11:

(1) Complete:

- (a) The additive inverse of 6 is \_\_\_\_\_
- (b) The additive inverse -3 is \_\_\_\_\_
- (c) The identity element for addition is \_\_\_\_\_
- (d) The identity element for multiplication is \_\_\_\_\_

(2) Find the value of:

- |   |                                    |                                    |
|---|------------------------------------|------------------------------------|
| (a) $2 + 0$                                     | (b) $1 \times -10$                 | (c) $6 \times 1$                   |
| (d) $-5 \div 0$                                 | (e) $-5 + 0$                       | (f) $(7) + (-7)$                   |
| (g) $0 \div 3$                                  | (h) $8 \div 1 + 0$                 | (i) $\frac{6}{5-5}$                |
| (j) $-20 \times -3 \times 0$                    | (k) $(6 \times 0) + (-5 \times 0)$ | (l) $(3 \div 3) \times (0 \div 3)$ |
| (m) $\frac{0 \times 8}{3-1} \times \frac{2}{1}$ | (n) $0 + 1 + 2 \times 0$           | (o) $-11 + 11$                     |

(3) For which values of  $x$  will the following be undefined?

- |                   |                      |                       |                      |
|-------------------|----------------------|-----------------------|----------------------|
| (a) $\frac{7}{x}$ | (b) $\frac{-3}{x-1}$ | (c) $\frac{0+1}{x+1}$ | (d) $\frac{-8}{4+x}$ |
|-------------------|----------------------|-----------------------|----------------------|

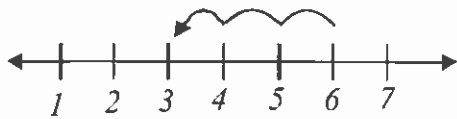
(4) Find the value of  $y$  in each of the following mathematical sentences:

- |                    |                    |
|--------------------|--------------------|
| (a) $y + 2 = 0$    | (c) $-11 + y = 0$  |
| (b) $(-4) + y = 0$ | (d) $y + (+7) = 0$ |

**A1.12 The addition of integers:**

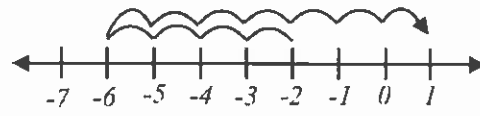
(Make use of a number line, if necessary.)

E.g.11  $6 + (-3)$



$\therefore \text{Answer} = \underline{3}$

E.g.12  $(-2) + (-4) + (7)$



$\therefore \text{Answer} = \underline{1}$

**Exercise 12:**

(1) Find the value of:

(a)  $(-5) + (6)$

(b)  $(-2) + (-4)$

(c)  $(7) + (3)$

(d)  $(-11) + (2)$

(e)  $(5) + (-8)$

(f)  $(-4) + (0)$

(g)  $(18) + (-7)$

(h)  $(-1) + (-1)$

(i)  $(9) + (-9)$

(j)  $(13) + (2)$

(k)  $(7 + (-7)) + (-2)$

(l)  $(-3) + 2 + (-6)$

(m)  $(-1) + (-2) + (-3)$

(n)  $4 + (-8) + 17$

(o)  $3 + 5 + 2 + 1$

(p)  $(-10) + (-6) + 2$

(q)  $(8) + (-5) + (-6)$

(r)  $-6 + 7 + 3 + 1$

(s)  $-1 + (-1) + 1 + 1$

(t)  $12 + (-12) + (-3)$

(2) Find the sum of each of the following:

(a) 3 and -8

(b) -2 ; 5 and -7

(c) 4 ; -5 and -1

(d) -2 ; -15 ; -1 and 4

(e) -2 ; -3 and 6

(f) 17 ; -5 ; 7 and -3

☺ Complete, by writing the next three numbers in the sequence:

(a) 2 ; 5 ; 8 ; 11 ; \_\_\_\_\_

(b) 1 ; 4 ; 9 ; 16 ; \_\_\_\_\_

(c) -17 ; -11 ; -6 ; -2 ; \_\_\_\_\_

(d) -2 ; -3 ; -5 ; -8 ; \_\_\_\_\_

**A1.13 Subtraction of integers:**

(Make use of a number line, if necessary.)

E.g.13  $(7) - (+11)$

$= (7) + (-11)$  [Remember that -11 is the additive inverse of +11.]

$= \underline{-4}$  [Use a number line to check your answer.]

E.g.14  $(-2) - (-9)$

$= (-2) + (+9)$

$= \underline{7}$

E.g.15  $6 - 3 + 7 - (-5)$

$= 6 + (-3) + 7 + (+5)$

$= 6 + 7 + 5 + (-3)$

$= \underline{15}$

Exercise 13:

(1) Calculate:

(a)  $(-6) - (9)$

(b)  $(-9) - (-9)$

(c)  $(5) - (-1) - (3)$

(d)  $13 - (-4)$

(e)  $11 - (-3) - (8)$

(f)  $(11) - (-6)$

(g)  $-4 - (-4) - 6$

(h)  $(-3) - (5)$

(i)  $16 - 12 - 4 - 1$

(j)  $(-6) - (-2) - (5)$

(k)  $(-2) - (-7) - (3)$

(l)  $-6 - (-2) - 5 - (-1)$

(m)  $(7) + (5) - (-2)$

(n)  $6 + 7 - (-3) - 8$

(o)  $(-15) - (-7) + (-3)$

(p)  $(-3) - (-2) + (2)$

(2) Subtract 11 from 6.

(3) Subtract -5 from -1.

(4) Subtract -11 from 2 and then add 6.

(5) Calculate the sum of 3 and -5 and subtract 2.

(6) Subtract the sum of -4 and -3 from the sum of 6 and -5.

☺ Complete, by writing the next six numbers in the sequence:

(a) 1 ; -2 ; -5 ; \_\_\_\_\_

(b) 1 ; 0 ; 2 ; -1 ; 3 ; -2 ; \_\_\_\_\_

**A1.14 Multiplication of integers:****Remember!**

$+$   $\times$   $+$  =  $+$

$+$   $\times$   $-$  =  $-$

$-$   $\times$   $-$  =  $+$

$-$   $\times$   $+$  =  $-$

*E.g. 16 (a)  $(-6) \times (-3) = +18$  because  $- \times - = +$  and  $6 \times 3 = 18$* *(b)  $(5)(-11) = (5) \times (-11) = -55$  because  $+ \times - = -$  and  $5 \times 11 = 55$* *(c)  $(-4) \times (-3) \times (-2) = -24$  because  $- \times - \times - = -$  and  $4 \times 3 \times 2 = 24$* Exercise 14:

Calculate:

(1)  $(-2) \times (5)$

(2)  $(6) \times (-7)$

(3)  $(-4) \times (-7)$

(4)  $(-9) \times (0)$

(5)  $(11) \times (3)$

(6)  $(-4)(-6)(2)$

(7)  $(12) \times (-6)$

(8)  $(-20) \times (3)$

(9)  $(-1) \times (-7)$

(10)  $(14) \times (4)$

(11)  $(-2) \times (-5) \times (6) \times (1)$

(12)  $(-1) \times (-1) \times (-1) \times (-1) \times (-1)$

(13)  $(9) \times (-12) \times (0) \times (18) \times (67) \times (-16)$

(14)  $(-3)(-6)(3)$

(15)  $(-1)(-1)(-1)(-1)(-2)(-1)(-1)(-1)(-1)$

(16)  $(9) \times (-1) \times (-2) \times (1) \times (2)$

☺ Calculate: (a)  $(-1)^{100} \times (-2)^5$   
 (b)  $\{-2[-3(2 \times -1)]\}$

### A1.15 Division of integers:

**Remember!**

$$\begin{array}{l} + \div + = + \\ + \div - = - \\ - \div - = + \\ - \div + = - \end{array}$$

E.g. 17 (a)  $\frac{-14}{-7} = \underline{+2}$  because  $- \div - = +$  and  $14 \div 7 = 2$

(b)  $100 \div -20 = \underline{-5}$  because  $+ \div - = -$  and  $100 \div 20 = 5$

#### Exercise 15:

Calculate the following:

(1)  $15 \div (-3)$

(2)  $(-55) \div (-11)$

(3)  $(144) \div 12$

(4)  $\frac{35}{-7}$

(5)  $(-22) \div (0)$

(6)  $(-24) \div (-8)$

(7)  $\frac{64}{4}$

(8)  $0 \div (14)$

(9)  $\frac{-48}{8} \div -3$

(10)  $-28 \div -7$

(11)  $\frac{-20-2}{11}$

(12)  $-88 \div -2 \div -4$

(13)  $(36) \div (-3)$

(14)  $\frac{-27}{-9}$

(15)  $(-40) \div (-2) \div (-4)$

(16)  $64 \div 8$

### A1.16 Combination of operations:

Order of operations: See p.7

E.g. 18 (a)  $(4) \times (-3) + (-8) \div (-2)$   
 $= (-12) + (+4)$   
 $= \underline{-8}$

(b)  $(-3 + 5 \times 2)^2$   
 $= (-3 + 10)^2$   
 $= (7)^2$   
 $= \underline{49}$

$$(c) \quad (-4)^3 \div [(8) \times (-8)]$$

$$= (-4)(-4)(-4) \div [-64]$$

$$= (-64) \div (-64)$$

$$= \underline{1}$$

$$(d) \quad \frac{(-11)(2 - 6)}{(-2)(-1)}$$

$$= \frac{(-11)(-4)}{+2}$$

$$= \frac{+44}{+2}$$

$$= \underline{22}$$

### Exercise 16:

Calculate:

$$(1) \quad [15 - (-2)] \times (-3)$$

$$(2) \quad -11 - (-16 \div 4) + 3$$

$$(3) \quad [4 + (-3)]^3 + 3^2$$

$$(4) \quad 4 \times 0 + (-3)(1) - 15(0)$$

$$(5) \quad -4 \times (25 \div -5) \div 2$$

$$(6) \quad [-3 \times (-8) \div (-4)] \times 2 - (-7)$$

$$(7) \quad \sqrt{(-8)(-2) + 18} \div 2$$

$$(8) \quad -50 \times 3 \div (-5) + (-1 - 2)$$

$$(9) \quad -37 \div 37$$

$$(10) \quad 14 \div 1 + 0 \times -6$$

$$(11) \quad \frac{36}{6} - \frac{-42}{-2}$$

$$(12) \quad \frac{(-14)(7)}{50 - 1}$$

$$(13) \quad (5 - 3)^3 - (-4 - 2) + 0 \times 16$$

$$(14) \quad 17 - [-7 + 8 - (-3)]$$

$$(15) \quad -5 - [2 \times (-4)] - (-1)$$

$$(16) \quad 3[15 \div (-3)] \times (-2)$$

$$(17) \quad \frac{60 \div 15}{12 - 16}$$

$$(18) \quad \sqrt[3]{(13 - 9) \times (-10 \div -5)}$$

$$(19) \quad (-3)^3 + 2[3 - 6]$$

$$(20) \quad (3)^3 - (-2)^2 + (-1)^7$$

$$(21) \quad \sqrt{(-1 - 2)^2 + [1 - (-3)]^2}$$

$$(22) \quad \frac{5 - (-3) + 2^2}{(2)(-3)}$$

- ☺ Twelve people owe the bank a total amount of R12 430. They have paid back R620. A further loan of R3 730 is granted to them. If they are all responsible for the loan, how much does each person owe?

### A1.17 Substitution:

E.g. 19 If  $a = -1$  and  $b = 3$ , Calculate the value of the following:

$$\begin{aligned} (a) \quad ab \\ &= (-1)(3) \\ &= \underline{-3} \end{aligned}$$

$$\begin{aligned} (b) \quad (b - a)^2 \\ &= [(3) - (-1)]^2 \\ &= [3 + 1]^2 \\ &= [4]^2 \\ &= \underline{16} \end{aligned}$$

$$\begin{aligned} (c) \quad a^3 + 4 \\ &= (-1)^3 + 4 \\ &= -1 + 4 \\ &= \underline{3} \end{aligned}$$

#### Exercise 17:

(1) Calculate the value of the following expressions, if  $x = -2$  ;  $y = 4$  and  $z = 5$ :

$$(a) \quad xyz$$

$$(b) \quad x^2 - y^2$$

$$(c) \quad (x - y)^2$$

$$(d) \quad x^2y + zy + z$$

$$(e) \quad \frac{yz}{x}$$

$$(f) \quad z - x$$

(2) If  $a = 0$  ;  $b = -3$   $c = -1$  and  $d = 7$ , calculate:

$$(a) \quad 2(a + b + d)$$

$$(b) \quad abcd$$

$$(c) \quad c^3 + b^2 - d$$

$$(d) \quad 3b + 2c - 5a$$

$$(e) \quad (d - c)^2$$

$$(f) \quad \sqrt{-bd - 4c}$$

- ☺ The speed ( $v$ ) of a motor after  $t$  seconds is determined by the formula:  $v = u + at$ . If  $u = -3$  and  $a = -5$ , determine the speed of the motor after 6 seconds.

### A1.18 REVISION EXERCISE:

(1) Tabulate:

- (a) the factors of 30.
- (b) the prime factors of 30.
- (c) the smallest multiple of 30.
- (d) the factors of 30 which are perfect squares.

(2) (i) Determine the prime factors of:

- |        |        |        |
|--------|--------|--------|
| (a) 25 | (b) 40 | (c) 60 |
| (d) 12 | (e) 30 | (f) 98 |
| (g) 35 | (h) 64 | (i) 90 |

(ii) Use (i) and calculate the following:

- (a) The HCF of 25; 40 and 60.  
 (b) The LCM of 12; 30 and 64.  
 (c) The HCF and the LCM of 30; 35 and 90.

(3) Calculate the following, without using a calculator. If necessary, use prime factors:

- |   |  |
|---|--|
| (a) $\sqrt{1\ 296}$                       | (b) $\sqrt{144} \div \sqrt{9}$             |
| (c) $\sqrt[3]{(1 + 26)^3}$                | (d) $(4 + 2)^2 - 3^3$                      |
| (e) $\sqrt{\sqrt{6 \times 18 \times 12}}$ | (f) $\sqrt{1\ 089}$                        |
| (g) $1^2 + 7^2 - \sqrt[3]{64}$            | (h) $\sqrt{10^2 + 3^2 + 3 \times 2^2}$     |
| (i) $\sqrt[3]{35\ 937}$                   | (j) $\sqrt{23 + \sqrt[3]{8}} + \sqrt{144}$ |

(4) Calculate the following:

- |                                     |                                      |
|-------------------------------------|--------------------------------------|
| (a) $5 - (28 \div 7) + (-3)$        | (b) $18 \times 0 \div 3$             |
| (c) $\frac{-12}{3} + \frac{18}{-9}$ | (d) $\frac{(-12)(3)}{1 - 10}$        |
| (e) $-11 + 3 \times 2 - (6)$        | (f) $5 \div (-1) + 0 \div 17$        |
| (g) $[-16 \div (-4)] + [12 - (-2)]$ | (h) $(2 - 3)(4) - 5(2 \times 3 - 6)$ |
| (i) $12 - [-6 + (-3) - 4]$          | (j) $(4 - 7)^2 - 3 \times 5$         |
| (k) $6 + 3 - 3 + 2 \times 0$        | (l) $18 \div -6 + 3 \div -1$         |

(5) Subtract 14 from -3.

(6) Subtract the sum of 2 and -4 from 16.

(7) Find the sum of 6 ; -11 and 7.

(8) Determine the product of 5 and -6.

(9) Determine the difference between 8 and 3.

(10) Subtract 5 from 13 and then add -3.

(11) Calculate the value of the following expressions if  $m = 3$  ;  $n = -1$  and  $p = 5$ :

(a)  $nm - p$

(b)  $p^2 - 3m$

(c)  $(m - n)(m + n)$

(d)  $\frac{m + p}{2n}$

(e)  $3(n - m)$

(f)  $m + n + p$

(g)  $m^3 - 3m - 3$

(h)  $n(m + p) - m(p - n)$

(i)  $m^2 + 2n^2 - p^2$

(j)  $4 \div \sqrt{mp - n}$

(12) Are the following statements true or false?

- (a) All integers are also natural numbers.
- (b) The largest multiple of 12 is 144.
- (c) The smallest prime number is 1.
- (d) The smallest whole number is 0.
- (e) 1 is the identity element of multiplication.

(13) Insert  $<$  ;  $>$  or  $=$  in the middle column in order to make each statement true:

(a)	$(-7) + (-3)$		$(-7) - (-3)$
(b)	$(-2) + 6 - (18)$		$14 - 7 + (-8)$
(c)	$22 - 40 + 12$		$22 - 40 - 12$
(d)	$40 + (-23) - (14)$		$37 + (-51) - (-35)$
(e)	$-331 + 82 + 58$		$-499 + 158 + 82$

\*\*\*\*\*



## Chapter A2

### Number patterns

#### A2.1 Completing number patterns:

*E.g.1 Complete the next four numbers in the following sequences:*

(a)  $-3 ; -7 ; -11 ; \dots\dots\dots$

(b)  $2 ; 4 ; 8 ; 16 ; \dots\dots\dots$

\*\*\*\*\*

(a)  $-3 ; -7 ; -11 ; \underline{-15} ; \underline{-19} ; \underline{-23} ; \underline{-27}$

(b)  $2 ; 4 ; 8 ; 16 ; \underline{32} ; \underline{64} ; \underline{128} ; \underline{256}$

#### Exercise 1:

Complete the next three numbers in the following sequences:

- (1)  $3 ; 6 ; 9 ; 12 ; \dots\dots$
- (2)  $-1\ 000\ 000 ; -100\ 000 ; -10\ 000 ; \dots\dots$
- (3)  $1 ; 1 ; 2 ; 3 ; 5 ; 8 ; \dots\dots$
- (4)  $44 ; 39 ; 34 ; 29 ; \dots\dots$
- (5)  $1 ; 4 ; 9 ; 16 ; 25 ; \dots\dots$
- (6)  $103 ; 105 ; 107 ; 109 ; \dots\dots$
- (7)  $1 ; -2 ; 4 ; -8 ; \dots\dots$
- (8)  $3 ; 4 ; 6 ; 9 ; 13 ; \dots\dots$
- (9)  $-2 ; 2 ; -2 ; 2 ; \dots\dots$
- (10)  $300 ; 600 ; 1\ 200 ; 2\ 400 ; \dots\dots$
- (11)  $1\ 009 ; 1\ 007 ; 1\ 005 ; \dots\dots$
- (12)  $3 ; 5 ; 9 ; 15 ; 23 ; \dots\dots$

*E.g.2 Complete the next four numbers in the following sequences and write the pattern in words:*

(a)  $8 ; 12 ; 16 ; 20 ; \dots\dots\dots$

(b)  $3 ; -6 ; 12 ; -24 ; \dots\dots\dots$

\*\*\*\*\*

(a)  $8 ; 12 ; 16 ; 20 ; \underline{24} ; \underline{28} ; \underline{32} ; \underline{36}$

*Add 4.                    or                    The multiples of 4 greater and equal to 8.*

(b)  $3 ; -6 ; 12 ; -24 ; \underline{48} ; \underline{-96} ; \underline{192} ; \underline{-384}$

*Multiply with -2.*

Exercise 2:

Complete the next five numbers in the following sequences and write the pattern in words:

- (1) 17 ; 20 ; 23 ; 26 ; .....
- (2) -10 ; -11 ; -13 ; -16 ; .....
- (3) 1 ; 3 ; 9 ; 27 ; .....
- (4) 600 ; 800 ; 1 000 ; 1 200 ; .....
- (5) 3 ; 6 ; 10 ; 15 ; 21 ; .....
- (6) -12 ; -9 ; -6 ; .....
- (7) 1 ; 8 ; 27 ; 64 ; .....
- (8) 2 ; 3 ; 5 ; 7 ; 11 ; .....
- (9) -3 ; 9 ; -27 ; 81 ; .....
- (10) 144 ; 121 ; 100 ; 81 ; .....

**A2.2 Tables:**

*E.g.3 Complete the table by applying the given relationship.*

	<b>5</b>				<b>11</b>			
<i>n</i>	1	2	3		7	8		20
$n^2 + 2$	3	6	11	27			123	
					<b>51</b>	<b>66</b>		<b>402</b>

*E.g.4 Write the die relationship between the numbers in these two rows and complete the table.*

	<b>6</b>				<b>9</b>			
<i>n</i>	1	2	3		7	8		20
$T_n$	2	4	6	12			18	
					<b>14</b>	<b>16</b>		<b>40</b>

*Relationship / Rule:  $T_n = 2n \therefore n$  is multiplied by 2 each time to obtain the value of  $T_n$ !*

*E.g.5 Complete the following table and answer the questions below:*

(a)

<i>Position</i>	1	2	3	4		6	8	11
<i>Term</i>	4	7	10		16			

- (b) Describe the pattern of the sequence in words.
- (c) If the position of the term in the sequence is represented by *n*, write the general term in terms of *n*.
- (d) Determine the 100<sup>th</sup> term.

\*\*\*\*\*

(a)

	<b>5</b>							
<i>Position</i>	1	2	3	4		6	8	11
<i>Term</i>	4	7	10		16			
				<b>13</b>		<b>19</b>	<b>25</b>	<b>34</b>

- (b) Add 3.
- (c) General term ( $T_n$ ) =  $3n + 1$ . The 3 is the constant difference which is added each time. The constant 1 is obtained by checking the terms  $\rightarrow T_1 = 3(1) = 3 + 1 = 4$
- (d) For 100<sup>th</sup> term, substitute *n* with 100:  $\therefore$  Term 100:  $T_{100} = 3(100) + 1 = 300 + 1 = \underline{301}$

Exercise 3:

(1) Redraw the following tables and complete:

(a) 

$n$	1	2	3	4		8	9
$5n$	5	10			30		

(b) 

$m$	1	2	3			20	31
$m - 4$	-3	-2		12	14		

(c) 

$t$	1	2	3		7		12
$2t - 1$	1	3		11		21	

(d) 

$k$	-3	-2	-1	0	1	2	3
$k + 2$	-1	0	1				

(e) 

$n$	2	4	6	7	50		92
$n \div 2$	1	2	3			34	

(f) 

$y$	1	2	3		8	9	
$y + 11$	12			15			36

(g) 

$x$	1	2	3	4		8	9
$x^3$	1	8			125		

(h) 

$n$	1	2	3				7
$-n^2$			-9	-16	-36		

(i) 

$n$	-3	-2	0		12		50
$7 - n$	10	6		6		-6	

(j) 

$n$	2	4		8		12	
$-3 \times n$	-3		-18		-33		

(2) Write the relationship between  $x$  and the number in the second row and complete the tables:

(a) 

$x$	1	2	3	4	5	6	7
?	4	8	12	16			

(b) 

$x$	1	2	3	5	7	11	20
?	-3		-1		3		16

(c) 

$x$	1	2	6	7		11	12
?	5	10	30		45		

(d) 

$x$	1	2	3	4	5	6	
?	3	5	7				17

(e) 

$x$	1	2	3	4	5		
?	-1	-8	-27				-1000

(f) 

$x$	1	3	5	7			30
?	-7	-9	-11		-15	-20	

(g) 

$x$	1	2	3			8	9
?	5	8	11	14	20		

(h) 

$x$	1	2	3	4	5	7	11
?	2	5	10	17			

(i) 

$x$	1	2	3	4	5	8	17
?	-2	-6	-10				

(j) 

$x$	1	2	3	4	6		11
?	-3	-6	-9	-12		-21	

(3) Complete the table and answer the questions below:

(a) 

Position	1	2	3	4		10	15	20
Term	6	8	10		18			

(b) Describe the pattern of the sequence in words.

(c) If the position of the term is represented by  $n$ , write the general term in terms of  $n$ .(d) Determine the 50<sup>th</sup> term.

(4) Complete the table and answer the questions below:

(a)	Position	1	2	3	4		10	15	20
	Term	-3	-8	-13		-33			

(b) Describe the pattern of the sequence in words.

(c) If the position of the term is represented by  $n$ , write the general term in terms of  $n$ .

(d) Determine the 80<sup>th</sup> term.

### A2.3 Other number patterns:

#### Exercise 4:

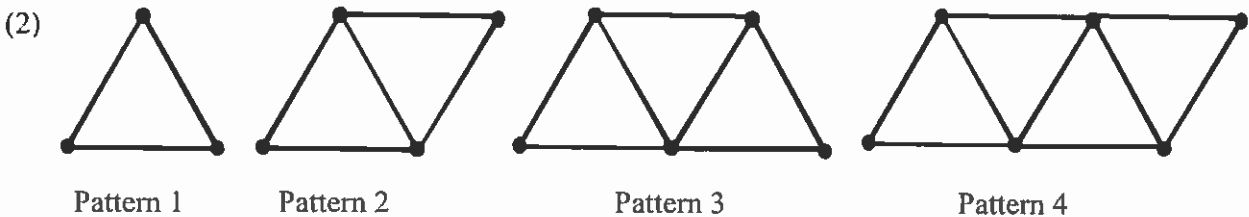
(1) (a) Complete the next three numbers in the following sequences:  
[Pay specific attention to the difference between the terms.]

(i) 3 ; 6 ; 11 ; 18 ; 27 ; .....

(ii) 1 ; 6 ; 15 ; 28 ; 45 ; .....

(iii) 2 ; 2 ; 0 ; -4 ; -10 ; .....

(b) What is the similarity between all the sequences in (a)?



(a) Use the patterns above and complete the following table:

Pattern	1	2	3	4	5	6	7
Number of lines							
Number of dots							

(b) Describe the sequences for the lines and dots in words.

(3) Consider the following number pattern:

$$\begin{aligned}
 11^2 &= 121 \\
 111^2 &= 12321 \\
 1111^2 &= 1234321 \\
 11111^2 &= \text{(i)} \\
 111111^2 &= \text{(ii)}
 \end{aligned}$$

(a) Complete (i) by using a calculator.

(b) Determine the value of (ii) without using a calculator. Check your answer with a calculator.