

Graad 10 – Boek C **(Hersiene KABV uitgawe)**

INHOUD:

	<u>Bladsy:</u>
C1. Trigonometrie	3
C2. Euklidiese meetkunde	49
C3. Analitiese meetkunde	123
C4. Buite-oppervlaktes en volumes	159

Hierdie boek is opgestel en verwerk deur E.J. Du Toit in 2011.

Hersiene uitgawe 2023.

Webtuiste: www.abcbooks.co.za

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ISBN 978-1-919957-07-4

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Hoofstuk C1

Trigonometrie

C1.1 Inleiding tot Trigonometrie:

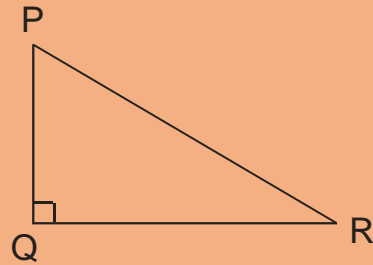
Trigonometrie is die studie van die verhouding tussen die hoeke en sye van driehoeke. In graad 9 het ons reeds gelykvormigheid bestudeer. Gelykvormige driehoeke is driehoeke waarvan al drie pare ooreenstemmende hoeke gelyk is aan mekaar óf as die ooreenstemmende pare sye eweredig is (in dieselfde verhouding is.) Gelykvormige driehoeke het dus dieselfde vorm, maar nie noodwendig dieselfde grootte nie!

Terme: In 'n reghoekige driehoek word die sye en hoeke as volg benoem:

PR is die skuinssy (s).

PQ is die teenoorstaande (t) sy van \hat{R} .

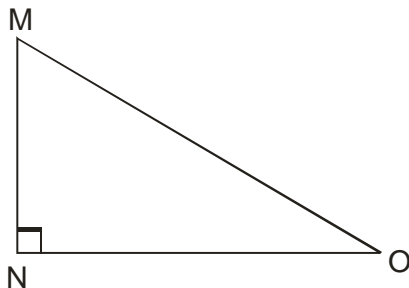
QR is \hat{R} se aangrensende (a) sy.



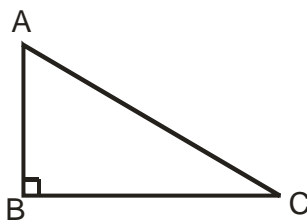
Oefening 1:

Datum: _____

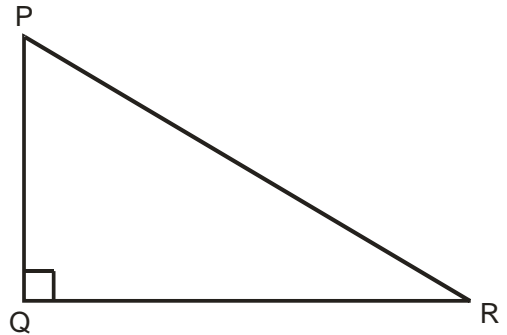
(1) Driehoek 1:



Driehoek 2:



Driehoek 3:



Meet die lengte van elke sy en die grootte van elke hoek van al die driehoeke en vervang dit as volg: (Rond af tot 2 des.)

Driehoek 1:

(a) $\frac{MN}{OM} = \text{---} \approx \text{---}$

(b) $\frac{ON}{OM} = \text{---} \approx \text{---}$

(c) $\frac{MN}{NO} = \text{---} \approx \text{---}$

Driehoek 2:

$\frac{AB}{AC} = \text{---} \approx \text{---}$

$\frac{BC}{AC} = \text{---} \approx \text{---}$

$\frac{AB}{BC} = \text{---} \approx \text{---}$

Driehoek 3:

$\frac{PQ}{PR} = \text{---} \approx \text{---}$

$\frac{QR}{PR} = \text{---} \approx \text{---}$

$\frac{PQ}{QR} = \text{---} \approx \text{---}$

- (2) (a) Wat merk jy van die hoeke van die 3 driehoeke in (1)? _____
 (b) Wat is die verband tussen ΔMNO , ΔABC en ΔPQR ? _____
 (c) Wat merk jy i.v.m. die verhoudings van die ooreenstemmende sye soos gemeet in nr.1 a – c?

(3) Gebruik die figuur regs en voltooi die volgende:

(a) In ΔABC en ΔADE en ΔAFG :

Bewering:

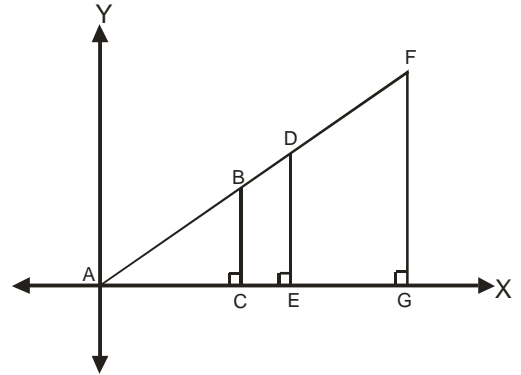
Rede:

* $\hat{A} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ [_____]

* $\hat{C} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ [_____]

* $\hat{B} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ [_____]

$\therefore \Delta ABC \text{ /// } \Delta \underline{\hspace{2cm}} \text{ /// } \Delta \underline{\hspace{2cm}}$ [_____]



(b) Uit (a) kan ons aflei dat: $\frac{AB}{AC} = \frac{AD}{AE} = \frac{AF}{AG}$ [Gelykvormige driehoeke]

Net so is: $\frac{AB}{BC} = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$ en $\frac{BC}{AC} = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

Uit oefening 1 het ons gesien dat die verhoudings van die sye van gelykvormige driehoeke dieselfde is. Hierdie verhouding van die sye hang dus af van die grootte van die driehoeke se hoeke.

Elk van die verskillende pare ooreenstemmende sye word as volg benoem:
 θ word die inklinasiehoek genoem en elk van die volgende verhoudings is dus afhanklik van θ !

Die sinusverhouding:

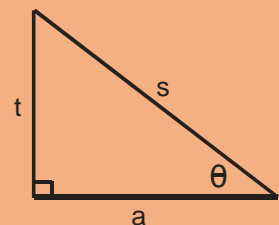
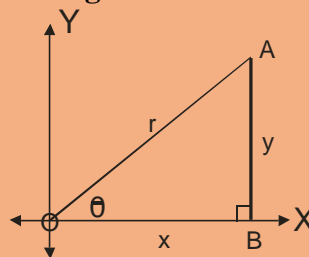
$$\sin \theta = \frac{AB}{OA} \left[\frac{\theta \text{ se teenoorstaande sy}}{\text{die skuinssy}} \right] = \frac{y}{r} = \frac{t}{s}$$

Die cosinusverhouding:

$$\cos \theta = \frac{OB}{OA} \left[\frac{\theta \text{ se aangrensende sy}}{\text{die skuinssy}} \right] = \frac{x}{r} = \frac{a}{s}$$

Die tangensverhouding:

$$\tan \theta = \frac{AB}{OB} \left[\frac{\theta \text{ se teenoorstaande sy}}{\theta \text{ se aangrensende sy}} \right] = \frac{y}{x} = \frac{t}{a}$$

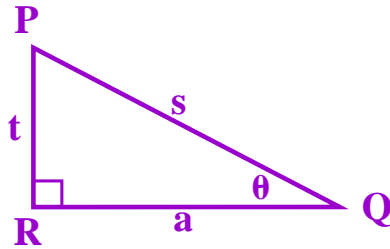


Vb.1 Skryf die volgende in terme van die sye van die driehoek:

(a) $\sin \theta = \frac{t}{s} = \frac{PR}{PQ}$

(b) $\cos \theta = \frac{a}{s} = \frac{RQ}{PQ}$

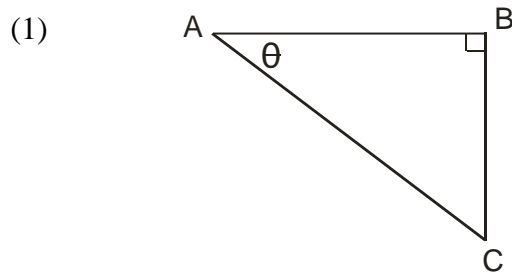
(c) $\tan \theta = \frac{t}{a} = \frac{PR}{RQ}$



Oefening 2:

Datum: _____

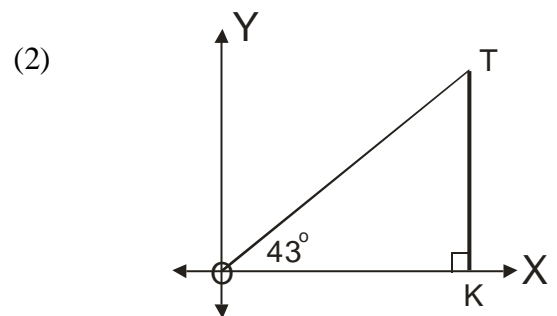
Skryf die volgende in terme van die sye van die gegewe driehoek:



(a) $\sin \theta = \underline{\hspace{2cm}}$

(b) $\cos \theta = \underline{\hspace{2cm}}$

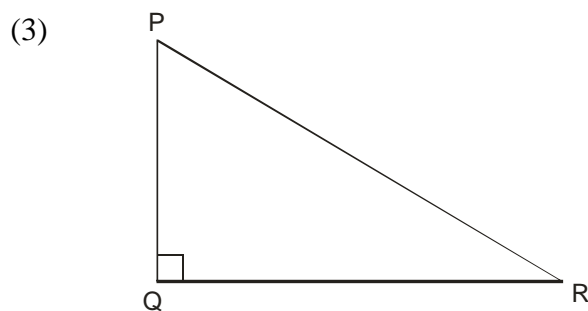
(c) $\tan \theta = \underline{\hspace{2cm}}$



(a) $\cos 43^\circ = \underline{\hspace{2cm}}$

(b) $\tan 43^\circ = \underline{\hspace{2cm}}$

(c) $\sin 43^\circ = \underline{\hspace{2cm}}$

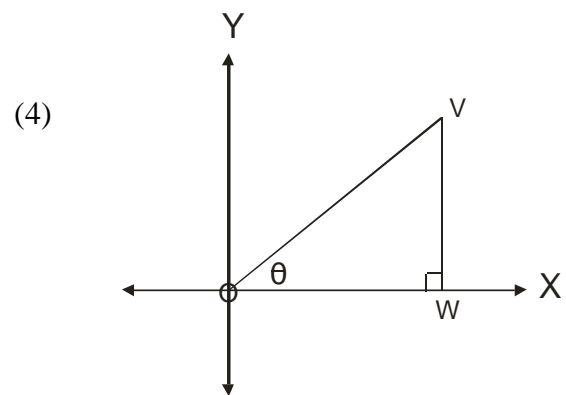


(a) $\sin \hat{R} = \underline{\hspace{2cm}}$

(b) $\cos \hat{R} = \underline{\hspace{2cm}}$

(c) $\tan \hat{R} = \underline{\hspace{2cm}}$

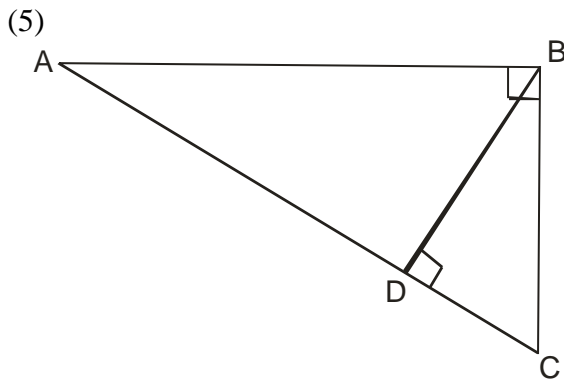
(d) $\sin \hat{P} = \underline{\hspace{2cm}}$



(a) $\cos \theta = \underline{\hspace{2cm}}$

(b) $\tan \theta = \underline{\hspace{2cm}}$

(c) $\sin \theta = \underline{\hspace{2cm}}$



(a) $\sin \hat{C}$ in $\Delta ABC =$ _____

(b) $\cos \hat{A}$ in $\Delta ABD =$ _____

(c) $\tan \hat{B}$ in $\Delta ABD =$ _____

(d) $\cos \hat{B}$ in $\Delta BDC =$ _____

(e) $\sin \hat{C}$ in $\Delta BDC =$ _____

(f) $\tan \hat{A}$ in $\Delta ABC =$ _____

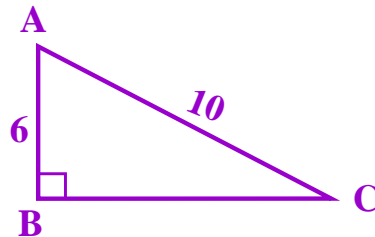
C1.2 Gebruik van Pythagoras:

Vb.2 Bereken die volgende verhoudings:

(a) $\sin \hat{A}$

(b) $\tan \hat{A}$

(c) $\cos \hat{C}$



Bereken eers die lengte van BC deur gebruik te maak van die stelling van Pythagoras.

$$AC^2 = AB^2 + BC^2$$

$$10^2 = 6^2 + BC^2$$

$$100 = 36 + BC^2$$

$$100 - 36 = BC^2$$

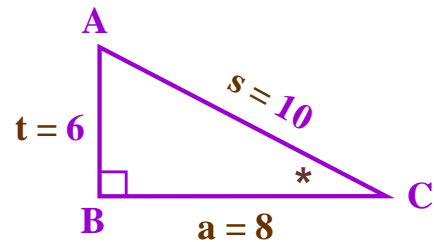
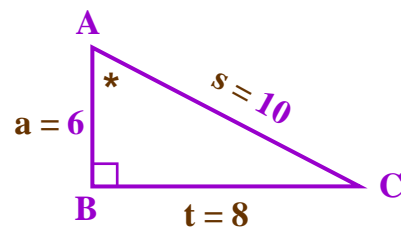
$$\therefore BC^2 = 64$$

$$\therefore BC = 8$$

$$\therefore \text{(a) } \sin \hat{A} = \frac{t}{s} = \frac{8}{10} = 0,8$$

$$\therefore \text{(b) } \cos \hat{A} = \frac{a}{s} = \frac{6}{10} = 0,6$$

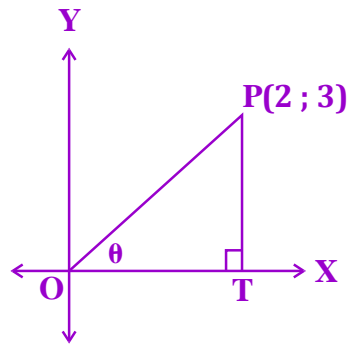
$$\therefore \text{(c) } \tan \hat{C} = \frac{t}{a} = \frac{6}{8} = 0,75$$



Vb.3 Bereken die volgende verhoudings:

(a) $\cos \theta$

(b) $\tan \theta$



Bereken eers weer die lengte van OP met behulp van die stelling van Pythagoras.

$$\therefore OP^2 = OT^2 + PT^2$$

$$OP^2 = 2^2 + 3^2$$

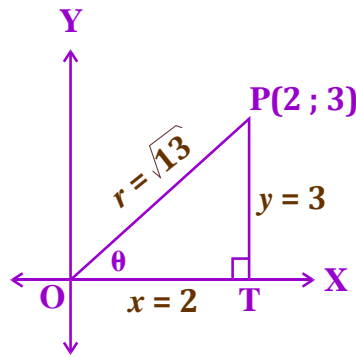
$$OP^2 = 4 + 9$$

$$OP^2 = 13$$

$$OP = \sqrt{13}$$

$$\therefore \text{(a) } \cos \theta = \frac{x}{r} = \frac{2}{\sqrt{13}}$$

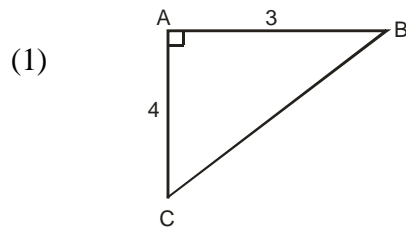
$$\therefore \text{(b) } \tan \theta = \frac{y}{x} = \frac{3}{2}$$



Oefening 3:

Datum: _____

Bereken:



(a) die lengte van BC.

(b) $\sin \hat{B}$

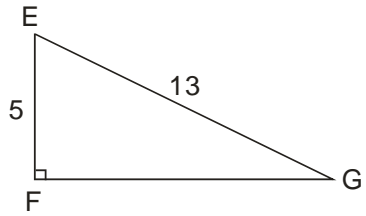
(c) $\tan \hat{B}$

(d) $\cos \hat{B}$

(e) $\cos \hat{C}$

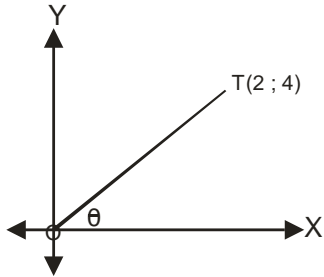
(f) $\sin \hat{C}$

(2)



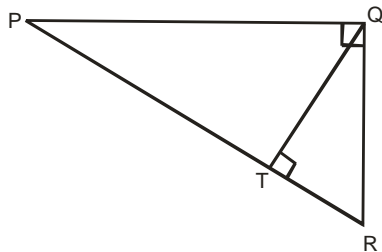
- (a) $\cos \hat{E}$
- (b) $\sin \hat{G}$
- (c) $\tan \hat{E}$
- (d) $\tan \hat{G}$

(3)



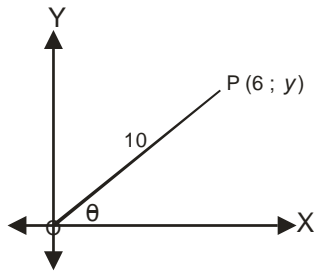
- (a) die lengte van OT.
- (b) $\cos \theta$
- (c) $\sin \theta$
- (d) $\tan \theta$

(4)



- As $PQ = 8$, $PT = 6,4$ en $TR = 3,6$; bereken:
- (a) die lengtes van QR en QT.
 - (b) $\cos R$ in ΔQTR
 - (c) $\tan P$ in ΔPQR
 - (d) $\sin Q$ in ΔPQT
 - (e) $\sin Q$ in ΔQTR

(5)



- (a) y
- (b) $\sin \theta$
- (c) $\cos \theta$
- (d) $\tan \theta$

C1.3 Gebruik van die sakrekenaar:

C1.3.1 Grade, minute en sekondes:

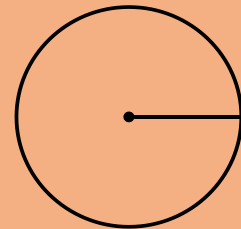
Indien opmeting gedoen word, word gebruik gemaak van afstande en hoekgroottes. Ons weet reeds dat 'n omwenteling, m.a.w. die volle draai van 'n sirkel, 360 grade (360°) is.

Elke graad is die hoek by die middelpunt van 'n sirkel wat die grootte van die boog beskryf wat dan 'n breukdeel van die omtrek van die sirkel voorstel.

$\therefore 1^\circ$ is gelykstaande aan $\frac{1}{360}$ ste van die omtrek van die sirkel.

Een minuut ($1'$) is gelykstaande aan $\frac{1}{60}$ ste van 'n graad.

Een sekonde ($1''$) is gelykstaande aan $\frac{1}{60}$ ste van 'n minuut.



Vb.4 (a) **Beskryf die volgende hoekgrootte: $13^\circ 24' 36''$**

13 grade, 24 minute en 36 sekondes.

(b) **Skakel die volgende om na grade alleenlik: $13^\circ 24' 36''$**

$$\begin{aligned}
 13^\circ 24' 36'' &= 13^\circ + 24' + \frac{36}{60}' = 13^\circ + 24' + 0,6' \\
 &= 13^\circ + 24,6' = 13^\circ + \frac{24,6}{60}^\circ \\
 &= 13^\circ + 0,41^\circ = \mathbf{13,41^\circ}
 \end{aligned}$$

(c) **Skakel die volgende om na grade en minute: $64,3^\circ$**

$$64,3^\circ = 64^\circ + 0,3^\circ = 64^\circ + (0,3 \times 60)' = \mathbf{64^\circ 18'}$$

Oefening 4:

Datum: _____

(1) Skakel die volgende om na grade alleenlik:

(a) $72^{\circ} 24'$

(b) $88^{\circ} 33'$

(c) $324^{\circ} 48'$

(d) $25^{\circ} 12' 36''$

(e) $112^{\circ} 36' 54''$

(f) $7^{\circ} 6' 18''$

(2) Skakel die volgende om na grade en minute:

(a) $38,5^{\circ}$

(b) $101,7^{\circ}$

(c) $16,45^{\circ}$

C1.3.2 Die sakrekenaar:**C1.3.2.1 Trigonometriese uitdrukkings:****ONTHOU:** Die sakrekenaar moet op “deg” wees!

Maak gebruik van 'n nie-programmeerbare, wetenskaplike sakrekenaar!

Vb.5 Bereken die volgende, korrek tot twee desimale:

<u>Uitdrukking:</u>	<u>Vertoon:</u>	<u>2 des. plekke:</u>	<u>Sleutels:</u>
(a) $\sin 12^{\circ}$	= 0,2079 ...	≈ 0,21	$\sin 12 =$
(b) $\cos 42^{\circ} 12'$	= 0,7408 ...	≈ 0,74	$\cos 42^{\circ} 12' =$
(c) $2 \tan 77^{\circ}$	= 8,6629 ...	≈ 8,66	$2 \tan 77 =$
(d) $\cos^2 44^{\circ}$	= 0,5174 ...	≈ 0,52	$\cos 44)^2$ of $(\cos 44)x^2 =$
(e) $4 - \tan 220^{\circ}$	= 3,1609 ...	≈ 3,16	$4 - \tan 220 =$
(f) $\frac{\sin 67^{\circ}}{3}$	= 0,3068 ...	≈ 0,31	$\sin 67 = \div 3 =$

Oefening 5:

Datum: _____

(1) Bereken die volgende, korrek tot twee desimale:

- | | |
|---------------------------------|----------------------------------|
| (a) $\sin 33^\circ =$ _____ | (b) $\cos 56^\circ =$ _____ |
| (c) $\tan 11,5^\circ =$ _____ | (d) $\sin 145^\circ =$ _____ |
| (e) $\sin 301^\circ =$ _____ | (f) $\cos 201^\circ 24' =$ _____ |
| (g) $\tan 88^\circ 56' =$ _____ | (h) $\cos 345^\circ =$ _____ |
| (i) $\sin 23,4^\circ =$ _____ | (j) $\tan 66^\circ 34' =$ _____ |
| (k) $\cos 64,1^\circ =$ _____ | (l) $\tan 6,6^\circ =$ _____ |
| (m) $\sin 12^\circ 12' =$ _____ | (n) $\cos 0,5^\circ =$ _____ |

(2) Bereken die volgende, korrek tot 1 desimaal: (Skryf jou sleutels neer!)

- | | |
|---|---|
| (a) $2 \sin 34^\circ =$
_____ | (b) $3,5 + \cos 200^\circ =$ _____
_____ |
| (c) $\tan^2 130^\circ =$
_____ | (d) $\frac{\cos 71^\circ}{2} =$ _____
_____ |
| (e) $\sin(32^\circ + 12^\circ) =$ _____
_____ | (f) $\cos 176^\circ - \cos 76^\circ =$ _____
_____ |
| (g) $\sqrt{\sin 16^\circ} =$ _____
_____ | (h) $\tan 100^\circ + 7,1 =$ _____
_____ |
| (i) $4 \div \sin 133^\circ 24' =$ _____
_____ | (j) $\sin^3 72,12^\circ =$ _____
_____ |
| (k) $7 + \frac{\tan 100^\circ}{2} =$ _____
_____ | (l) $\cos(4 \times 31,3^\circ) =$ _____
_____ |
| (m) $\sqrt{10 \cos 300^\circ} =$ _____
_____ | (n) $\sin 30^\circ \times \cos 30^\circ =$ _____
_____ |
| (o) $-7,1 - \sin 304^\circ =$ _____
_____ | (p) $1,6 - 2 \times \cos^2 123^\circ =$ _____
_____ |

C1.3.2.2 Trigonometriese vergelykings:

Ons het gesien dat bv. $\sin 30^\circ = 0,5$
 \therefore as $\sin x = 0,5$ kan ons aflei dat $x = 30^\circ$ as $x \in [0^\circ; 90^\circ]$

Vb.6 Los op vir x as $x \in [0^\circ; 90^\circ]$; korrek tot 1 desimaal:

(a) $\cos x = 0,34$

(b) $2 \tan x = 4,64$

(c) $\sin 3x = 0,7$

(a) $\cos x = 0,34$

$\therefore x \approx 70,1^\circ$

[Sleutels: Shift $\cos^{-1} 0.34 =$]

(b) $\tan x = \frac{4,64}{2}$

$\tan x = 2,32$

$\therefore x \approx 66,7^\circ$

[Sleutels: Shift $\tan^{-1} 2.32 =$]

(c) $\sin 3x = 0,7$

$\therefore 3x = 44,427 \dots$

$\therefore x \approx 14,8^\circ$

[Sleutels: Shift $\sin^{-1} 0.7 = \div 3 =$]

Oefening 6:

Datum: _____

Los op vir x as $x \in [0^\circ; 90^\circ]$; korrek tot 1 desimaal:

(1) $\sin x = 0,34$

(2) $\cos x = 0,551$

(3) $\tan x = 6,9$

(4) $\cos x = \frac{1}{2}$

(5) $\tan x = 44,4$

(6) $\sin x = 0,881$

(7) $\cos x = 0,401$

(8) $\sin x - 0,2 = 0$

(9) $\tan x = 2 \times 3$

(10) $4 \sin x = 0,1$

(11) $\tan 3x = 6$

(12) $\cos(x + 10^\circ) = 0,9$

(13) $\cos x + 2 = 2,444$

(14) $\tan^2 x = 0,64$

(15) $\frac{\sin x}{2} = 0,1$

(16) $\tan(x - 10^\circ) = 20$

(17) $\cos 3x = 0,688$

(18) $\cos x - 3 = -2,445$

(19) $-2,3 \tan x = -3,2$

(20) $\sin \frac{x}{2} = 0,5$

(21) $\frac{2}{3} \sin x = \frac{1}{2}$

(22) $\tan(2x - 15^\circ) = 2$

(23) $\frac{\cos 2x}{2} = 0,2$

(24) $\sin x = \tan 25^\circ$

C1.3.2.3 Kombinasies:

Vb.7 Bereken $5 \sin 2A$ as $3 + \tan A = 4,2$ en $A \in [0^\circ; 90^\circ]$. Rond A af korrek tot 1 desimaal en die antwoord korrek af tot 3 desimale.

$$\text{As } 3 + \tan A = 4,2$$

$$\therefore \tan A = 4,2 - 3$$

$$\tan A = 1,2$$

$$\therefore A = 50,2^\circ$$

$$\therefore 5 \sin 2A$$

$$= 5 \sin (2 \times 50,2^\circ)$$

$$= 5 \sin 100,4^\circ$$

$$\approx 4,918$$

Oefening 7:

Datum: _____

Rond alle hoeken af tot 1 desimaal en elke funktiewaarde af tot drie desimale!

(1) Bereken $\sin^2\theta$ as $2 \cos \theta = 0,31$ en $\theta \in [0^\circ; 90^\circ]$.

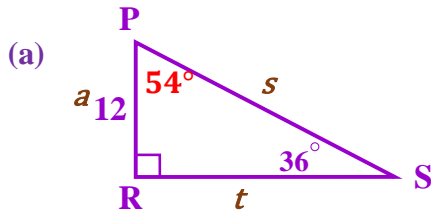
(2) As $-2 \tan A = -2$ en $0^\circ \leq A \leq 90^\circ$, bereken $\cos(A + 12^\circ)$.

(3) As $\cos 2x = 0,4$ en $x \in [0^\circ; 90^\circ]$, bereken $\cos^2 x + 3 \tan x$.

(4) Bereken $\frac{\sin \theta + \cos \theta}{-3,1}$ as $0^\circ \leq \theta \leq 90^\circ$ en $\tan(\theta - 25^\circ) = 2,1$.

C1.4 Oplos van reghoekige driehoeke:

Vb.8 Bereken die onbekende hoek en sye in elk van die volgende driehoeke:
Rond korrek af tot een desimaal!



$$* \hat{P} = 90^\circ - 36^\circ = 54^\circ$$

$$* \tan \hat{P} = \frac{t}{a}$$

$$\tan 54^\circ = \frac{RS}{12}$$

$$12 \tan 54^\circ = RS$$

$$\therefore 16,5 = RS$$

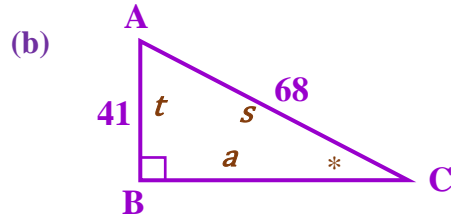
$$* PS^2 = PR^2 + RS^2$$

$$PS^2 = 12^2 + 16,5^2$$

$$PS^2 = 144 + 272,25$$

$$PS^2 = 416,25$$

$$PS = 20,4$$



$$* \sin \hat{C} = \frac{t}{s}$$

$$\sin \hat{C} = \frac{41}{68}$$

$$\sin \hat{C} = 0,602 \dots \dots$$

$$\therefore \hat{C} = 37,080 \dots \approx 37,1^\circ$$

$$* AC^2 = AB^2 + BC^2$$

$$68^2 = 41^2 + BC^2$$

$$4624 = 1681 + BC^2$$

$$4624 - 1681 = BC^2$$

$$2943 = BC^2$$

$$54,2 = BC$$

$$* \hat{A} = 90^\circ - 37,1^\circ = 52,9^\circ$$

Oefening 8:

Datum: _____

Los die volgende driehoeke op, korrek tot een desimaal:

(1)

