

Graad 10 – Handboek Antwoorde

(Hersiene KABV uitgawe)

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Hoofstuk A1

Getalle stelsels

GEEN SAKREKENAAR MAG IN HIERDIE HOOFSTUK GEBRUIK WORD NIE!

A1.1 Getallestelsels:

Oefening 1:

Voltooi:	*Natuurlike getalle:	\mathbb{N}	=	$\{1; 2; 3; \dots\}$
	* Telgetalle:	\mathbb{N}_0	=	$\{0; 1; 2; 3; \dots\}$
	* Heelgetalle:	\mathbb{Z}	=	$\{\dots; -2; -1; 0; 1; 2; 3; \dots\}$
	* Rasionale getalle:	\mathbb{Q}	=	$\left\{\frac{a}{b} / a, b \in \mathbb{Z}; b \neq 0\right\}$

A1.2 Rasionale getalle:

A1.2.1 Ekwivalente breuke:

Oefening 2:

Hierdie is moontlike antwoorde!

(1) Skryf drie ekwivalente breuke neer vir elk van die volgende rasionale getalle:

$$(a) \quad \frac{-1}{4} = \frac{-2}{8} = \frac{-3}{12} = \frac{-10}{40}$$

$$(b) \quad \frac{3}{7} = \frac{6}{14} = \frac{9}{21} = \frac{15}{35}$$

$$(c) \quad \frac{1}{6} = \frac{2}{12} = \frac{10}{60} = \frac{100}{600}$$

$$(d) \quad \frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{20}{30}$$

$$(e) \quad \frac{12}{14} = \frac{6}{7} = \frac{24}{28} = \frac{120}{140}$$

$$(f) \quad \frac{-36}{-9} = \frac{-4}{-1} = \frac{4}{1} = \frac{12}{3}$$

$$(g) \quad 2\frac{6}{11} = \frac{28}{11} = 2\frac{12}{22} = 2\frac{60}{110}$$

$$(h) \quad 5 = \frac{5}{1} = \frac{10}{2} = \frac{25}{5}$$

(2) Is die volgende ekwivalente breuke of nie? (Antwoord net ja of nee.)

$$(a) \quad \frac{12}{5} = \frac{24}{10} ? : \text{Ja}$$

$$(b) \quad \frac{7}{3} = \frac{3}{7} ? : \text{Nee}$$

$$(c) \quad \frac{3}{-2} = \frac{6}{4} ? : \text{Nee}$$

$$(d) \quad \frac{3}{-5} = \frac{-9}{15} ? : \text{Ja}$$

$$(e) \quad \frac{2}{3} = \frac{4}{9} ? : \text{Nee}$$

$$(f) \quad \frac{3}{1} = \frac{48}{16} ? : \text{Ja}$$

$$(g) \quad \frac{4}{3} = \frac{-12}{-9} ? : \text{Ja}$$

$$(h) \quad \frac{25}{10} = \frac{5}{2} ? : \text{Ja}$$

$$(i) \quad \frac{5}{4} = \frac{4}{3} ? : \text{Nee}$$

A1.2.2 Ordering van rationale getalle:

Oefening 3:

(1) Rangskik die volgende breuke in stygende volgorde:

$$(a) \frac{3}{4} ; \frac{2}{3} \text{ en } \frac{4}{5} : \frac{3}{4} = \frac{45}{60} ; \frac{2}{3} = \frac{40}{60} ; \frac{4}{5} = \frac{48}{60} \quad \therefore \frac{2}{3} < \frac{3}{4} < \frac{4}{5}$$

$$(b) \frac{2}{3} ; \frac{5}{7} \text{ en } \frac{4}{6} : \frac{2}{3} = \frac{28}{42} ; \frac{5}{7} = \frac{30}{42} ; \frac{4}{6} = \frac{28}{42} \quad \therefore \frac{2}{3} = \frac{4}{6} < \frac{5}{7}$$

(2) Rangskik die volgende breuke in dalende volgorde:

$$(a) \frac{5}{8} ; \frac{2}{3} \text{ en } \frac{3}{4} : \frac{5}{8} = \frac{15}{24} ; \frac{2}{3} = \frac{16}{24} ; \frac{3}{4} = \frac{18}{24} \quad \therefore \frac{3}{4} > \frac{2}{3} > \frac{5}{8}$$

$$(b) -1\frac{1}{2} ; -1\frac{2}{3} \text{ en } \frac{-7}{5} : \frac{-3}{2} = \frac{-45}{30} ; \frac{-5}{3} = \frac{-50}{30} ; \frac{-7}{5} = \frac{-42}{30} \quad \therefore \frac{-7}{5} > -1\frac{1}{2} > -1\frac{2}{3}$$

(3) Plaas telkens 'n rationale getal tussen die volgende getalle:

$$(a) \frac{-1}{3} \text{ en } \frac{-3}{5} : \frac{-1}{3} = \frac{-5}{15} \text{ en } \frac{-3}{5} = \frac{-9}{15} \quad \therefore \frac{-3}{5} < \frac{-8}{15} \text{ of } \frac{-7}{15} \text{ of } \frac{-6}{15} < \frac{-1}{3}$$

$$(b) \frac{3}{4} \text{ en } \frac{7}{10} : \frac{3}{4} = \frac{15}{20} = \frac{30}{40} \text{ en } \frac{7}{10} = \frac{14}{20} = \frac{28}{40} \quad \therefore \frac{7}{10} < \frac{29}{40} < \frac{3}{4}$$

A1.2.3 Herleiding van gewone breuke na desimale breuke:

Oefening 4:

Druk die volgende uit as desimale breuke, sonder om 'n sakrekenaar te gebruik:

$$(1) \frac{22}{7} = \frac{22,000\dots}{7} = \frac{22,10^3 0^2 0}{7} = 3,142\dots$$

$$(2) 4\frac{2}{3} = 4\frac{2,20^2 0^2 0}{3} = 4,66\dots \approx 4,6$$

$$(3) \frac{1}{8} = \frac{1,000\dots}{8} = \frac{1,10^2 0^4 0}{8} = 0,125$$

$$(4) \frac{7}{9} = \frac{7,000\dots}{9} = \frac{7,70^7 0^7 0}{9} = 0,77\dots \approx 0,7$$

$$(5) \frac{17}{25} = \frac{17,000\dots}{25} = \frac{17,170^2 0}{25} = 0,68$$

$$(6) \frac{5}{100} = \frac{5,000\dots}{100} = \frac{5,50^5 0}{100} = 0,05$$

$$(7) \frac{4}{11} = \frac{4,40^7 0^4 0^7 0^4 0}{11} = 0,3636\dots \approx 0,3\bar{6}$$

$$(8) -2\frac{6}{7} = -2\frac{6,60^4 0^5 0}{7} = -2,857\dots$$

$$(9) -5\frac{5}{6} = -5\frac{5,50^2 0^2 0}{6} = -5,833\dots \approx -5,8\bar{3}$$

$$(10) \frac{33}{8} = \frac{33,000\dots}{8} = \frac{33,10^2 0^4 0}{8} = 4,125$$

A1.2.4 Afronding van desimale breuke:

Oefening 5:

(1) Rond die volgende breuke af tot die aantal desimale plekke aangedui in hakies:

(a) $3,57\text{\textcircled{3}}$ (tot 2 des)

$$\approx 3,57$$

(b) $12,008\text{\textcircled{73}}$ (tot 3 des)

$$\approx 12,009$$

(c) $0,00384$ (tot 5 des)

$$= 0,00384$$

(d) $7,3\text{\textcircled{226}}$ (tot 1 des)

$$\approx 7,3$$

(e) $8,3\text{\textcircled{9999}}$ (tot 1 des)

$$\approx 8,4$$

(f) $90,\text{\textcircled{9}}023$ (tot die naaste heelgetal)

$$\approx 91$$

(g) $0,4332\text{\textcircled{18}}$ (tot 4 des)

$$\approx 0,4332$$

(h) $1\,456,679\text{\textcircled{9}}$ (tot 3 des)

$$\approx 1\,456,680$$

(i) $66,66\text{\textcircled{6}}$ (tot 2 des)

$$\approx 66,67$$

(j) $13,000\text{\textcircled{34}}$ (tot 3 des)

$$\approx 13,000$$

(2) Beskou die volgende afronding en kies die korrekte woord in die hakies:

(a) $3,47653 \approx 3,477$ korrek tot die naaste (tiende, honderdste of **duisendste**)

(b) $96\,995,31956 \approx 96\,995,32$ korrek tot die naaste (tiende, **honderdste** of duisendste)

A1.2.5 Herleiding van desimale breuke na gewone breuke:

Oefening 6:

Druk die volgende uit as gewone breuke in eenvoudigste vorm:

$$(1) \quad 0,125 = \frac{125}{1\,000} = \frac{125}{1\,000} \div \frac{125}{125} \\ = \frac{1}{8}$$

$$(2) \quad 1,25 = 1 \frac{25}{100} = 1 \frac{25}{100} \div \frac{25}{25} \\ = 1 \frac{1}{4}$$

$$(3) \quad 14,6 = 14 \frac{6}{10} = 14 \frac{6}{10} \div \frac{2}{2} \\ = 14 \frac{3}{5}$$

$$(4) \quad -0,5 = -\frac{5}{10} = -\frac{5}{10} \div \frac{5}{5} \\ = -\frac{1}{2}$$

$$(5) \quad -1,2 = -1 \frac{2}{10} = -1 \frac{2}{10} \div \frac{2}{2} \\ = -1 \frac{1}{5}$$

$$(6) \quad 23,5 = 23 \frac{5}{10} = 23 \frac{5}{10} \div \frac{5}{5} \\ = 23 \frac{1}{2}$$

$$(7) \quad 3,04 = 3 \frac{4}{100} = 3 \frac{4}{100} \div \frac{4}{4} \\ = 3 \frac{1}{25}$$

$$(8) \quad 7,3 = 7 \frac{3}{10}$$

$$(9) \quad 100,75 = 100 \frac{75}{100} = 100 \frac{75}{100} \div \frac{25}{25} \\ = 100 \frac{3}{4}$$

$$(10) \quad 0,00005 = \frac{5}{100\,000} = \frac{5}{100\,000} \div \frac{5}{5} \\ = \frac{1}{20\,000}$$

A1.2.6 Herleiding van repeterende breuke na gewone breuke:

Oefening 7:

Herlei die volgende na gewone breuke in eenvoudigste vorm: (Sonder 'n sakrekenaar.)

(1) $3,\dot{6}$

$$= 3\frac{6}{9}$$

$$= 3\frac{6}{9} \div \frac{3}{3}$$

$$= 3\frac{2}{3}$$

(2) $0,\dot{1}\dot{3}$

$$= \frac{13}{99}$$

(3) $22,3\dot{9}$

$$= 22 + 0,3 + 0,0\dot{9}$$

$$= 22 + \frac{3}{10} + 0,9 \div 10$$

$$= 22 + \frac{3}{10} + \frac{9}{9} \times \frac{1}{10}$$

$$= 22 + \frac{3}{10} + 1 \times \frac{1}{10}$$

$$= 22 + \frac{3}{10} + \frac{1}{10}$$

$$= 22 + \frac{3+1}{10} = 22\frac{4}{10}$$

$$= 22\frac{2}{5}$$

(4) $-1,\dot{1}\dot{3}\dot{5}$ of $-1,\overline{135}$

$$= -1\frac{135}{999}$$

$$= -1\frac{135}{999} \div \frac{27}{27}$$

$$= -1\frac{5}{37}$$

(5) $0,\dot{7}$

$$= \frac{7}{9}$$

(6) $0,00\dot{3}$

$$= 0,3 \div 100$$

$$= \frac{3}{9} \times \frac{1}{100} = \frac{3}{900}$$

$$= \frac{1}{300}$$

(7) $1,\overline{214}$

$$= 1\frac{214}{999}$$

(8) $3,2\dot{5}\dot{8}$

$$= 3 + 0,2 + 0,0\dot{5}\dot{8}$$

$$= 3 + \frac{2}{10} + 0,5\dot{8} \div 10$$

$$= 3 + \frac{2}{10} + \frac{58}{99} \times \frac{1}{10}$$

$$= 3 + \frac{2}{10} \times \frac{99}{99} + \frac{58}{990}$$

$$= 3 + \frac{198}{990} + \frac{58}{990} = 3\frac{256}{990}$$

$$= 3\frac{128}{495}$$

☺ Bereken die volgende sonder die gebruik van 'n sakrekenaar: $0,4 + \frac{2}{3}$

$$\begin{aligned} 0,4 + \frac{2}{3} &= \frac{4}{9} + \frac{2}{3} \\ &= \frac{4}{9} + \frac{2}{3} \times \frac{3}{3} \\ &= \frac{4}{9} + \frac{6}{9} = \frac{10}{9} = 1\frac{1}{9} \end{aligned}$$

A1.3 Irrasionale en Reële getalle:

Oefening 8:

(1) Watter van die volgende getalle is Rasionale getalle (\mathbb{Q}) en watter is Irrasionale getalle (\mathbb{Q}')?

(2)

- (a) 14 : \mathbb{Q} (b) $\frac{1}{5}$: \mathbb{Q} (c) $\sqrt{81}$: \mathbb{Q}
 (d) 0,12 : \mathbb{Q} (e) $\sqrt{18}$: \mathbb{Q}' (f) $12,2\dot{3}$: \mathbb{Q}
 (g) $-\sqrt{\frac{12}{3}}$: \mathbb{Q} (h) 0,2945 ... : \mathbb{Q}' (i) $\sqrt[3]{64}$: \mathbb{Q}
 (j) π : \mathbb{Q}' (k) $\sqrt[5]{32}$: \mathbb{Q} (l) $\frac{11}{7}$: \mathbb{Q}

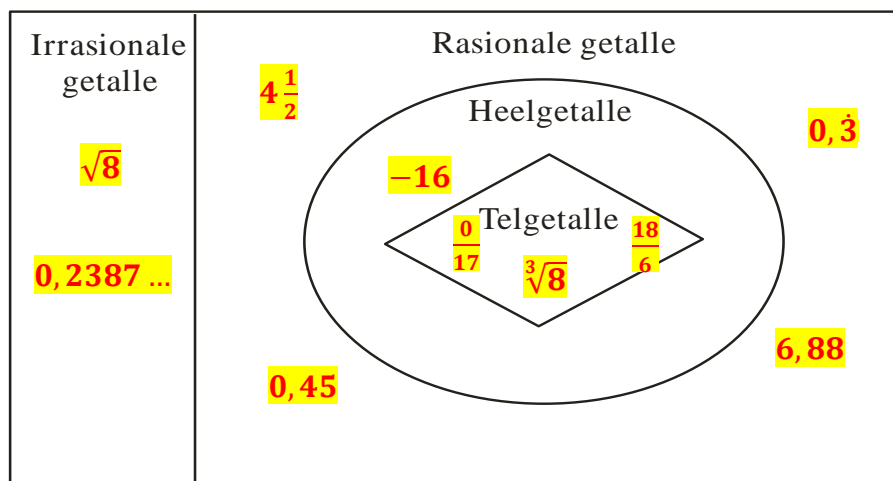
Tussen watter twee heelgetalle lê die volgende irrasionale getalle:

- (a) $-\sqrt{12}$ (b) $\sqrt{66}$
 $-\sqrt{16} < -\sqrt{12} < -\sqrt{9}$ $\sqrt{64} < \sqrt{66} < \sqrt{81}$
 $-4 < -\sqrt{12} < -3$ $8 < \sqrt{66} < 9$
- (c) $\sqrt[3]{5}$ (d) $\sqrt[5]{2}$
 $\sqrt[3]{1} < \sqrt[3]{5} < \sqrt[3]{8}$ $\sqrt[5]{1} < \sqrt[5]{2} < \sqrt[5]{32}$
 $1 < \sqrt[3]{5} < 2$ $1 < \sqrt[5]{2} < 2$

(3) Die onderstaande diagram is 'n opsomming van al die getalle wat op skoolvlak gebruik word. Plaas die volgende getalle op die regte plek in die tabel; waar nodig, vereenvoudig die getal:

$4\frac{1}{2}$; $\sqrt[3]{8} = 2$; $\sqrt{8}$; -16 ; 0,45 ; $0,\dot{3}$; $\frac{18}{6} = 3$; 0,2387 ... ; $\frac{0}{17} = 0$; 6,88

Reële getalle:



☺ (1) Buiten die reële getalle kry ons ook nog die nie-reële getalle.

Gee 'n voorbeeld van 'n nie-reële getal. $\sqrt{-1}$; $\sqrt[3]{-2}$;

(2) Wat word die versameling genoem wat alle reële en nie-reële getalle bevat?

Komplekse getalle

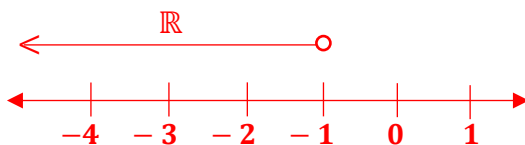
A1.4 Voostelling van versamelings van getalle:

Oefening 9:

(1) Skryf die volgende in interval notasie (waar van toepassing) en stel dit op 'n getallelyn voor:

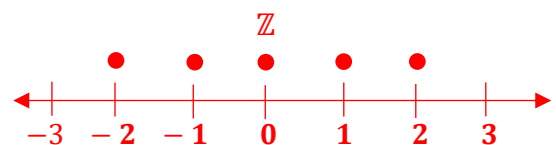
(a) $\{x : x < -1; x \in \mathbb{R}\}$

$x \in (-\infty; -1)$



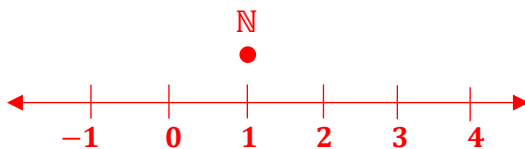
(b) $\{x : -3 < x < 3; x \in \mathbb{Z}\}$

Geen interval notasie



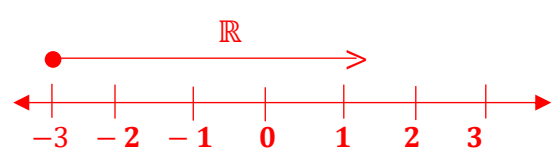
(c) $\{y : y < 2; y \in \mathbb{N}\}$

Geen interval notasie



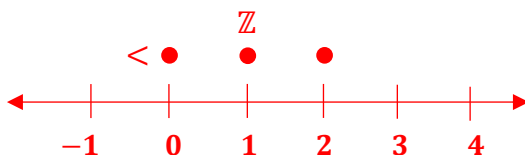
(d) $\{x : x \geq -3; x \in \mathbb{R}\}$

$x \in [-3; \infty)$



(e) $\{x/x < 3; x \in \mathbb{Z}\}$

Geen interval notasie



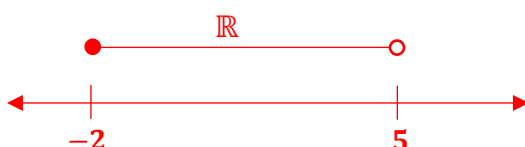
(f) $\{p/p \geq \frac{-5}{2}; p \in \mathbb{R}\}$

$p \in [-\frac{5}{2}; \infty)$



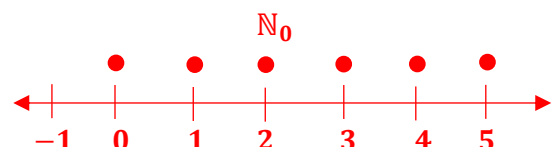
(g) $\{m : -2 \leq m < 5; m \in \mathbb{R}\}$

$m \in [-2; 5)$



(h) $\{x : x \leq 5; x \in \mathbb{N}_0\}$

Geen interval notasie

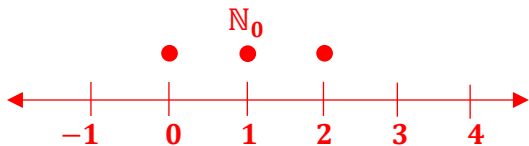


(2) Los op vir x en stel jou antwoord op 'n getallelyn voor:

(a) $x + 1 \leq 3; x \in \mathbb{N}_0$

$$x \leq 3 - 1$$

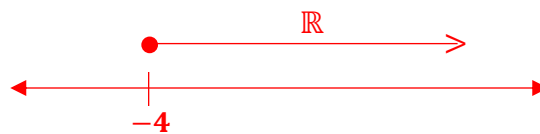
$$x \leq 2$$



(b) $2x \geq -8; x \in \mathbb{R}$

$$x \geq \frac{-8}{2}$$

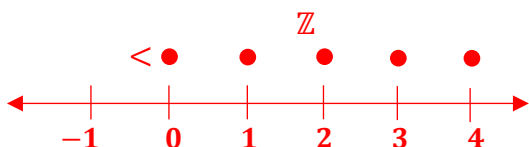
$$x \geq -4$$



(c) $x - 4 \leq 0; x \in \mathbb{Z}$

$$x \leq 0 + 4$$

$$x \leq 4$$

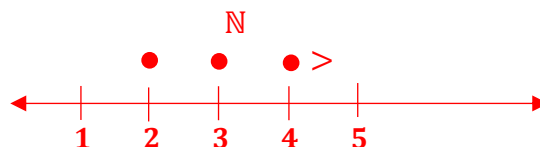


(d) $2x + 3 > 7; x \in \mathbb{N}$

$$2x > 7 - 3$$

$$x > \frac{4}{2}$$

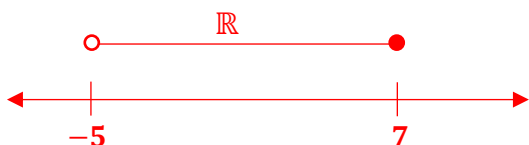
$$x > 2$$



(e) $-6 < x - 1 \leq 6; x \in \mathbb{R}$

$$-6 + 1 < x \leq 6 + 1$$

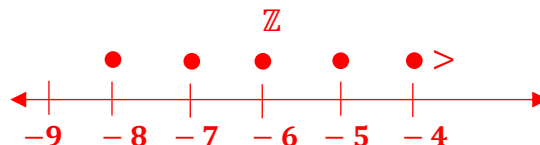
$$-5 < x \leq 7$$



(f) $x + 7 \geq -1; x \in \mathbb{Z}$

$$x \geq -1 - 7$$

$$x \geq -8$$



A1.5 HERSIENINGSOEFENING:

(1) Herlei die volgende na gewone breuke in eenvoudigste vorm: (Sonder 'n sakrekenaar.)

(a) $14,1\bar{7}$

$$= 14 + 0,1 + 0,0\bar{7}$$

$$= 14 + \frac{1}{10} + 0,7 \div 10$$

$$= 14 + \frac{1}{10} + \frac{7}{9} \times \frac{1}{10}$$

$$= 14 + \frac{1}{10} + \frac{7}{90}$$

$$= 14 + \frac{1 \times 9}{10 \times 9} + \frac{7}{90}$$

$$= 14 + \frac{9}{90} + \frac{7}{90} = 14 \frac{16}{90}$$

$$= 14 \frac{8}{45}$$

(b) $0,1\bar{234}$

$$= \frac{1234}{9999}$$

(c) 4,68

$$= 4 \frac{68}{100}$$

$$= 4 \frac{68}{100} \div \frac{4}{4}$$

$$= 4 \frac{17}{25}$$

(d) 5,1

$$= 5 \frac{1}{9}$$

(2) Merk die rationale getalle tussen 0 en 10 met 'n \checkmark :

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
$\sqrt{9} = 3$	-1	$\sqrt{8}$	$\frac{6}{3} = 2$	$\sqrt[3]{16}$	π	$\frac{0}{3} = 0$	$\sqrt{144} = 12$	4,124...	$\sqrt{\frac{24}{6}} = \sqrt{4} = 2$
\checkmark			\checkmark						\checkmark

(3) Rond die volgende breuke af tot die aantal desimale plekke aangedui in hakies:

(a) 7,199 (tot 1 des)

$$\approx 7,2$$

(b) 0,048561 (tot 4 des)

$$\approx 0,0486$$

(c) 234,34 (tot 1 des)

$$\approx 234,3$$

(d) 1 001,1989 (tot die naaste heelgetal)

$$\approx 1\ 001$$

(e) 3,997 (tot 2 des)

$$\approx 4,00$$

(f) 23,712 (tot die naaste heelgetal)

$$\approx 24$$

(4) Plaas enige twee irrasionale getalle tussen 2 en 3.

$$2 = \sqrt{4}$$

en

$$3 = \sqrt{9}$$

$$\therefore 2 < \sqrt{5} < \sqrt{6} < \sqrt{7} < \sqrt{8} < 3$$

Enige 2!

(5) Tussen watter twee rationale getalle lê die volgende irrasionale getalle:

(a) $\sqrt{\frac{1}{2}}$

$$\sqrt{\frac{1}{1}} > \sqrt{\frac{1}{2}} > \sqrt{\frac{1}{4}}$$

$$1 > \sqrt{\frac{1}{2}} > \frac{1}{2}$$

$$\frac{1}{2} < \sqrt{\frac{1}{2}} < 1$$

(b) $\sqrt[3]{57}$

$$\sqrt[3]{27} < \sqrt[3]{57} < \sqrt[3]{64}$$

$$3 < \sqrt[3]{57} < 4$$

$$4 > \sqrt[3]{57} > 3$$

of

(6) Voltooi die ontbrekende voorstellings in die onderstaande tabel:

	Versamelingskeurdernotasië:	Intervalnotasië:	Getalleglyn:
(1)	$\{x / -1 < x \leq 2; x \in \mathbb{R}\}$	$x \in (-1; 2]$	
(2)	$\{x : -2 \leq x \leq 5; x \in \mathbb{R}\}$	$x \in [-2; 5]$	
(3)	$\{y / y \leq 3; y \in \mathbb{R}\}$	$y \in (-\infty; 3]$	
(4)	$\{x / x \leq 3; x \in \mathbb{N}_0\}$	Nvt	
(5)	$\{y / y \geq 3; y \in \mathbb{N}\}$	Nvt	
(6)	$\{m : 0 < m \leq 4; m \in \mathbb{R}\}$	$m \in (0; 4]$	
(7)	$\{t : t \geq -5; t \in \mathbb{R}\}$	$t \in [-5; \infty)$	
(8)	$\{m : m \leq 6; m \in \mathbb{R}\}$	$m \in (-\infty; 6]$	
(9)	$\{x / -1 < x < 2; x \in \mathbb{Z}\}$	Nvt	
(10)	$\{x / x > -1; x \in \mathbb{R}\}$	$x \in (-1; \infty)$	

Hoofstuk A2

Algebraïese uitdrukkings

A2.1 Produkte:

A2.1.1 Distributiewe wet:

Oefening 1:

Bepaal die volgende produkte:

$$\begin{aligned} (1) \quad & (y - 4)(y + 3) \\ &= y^2 + 3y - 4y - 12 \\ &= y^2 - y - 12 \end{aligned}$$

$$\begin{aligned} (2) \quad & (p - 2)(p - 7) \\ &= p^2 - 7p - 2p + 14 \\ &= p^2 - 9p + 14 \end{aligned}$$

$$\begin{aligned} (3) \quad & (2x + 1)(x - 5) \\ &= 2x^2 - 10x + 1x - 5 \\ &= 2x^2 - 9x - 5 \end{aligned}$$

$$\begin{aligned} (4) \quad & (x - 2y)(2x - y) \\ &= 2x^2 - 1xy - 4xy + 2y^2 \\ &= 2x^2 - 5xy + 2y^2 \end{aligned}$$

$$\begin{aligned} (5) \quad & (4ab + 1)(2ab - 3) \\ &= 8a^2b^2 - 12ab + 2ab - 3 \\ &= 8a^2b^2 - 10ab - 3 \end{aligned}$$

$$\begin{aligned} (6) \quad & (5 - 7m)(2 - 3m) \\ &= 10 - 15m - 14m + 21m^2 \\ &= 10 - 29m + 21m^2 \end{aligned}$$

$$\begin{aligned} (7) \quad & (2a - 4b)(3a + 2b) \\ &= 6a^2 + 4ab - 12ab - 8b^2 \\ &= 6a^2 - 8ab - 8b^2 \end{aligned}$$

$$\begin{aligned} (8) \quad & (m + n)(2m - 1) \\ &= 2m^2 - 1m + 2mn - n \end{aligned}$$

$$\begin{aligned} (9) \quad & (d - 12)(12 + d) \\ &= 12d + d^2 - 144 - 12d \\ &= d^2 - 144 \end{aligned}$$

$$\begin{aligned} (10) \quad & (a^2 + 4)(a^2 + 2) \\ &= a^4 + 2a^2 + 4a^2 + 8 \\ &= a^4 + 6a^2 + 8 \end{aligned}$$

$$\begin{aligned} (11) \quad & \left(\frac{1}{2}m - 6\right)(8m - 3) \\ &= 4m^2 - \frac{3}{2}m - 48m + 18 \\ &= 4m^2 - 49\frac{1}{2}m + 18 \end{aligned}$$

$$\begin{aligned} (12) \quad & (-2k - 5)(5 + 3k) \\ &= -10k - 6k^2 - 25 - 15k \\ &= -6k^2 - 25k - 25 \end{aligned}$$

$$\begin{aligned}
 (13) \quad & \left(p + \frac{1}{p}\right)\left(8p - \frac{4}{p}\right) \\
 &= 8p^2 - \frac{4p}{p} + \frac{8p}{p} - \frac{4}{p^2} \\
 &= 8p^2 - 4 + 8 - \frac{4}{p^2} \\
 &= 8p^2 + 4 - \frac{4}{p^2}
 \end{aligned}$$

$$\begin{aligned}
 (15) \quad & (3r^3 + 2)(2r^2 - 5) \\
 &= 6r^5 - 15r^3 + 4r^2 - 10
 \end{aligned}$$

$$\begin{aligned}
 (17) \quad & \left(\frac{1}{p^3q^2} - \frac{2}{p^2q}\right)\left(\frac{1}{p} + \frac{2}{q}\right) \\
 &= \frac{1}{p^4q^2} + \frac{2}{p^3q^3} - \frac{2}{p^3q} - \frac{4}{p^2q^2}
 \end{aligned}$$

$$\begin{aligned}
 (14) \quad & (abc - 2ac)(abc + 3bc) \\
 &= a^2b^2c^2 + 3ab^2c^2 - 2a^2bc^2 - 6abc^2
 \end{aligned}$$

$$\begin{aligned}
 (16) \quad & 2x(x - 5y)(3x + 2y) \\
 &= 2x(3x^2 + 2xy - 15xy - 10y^2) \\
 &= 2x(3x^2 - 13xy - 10y^2) \\
 &= 6x^3 - 26x^2y - 20xy^2
 \end{aligned}$$

$$\begin{aligned}
 (18) \quad & \left(\frac{m^2n}{3} - \frac{6}{mn}\right)\left(\frac{mn}{2} - \frac{3}{mn^2}\right) \\
 &= \frac{m^3n^2}{6} - \frac{3m^2n}{3mn^2} - \frac{6mn}{2mn} + \frac{18}{m^2n^3} \\
 &= \frac{m^3n^2}{6} - \frac{m}{n} - 3 + \frac{18}{m^2n^3}
 \end{aligned}$$

Oefening 2:

Vereenvoudig:

$$\begin{aligned}
 (1) \quad & (abc - 2)(abc + 2) \\
 &= a^2b^2c^2 - 4
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & \left(\frac{1}{3} + 5t\right)\left(\frac{1}{3} - 5t\right) \\
 &= \frac{1}{9} - 25t^2
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad & (p - 9q)(9q + p) \\
 &= (p - 9q)(p + 9q) \\
 &= p^2 - 81q^2
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad & (n + 7k)(7n - k) \\
 &= 7n^2 - 1kn + 49kn - 7k^2 \\
 &= 7n^2 + 48kn - 7k^2
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad & (-a + 4b)(-a - 4b) \\
 &= a^2 + 4ab - 4ab - 16b^2 \\
 &= a^2 - 16b^2
 \end{aligned}$$

$$\begin{aligned}
 (7) \quad & (x^{2m} - 8)(x^{2m} + 8) \\
 &= x^{2m+2m} - 64 \\
 &= x^{4m} - 64
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad & (b^6c^3 + 6)(b^6c^3 + 6) \\
 &= b^{12}c^6 + 6b^6c^3 + 6b^6c^3 + 36 \\
 &= b^{12}c^6 + 12b^6c^3 + 36
 \end{aligned}$$

$$\begin{aligned}
 (11) \quad & (m - 2n)^2(m + 2n)^2 \\
 &= [(m - 2n)(m + 2n)]^2 \\
 &= [m^2 - 4n^2]^2 \\
 &= m^4 - 8m^2n^2 + 16n^4
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad & -x\left(\frac{1}{x} - x\right)\left(\frac{1}{x} + x\right) \\
 &= -x\left(\frac{1}{x^2} - x^2\right) \\
 &= \frac{-x}{x^2} + x\left(\frac{1}{x^2} - x^2\right) \\
 &= \frac{-1}{x} + x^3
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad & (0,3 + 3q)(0,3 - 3q) \\
 &= 0,09 - 9q^2
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad & (4xk^5 - 7)(7 + 4xk^5) \\
 &= (4xk^5 - 7)(4xk^5 + 7) \\
 &= 16x^2k^{10} - 49
 \end{aligned}$$

$$\begin{aligned}
 (12) \quad & \left(\frac{m}{n} + 2\right)\left(\frac{m^2}{n^2} + 4\right)\left(\frac{m}{n} - 2\right) \\
 &= \left(\frac{m}{n} + 2\right)\left(\frac{m}{n} - 2\right)\left(\frac{m^2}{n^2} + 4\right) \\
 &= \left(\frac{m^2}{n^2} - 4\right)\left(\frac{m^2}{n^2} + 4\right) \\
 &= \frac{m^4}{n^4} - 16
 \end{aligned}$$

A2.1.2 Kwadrering van 'n tweeterm:

Oefening 3:

Bepaal die volgende kwadrate:

$$\begin{aligned}
 (1) \quad & (y - 11)^2 \\
 &= (y - 11)(y - 11) \\
 &= y^2 - 11y - 11y + 121 \\
 &= y^2 - 22y + 121
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & (3p + 2q)^2 \\
 &= (3p + 2q)(3p + 2q) \\
 &= 9p^2 + 6pq + 6pq + 4q^2 \\
 &= 9p^2 + 12pq + 4q^2
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad & (-4 + 5c)^2 \\
 &= (-4 + 5c)(-4 + 5c) \\
 &= 16 - 20c - 20c + 25c^2 \\
 &= 16 - 40c + 25c^2
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad & (mn + 3)^2 \\
 &= (mn + 3)(mn + 3) \\
 &= m^2n^2 + 3mn + 3mn + 9 \\
 &= m^2n^2 + 6mn + 9
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad & (k^2 + 1)^2 \\
 &= (k^2 + 1)(k^2 + 1) \\
 &= k^4 + 1k^2 + 1k^2 + 1 \\
 &= k^4 + 2k^2 + 1
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad & (8 - 3b)^2 \\
 &= (8 - 3b)(8 - 3b) \\
 &= 64 - 24b - 24b + 9b^2 \\
 &= 64 - 48b + 9b^2
 \end{aligned}$$

$$\begin{aligned}
 (7) \quad & \left(x - \frac{1}{2}\right)^2 \\
 &= \left(x - \frac{1}{2}\right)\left(x - \frac{1}{2}\right) \\
 &= x^2 - \frac{1}{2}x - \frac{1}{2}x + \frac{1}{4} \\
 &= x^2 - 1x + \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad & \left(\frac{y}{5} - 3\right)^2 \\
 &= \left(\frac{y}{5} - 3\right)\left(\frac{y}{5} - 3\right) \\
 &= \frac{y^2}{25} - \frac{3y}{5} - \frac{3y}{5} + 9 \\
 &= \frac{y^2}{25} - \frac{6y}{5} + 9
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad & (5p - 2p^2)^2 \\
 &= (5p - 2p^2)(5p - 2p^2) \\
 &= 25p^2 - 10p^3 - 10p^3 + 4p^4 \\
 &= 25p^2 - 20p^3 + 4p^4
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad & \left(4 + \frac{3}{n}\right)^2 \\
 &= \left(4 + \frac{3}{n}\right)\left(4 + \frac{3}{n}\right) \\
 &= 16 + \frac{12}{n} + \frac{12}{n} + \frac{9}{n^2} \\
 &= 16 + \frac{24}{n} + \frac{9}{n^2}
 \end{aligned}$$

$$\begin{aligned}
 (11) \quad & (0,2 + 6y)^2 \\
 &= (0,2 + 6y)(0,2 + 6y) \\
 &= 0,04 + 1,2y + 1,2y + 36y^2 \\
 &= 0,04 + 2,4y + 36y^2
 \end{aligned}$$

$$\begin{aligned}
 (12) \quad & \left(\frac{2m}{p} + \frac{p^2}{3m}\right)^2 \\
 &= \left(\frac{2m}{p} + \frac{p^2}{3m}\right)\left(\frac{2m}{p} + \frac{p^2}{3m}\right) \\
 &= \frac{4m^2}{p^2} + \frac{2mp^2}{3mp} + \frac{2mp^2}{3mp} + \frac{p^4}{9m^2} \\
 &= \frac{4m^2}{p^2} + \frac{4p}{3} + \frac{p^4}{9m^2}
 \end{aligned}$$

Oefening 4:

Vereenvoudig(Gebruik die korter metode!)

(1) $(x - 3)^2$

$$= x^2 - 6x + 9$$

(3) $(3y + 7)^2$

$$= 9y^2 + 42y + 49$$

(5) $(5t^2 + 8)^2$

$$= 25t^4 + 80t^2 + 64$$

(7) $(-2k - 5)^2$

$$= 4k^2 + 20k + 25$$

(9) $(4x^2 + 10y^2)^2$

$$= 16x^4 + 80x^2y^2 + 100y^4$$

(11) $(8 - 3y)(8 + 3y)$

$$= 64 - 9y^2$$

(2) $(6m - 1)^2$

$$= 36m^2 - 12m + 1$$

(4) $(3 + pq)^2$

$$= 9 + 6pq + p^2q^2$$

(6) $\left(\frac{2}{3} - 6y\right)^2$

$$= \frac{4}{9} - 8y + 36y^2$$

(8) $\left(\frac{3p - 2q}{5m}\right)^2$

$$= \frac{9p^2 - 12pq + 4q^2}{25m^2}$$

(10) $(2mn + 7)(7 + 2mn)$

$$= (2mn + 7)(2mn + 7)$$

$$= 4m^2n^2 + 28mn + 49$$

(12) $-2(abc - 11)^2$

$$= -2(a^2b^2c^2 - 22abc + 121)$$

$$= -2a^2b^2c^2 + 44abc - 242$$

A2.1.3 Tweeterme en drieterme:Oefening 5:

Vereenvoudig die volgende:

(1) $(2a - 3)(a^2 + 5a - 4)$

$$= 2a^3 + 10a^2 - 8a - 3a^2 - 15a + 12$$

$$= 2a^3 + 7a^2 - 23a + 12$$

(2) $(m + 7)(2m^2 + 3m + 3)$

$$= 2m^3 + 3m^2 + 3m + 14m^2 + 21m + 21$$

$$= 2m^3 + 17m^2 + 24m + 21$$

$$(3) \quad (1 + x)(1 - x + x^2)$$

$$= 1 - 1x + 1x^2 + 1x - 1x^2 + x^3$$

$$= 1 + x^3$$

$$(4) \quad (3y - 2)(9y^2 + 6y + 4)$$

$$= 27y^3 + 18y^2 + 12y - 18y^2 - 12y - 8$$

$$= 27y^3 - 8$$

$$(5) \quad \left(2m + \frac{1}{2}\right)\left(\frac{m^2}{4} + 4 - 4m\right)$$

$$= \frac{2m^3}{4} + 8m - 8m^2 + \frac{m^2}{8} + 2 - \frac{4m}{2}$$

$$= \frac{m^3}{2} + 8m - \frac{8m^2}{1} + \frac{1m^2}{8} + 2 - 2m$$

$$= \frac{m^3}{2} + 6m - 7\frac{7}{8}m^2 + 2$$

$$(6) \quad (m^2n^2 - 5)(25 + 5m^2n^2 + m^4n^4)$$

$$= 25m^2n^2 + 5m^4n^4 + m^6n^6 - 125$$

$$\quad - 25m^2n^2 - 5m^2n^2$$

$$= m^6n^6 - 125$$

A2.1.4 Die som en verskil van twee derdemagte:

Oefening 6:

Skryf die volgende produkte direk neer, indien moontlik:

$$(1) \quad (a + 3)(a^2 - 3a + 9)$$

$$= a^3 + 27$$

$$(2) \quad (2y^3 + 4)(4y^6 - 8y^3 + 16)$$

$$= 8y^9 + 64$$

$$(3) \quad \left(\frac{x}{3} - 1\right)\left(\frac{1}{9}x^2 + \frac{1}{3}x + 1\right)$$

$$= \frac{x^3}{27} - 1$$

$$(4) \quad \left(6a^2 - \frac{1}{2}\right)\left(36a^4 + 3a^2 + \frac{1}{4}\right)$$

$$= 216a^6 - \frac{1}{8}$$

$$(5) \quad (5q + 7)(25q^2 - 35q + 49)$$

$$= 125q^3 + 343$$

$$(6) \quad (8 - 3m)(9m^2 + 24m + 64)$$

$$= (8 - 3m)(64 + 24m + 9m^2)$$

$$= 512 - 27m^3$$

$$(7) \quad (x - 5)(x^2 - 5x + 25)$$

$$= x^3 - 5x^2 + 25x - 5x^2 + 25x - 125$$

$$= x^3 - 10x^2 + 50x - 125$$

$$(8) \quad (0,1 + 0,2y)(0,01 - 0,02y + 0,04y^2)$$

$$= 0,001 + 0,008y^3$$

$$\begin{aligned}
 (9) \quad & (9a^4 + 6a^2b + 4b^2)(3a^2 - 2b) \\
 &= (3a^2 - 2b)(9a^4 + 6a^2b + 4b^2) \\
 &= 27a^6 - 8b^3
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad & 2(-1 + 5m)(25m^2 + 5m + 1) \\
 &= 2(5m - 1)(25m^2 + 5m + 1) \\
 &= 2(125m^3 - 1) \\
 &= 250m^3 - 2
 \end{aligned}$$

A2.1.5 Vereenvoudiging van uitdrukkingen:

Oefening 7:

Vereenvoudig:

$$\begin{aligned}
 (1) \quad & 2a(a + 1) - 3(a - 1)^2 \\
 &= 2a^2 + 2a - 3(a^2 - 2a + 1) \\
 &= 2a^2 + 2a - 3a^2 + 6a - 3 \\
 &= -1a^2 + 8a - 3
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & (m^3 + 3m^2 - 2m - 2)m + 4(m^3 + 5m^2 - 6) \\
 &= m(m^3 + 3m^2 - 2m - 2) + 4(m^3 + 5m^2 - 6) \\
 &= m^4 + 3m^3 - 2m^2 - 2m + 4m^3 + 20m^2 - 24 \\
 &= m^4 + 7m^3 + 18m^2 - 2m - 24
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad & (x - 1)(x^2 - x + 1) + 3(x - 2) - x^3 \\
 &= x^3 - x^2 + x - x^2 + x - 1 + 3x - 6 - x^3 \\
 &= -2x^2 + 5x - 7
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad & (y - 1)(y + 1)(y^2 + 1) \\
 &= (y^2 - 1)(y^2 + 1) \\
 &= y^4 - 1
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad & -2(2c + 1)(c - 2) - 2(2c - 1)^2 \\
 &= -2(2c^2 - 3c - 2) - 2(4c^2 - 4c + 1) \\
 &= -4c^2 + 6c + 4 - 8c^2 + 8c - 2 \\
 &= -12c^2 + 14c + 2
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad & \frac{1}{2}(4p - 3)^2 - \left(\frac{p}{2} + 2\right)^2 \\
 &= \frac{1}{2}(16p^2 - 24p + 9) - \left(\frac{p^2}{4} + 2p + 4\right) \\
 &= 8p^2 - 12p + \frac{9}{2} - \frac{1}{4}p^2 - 2p - 4 \\
 &= 7\frac{3}{4}p^2 - 14p + \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 (7) \quad & a^2 + b^2 - (2a - b)^2 + (a + 2b)^2 \\
 &= a^2 + b^2 - (4a^2 - 4ab + b^2) + a^2 + 4ab + 4b^2 \\
 &= \underline{a^2} + \underline{b^2} - \underline{4a^2} + \underline{4ab} - \underline{b^2} + \underline{a^2} + \underline{4ab} + \underline{4b^2} \\
 &= -2a^2 + 4b^2 + 8ab
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad & (3y + 1)(9y^2 + 1)(3y - 1) \\
 &= (3y + 1)(3y - 1)(9y^2 + 1) \\
 &= (9y^2 - 1)(9y^2 + 1) \\
 &= 81y^4 - 1
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad & 7 - 3(5t - 6) + 2t - (t - 1)(t + 1) \\
 &= 7 - 15t + 18 + 2t - (t^2 - 1) \\
 &= 7 - 15t + 18 + 2t - t^2 + 1 \\
 &= -1t^2 - 13t + 26
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad & (x + 3y)(x^2 - 3xy + 9y^2) + (x + 3y)(x - 3y) \\
 &= x^3 + 27y^3 + x^2 - 9y^2
 \end{aligned}$$

$$(11) \quad (2p - 3)(4p^2 + 6p + 9)(8p^3 + 27)$$

$$= (8p^3 - 27)(8p^3 + 27)$$

$$= 64p^6 - 729$$

$$(12) \quad (x - y + k)(x - y - k)$$

$$= [(x - y) + k][(x - y) - k]$$

$$= [(x - y)^2 - k^2]$$

$$= x^2 - 2xy + y^2 - k^2$$

$$(13) \quad (p - 3)^3 = (p - 3)(p - 3)(p - 3)$$

$$= (p - 3)(p^2 - 6p + 9)$$

$$= p^3 - 6p^2 + 9p - 3p^2 + 18p - 27$$

$$= p^3 - 9p^2 + 27p - 27$$

$$(14) \quad [(2m + n)(4m^2 - 2mn + n^2)]^2$$

$$= [8m^3 + n^3]^2$$

$$= 64m^6 + 16m^3n^3 + n^6$$

$$(15) \quad (x + y)(x - y) - (x - y)^2 + (x + y)^2 - x(x + y) + (x - y)(x^2 + 2xy + y^2)$$

$$= x^2 - y^2 - (x^2 - 2xy + y^2) + (x^2 + 2xy + y^2) - x^2 - xy + x^3 + 2x^2y + xy^2 - x^2y - 2xy^2 - y^3$$

$$= \underline{x^2} - \underline{y^2} - \underline{x^2} + \underline{2xy} - \underline{y^2} + \underline{x^2} + \underline{2xy} + \underline{y^2} - \underline{x^2} - \underline{xy} + \underline{x^3} + \underline{2x^2y} + \underline{1xy^2} - \underline{1x^2y} - \underline{2xy^2} - \underline{y^3}$$

$$= -y^2 + 3xy + x^3 + 1x^2y - 1xy^2 - y^3$$

☺ Vereenvoudig: (a) $(x^m + y^n)^2$ (b) $(a^{x+1} - 2)(a^{x+1} + 2)$

$$= (x^m + y^n)(x^m + y^n) \quad = (a^{x+1})^2 - 4$$

$$= x^{2m} + x^m y^n + x^m y^n + y^{2n} \quad = a^{2x+2} - 4$$

$$= x^{2m} + 2x^m y^n + y^{2n}$$

A2.2 Faktorisering:

A2.2.1 Gemeenskaplike faktor:

Oefening 8:

Faktoriseer volledig:

(1) $4am + 3a^2m^4$	(2) $12x^2 - 132y^2$	(3) $-p^5q^3 + pq$
$= am(4 + 3am^3)$	$= 12(x^2 - 11y^2)$	$= pq(-p^4q^2 + 1)$
(4) $4x^2 - 8x + 1$	(5) $\frac{1}{2}abc + \frac{1}{2}a^2bc^2$	(6) $5r^7R^4 + 15r^5R^2$
$= (4x^2 - 8x + 1)$	$= \frac{1}{2}abc(1 + ac)$	$= 5r^5R^2(r^2R^2 + 3)$

$$(7) \quad 7x(3y - 1) - 2(3y - 1)$$

$$= (3y - 1)(7x - 2)$$

$$(9) \quad 2gh + 18g^2h + 3g^3h$$

$$= gh(2 + 18g + 3g^2)$$

$$(11) \quad 3g(3g + h) - 1(3g + h)$$

$$= (3g + h)(3g - 1)$$

$$(13) \quad r(x - y) + (y - x) + 2t(x - y)$$

$$= r(x - y) - 1(x - y) + 2t(x - y)$$

$$= (x - y)(r - 1 + 2t)$$

$$(15) \quad 4xy + 2 - k(2xy + 1)$$

$$= 2(2xy + 1) - k(2xy + 1)$$

$$= (2xy + 1)(2 - k)$$

$$(8) \quad 12x^3t - 14xt^3 + 16x^2t^2$$

$$= 2xt(6x^2 - 7t^2 + 8xt)$$

$$(10) \quad a(2m + 3) + b(3 + 2m)$$

$$= (2m + 3)(a + b)$$

$$(12) \quad 1(4b + 5) + 7a(4b + 5)$$

$$= (4b + 5)(1 + 7a)$$

$$(14) \quad 5ab(d - 4c) - 7a(4c - d)$$

$$= 5ab(d - 4c) + 7a(d - 4c)$$

$$= (d - 4c)(5ab + 7a)$$

$$= (d - 4c)(a)(5b + 7)$$

$$(16) \quad y^4(x^2 - 5) + y(x^2 - 5)$$

$$= (x^2 - 5)(y^4 + y)$$

$$= y(x^2 - 5)(y^3 + 1)$$

A2.2.2 Groepering:

Oefening 9:

Faktoriseer volledig:

$$(1) \quad r - s - 5r + 5s$$

$$= 1(r - s) - 5(r - s)$$

$$= (r - s)(1 - 5) = -4(r - s)$$

$$(3) \quad pq - pr + q^2 - qr$$

$$= p(q - r) + q(q - r)$$

$$= (q - r)(p + q)$$

$$(5) \quad aw - bw + 2bw - 2aw$$

$$= -1aw + 1bw$$

$$= -1w(a - b) \quad \text{of}$$

$$= w(b - a)$$

$$(2) \quad 3am^3 + 6am + 7m^2n + 14n$$

$$= 3am(m^2 + 2) + 7n(m^2 + 2)$$

$$= (m^2 + 2)(3am + 7n)$$

$$(4) \quad 4x - 8k - rtx + 2rtk$$

$$= 4(x - 2k) - rt(x - 2k)$$

$$= (x - 2k)(4 - rt)$$

$$(6) \quad p^2 + p(2 + q) + 2q$$

$$= p^2 + 2p + pq + 2q$$

$$= p(p + 2) + q(p + 2)$$

$$= (p + 2)(p + q)$$

$$\begin{aligned}
 (7) \quad mx + nx + rx - my - ny - ry \\
 = x(m + n + r) - y(m + n + r) \\
 = (m + n + r)(x - y)
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad pq - 1 + p - q \\
 = pq + p - 1 - q \\
 = p(q + 1) - 1(1 + q) \\
 = (q + 1)(p - 1)
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad g(h - j) + g^2 - hj \\
 = gh - gj + g^2 - hj \\
 = gh + g^2 - gj - hj \\
 = g(h + g) - j(g + h) = (g + h)(g - j)
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad 3mn^3 + 2mt - 7m + 3kn^3 + 2kt - 7k \\
 = m(3n^3 + 2t - 7) + k(3n^3 + 2t - 7) \\
 = m(3n^3 + 2t - 7) + k(3n^3 + 2t - 7) \\
 = (3n^3 + 2t - 7)(m + k)
 \end{aligned}$$

A2.2.3 Verskil tussen twee vierkante:

Oefening 10:

Faktoriseer volledig:

$$\begin{aligned}
 (1) \quad c^2 - 81 \\
 = (c - 9)(c + 9)
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad 1 - p^2 \\
 = (1 + p)(1 - p)
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad x^2y^2 - 25 \\
 = (xy - 5)(xy + 5)
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad 9 + k^2 \\
 = (9 + k^2)
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad m^{10} - 16 \\
 = (m^5 - 4)(m^5 + 4)
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad r^6 - k^2 \\
 = (r^3 - k)(r^3 + k)
 \end{aligned}$$

$$\begin{aligned}
 (7) \quad (xyz)^2 - 121 \\
 = (xyz - 11)(xyz + 11)
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad n^2 - 50 \\
 = (n^2 - 50)
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad -9 + t^4 = t^4 - 9 \\
 = (t^2 + 3)(t^2 - 3)
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad 2x^2 - 8 \\
 = 2(x^2 - 4) \\
 = 2(x - 2)(x + 2)
 \end{aligned}$$

$$\begin{aligned}
 (11) \quad -y^2 - 49 \\
 = -1(y^2 + 49)
 \end{aligned}$$

$$\begin{aligned}
 (12) \quad (x + y)^2 - 100 \\
 = (x + y - 10)(x + y + 10)
 \end{aligned}$$

$$\begin{aligned}
 (13) \quad p^2m + p^2 - m - 1 \\
 = p^2(m + 1) - 1(m + 1) \\
 = (m + 1)(p^2 - 1) \\
 = (m + 1)(p - 1)(p + 1)
 \end{aligned}$$

$$\begin{aligned}
 (14) \quad (m - 3)^2 - (m + 2)^2 \\
 = [(m - 3) - (m + 2)][(m - 3) + (m + 2)] \\
 = [-5][2m - 1] \quad \text{of} \\
 = -5(2m - 1)
 \end{aligned}$$

$$\begin{aligned}
 (15) \quad x^3 - 9x \\
 = x(x^2 - 9) \\
 = x(x - 3)(x + 3)
 \end{aligned}$$

$$\begin{aligned}
 (16) \quad m^4 - n^4 \\
 = (m^2 - n^2)(m^2 + n^2) \\
 = (m - n)(m + n)(m^2 + n^2)
 \end{aligned}$$

$$\begin{aligned}
 (17) \quad & ab^7x - ab^3x \\
 &= ab^3x(b^4 - 1) \\
 &= ab^3x(b^2 - 1)(b^2 + 1) \\
 &= ab^3x(b - 1)(b + 1)(b^2 + 1)
 \end{aligned}$$

$$\begin{aligned}
 (18) \quad & y^2 - \frac{1}{4} \\
 &= \left(y + \frac{1}{2}\right)\left(y - \frac{1}{2}\right)
 \end{aligned}$$

$$\begin{aligned}
 (19) \quad & 3b(q^2 - 9) + 2(q^2 - 9) \\
 &= (q^2 - 9)(3b + 2) \\
 &= (q - 3)(q + 3)(3b + 2)
 \end{aligned}$$

$$\begin{aligned}
 (20) \quad & a^3b - 8ab \\
 &= ab(a^2 - 8)
 \end{aligned}$$

$$\begin{aligned}
 (21) \quad & 5k(p^2 - 4) + 2a(p^2 - 4) - 1(p^2 - 4) \\
 &= (p^2 - 4)(5k + 2a - 1) \\
 &= (p - 2)(p + 2)(5k + 2a - 1)
 \end{aligned}$$

$$\begin{aligned}
 (22) \quad & a^{12} - 81 \\
 &= (a^6 - 9)(a^6 + 9) \\
 &= (a^3 - 3)(a^3 + 3)(a^6 + 9)
 \end{aligned}$$

$$\begin{aligned}
 (23) \quad & (3r - 2)^2 - 4r^2 \\
 &= [(3r - 2) - 2r][(3r - 2) + 2r] \\
 &= [r - 2][5r - 2]
 \end{aligned}$$

$$\begin{aligned}
 (24) \quad & q^4(x^2 - 2x - 5) - k^2(x^2 - 2x - 5) \\
 &= (x^2 - 2x - 5)(q^4 - k^2) \\
 &= (x^2 - 2x - 5)(q^2 - k)(q^2 + k)
 \end{aligned}$$

$$\begin{aligned}
 (25) \quad & 4m^2 - n^2 + 2m + n \\
 &= (2m + n)(2m - n) + 1(2m + n) \\
 &= (2m + n)[(2m - n) + 1] \\
 &= (2m + n)(2m - n + 1)
 \end{aligned}$$

$$\begin{aligned}
 (26) \quad & 3p^2 - 2q - 3q^2 + 2p \\
 &= 3p^2 - 3q^2 + 2p - 2q \\
 &= 3(p^2 - q^2) + 2(p - q) \\
 &= 3(p - q)(p + q) + 2(p - q) \\
 &= (p - q)(3p + 3q + 2)
 \end{aligned}$$

$$\begin{aligned}
 (27) \quad & 16(x - y)^2 - 25(2x + 3y)^2 \\
 &= [4(x - y) - 5(2x + 3y)][4(x - y) + 5(2x + 3y)] \\
 &= [4x - 4y - 10x - 15y][4x - 4y + 10x + 15y] \\
 &= [-6x - 19y][14x + 11y]
 \end{aligned}$$

$$\begin{aligned}
 (28) \quad & (a - b + c)^2 - (a + b - c)^2 \\
 &= [(a - b + c) - (a + b - c)][(a - b + c) + (a + b - c)] \\
 &= [a - b + c - a - b + c][a - b + c + a + b - c] \\
 &= [-2b + 2c][2a] = -2(b - c)(2a) \\
 &= -4a(b - c)
 \end{aligned}$$

A2.2.4 Drieterme:Oefening 11:

Faktoriseer volledig:

(1) $x^2 + 5x + 6$

$= (x + 3)(x + 2)$

(2) $y^2 + 3y - 4$

$= (y + 4)(y - 1)$

(3) $m^2 - 3m - 10$

$= (m - 5)(m + 2)$

(4) $p^2 - 2p - 8$

$= (p - 4)(p + 2)$

(5) $k^2 + k - 20$

$= (k + 5)(k - 4)$

(6) $b^2 + 11b + 10$

$= (b + 10)(b + 1)$

(7) $a^2 - 5a - 14$

$= (a - 7)(a + 2)$

(8) $x^4 + 6x^2 + 9$

$= (x^2 + 3)(x^2 + 3)$

(9) $7 - 8q + q^2$
 $= q^2 - 8q + 7$

$= (q - 1)(q - 7)$

(10) $s^2 + 10s - 24$

$= (s + 12)(s - 2)$

(11) $p^2 - p - 6$

$= (p - 3)(p + 2)$

(12) $d^2 + 10d + 25$

$= (d + 5)(d + 5)$

(13) $p^2 - 2p + 1$

$= (p - 1)(p - 1)$

(14) $c^6 + 5c^3 - 24$

$= (c^3 - 3)(c^3 + 8)$

(15) $12 + 8s + s^2$
 $= s^2 + 8s + 12$

$= (s + 6)(s + 2)$

(16) $r^2 - 11r - 12$

$= (r - 12)(r + 1)$

(17) $k^2 - 15 - 10$
 $= k^2 - 25$

$= (k - 5)(k + 5)$

(18) $g^2 + 14g - 72$

$= (g + 18)(g - 4)$

(19) $m^2 - 18m - 144$

$= (m - 24)(m + 6)$

(20) $a^2 + 2a - 63$

$= (a + 9)(a - 7)$

(21) $t^2 - 13t + 42$

$= (t - 6)(t - 7)$

(22) $2t^2 - 12t + 10$

$= 2(t^2 - 6t + 5)$

$= 2(t - 1)(t - 5)$

(23) $r^4 + 2r^2 - 24$

$= (r^2 + 6)(r^2 - 4)$

$= (r^2 + 6)(r - 2)(r + 2)$

(24) $100 - 21y - y^2$

$= -1(y^2 + 21y - 100)$

$= -1(y + 25)(y - 4)$

(25) $7k - 60 + k^2$

$= k^2 + 7k - 60$

$= (k - 5)(k + 12)$

(26) $y^3 - 3y^2 - 18y$

$= y(y^2 - 3y - 18)$

$= y(y - 6)(y + 3)$

(27) $m^2 + m + \frac{1}{4}$

$= \left(m + \frac{1}{2}\right)\left(m + \frac{1}{2}\right)$

(28) $x^2 - 4xy + 3y^2$

$= (x - 3y)(x - y)$

(29) $15y^2 - 8y + 1$

$= (5y - 1)(3y - 1)$

(30) $15 - 2d - d^2$

$= -1(d^2 + 2d - 15)$

$= -1(d + 5)(d - 3)$

$$(31) \quad n^3 + 8n^2 + 12n = n(n^2 + 8n + 12)$$

$$= n(n+2)(n+6)$$

$$(32) \quad p^2 + 3pq - 18q^2 = (p+6q)(p-3q)$$

$$(33) \quad -8 + 4x + 4x^2 = 4(x^2 + 1x - 2)$$

$$= 4(x+2)(x-1)$$

$$(34) \quad p^2q^4 - pq^2 - 12 = (pq^2 - 4)(pq^2 + 3)$$

$$(35) \quad -k^2 + 12k - 35 = -1(k^2 - 12k + 35)$$

$$(36) \quad 8m^2 - 7m - 1 = (8m+1)(m-1)$$

$$= -1(k-5)(k-7)$$

$$(37) \quad x^2(a^2 + 5a + 6) - y^2(a^2 + 5a + 6) \quad (38) \quad 2t^4 - 26t^2 + 72$$

$$= (a^2 + 5a + 6)(x^2 - y^2)$$

$$= (a+2)(a+3)(x-y)(x+y)$$

$$= 2(t^4 - 13t^2 + 36)$$

$$= 2(t^2 - 4)(t^2 - 9)$$

$$= 2(t-2)(t+2)(t-3)(t+3)$$

$$\text{Stel } \underline{x+1} = \underline{k}$$

$$(39) \quad (x+1)^2 + 3(x+1) - 28$$

$$= k^2 + 3k - 28$$

$$= (k+7)(k-4)$$

$$= (x+1+7)(x+1-4)$$

$$= (x+8)(x-3)$$

$$(40) \quad x^2(y^2 - 9) - 4x(y^2 - 9) - 12(y^2 - 9)$$

$$= (y^2 - 9)(x^2 - 4x - 12)$$

$$= (y-3)(y+3)(x-6)(x+2)$$

Vergelyk nr.39 met nr.42

$$(41) \quad m^4n^4 + 4m^2n^2 - 5$$

$$= (m^2n^2 + 5)(m^2n^2 - 1)$$

$$= (m^2n^2 + 5)(mn-1)(mn+1)$$

$$(42) \quad (2c-1)^2 + 8(2c-1) + 16$$

$$= [(2c-1) + 4][(2c-1) + 4]$$

$$= [2c+3][2c+3]$$

$$= (2c+3)^2$$

A2.2.5 Nog drieterme:

Oefening 12:

Faktoriseer volledig:

$$(1) \quad 3x^2 + 10x + 3 \quad \begin{array}{l} 3x \nearrow +1 = +1x \\ x \searrow +3 = +9x \\ \hline +10x \end{array}$$

$$= (3x+1)(x+3)$$

$$(2) \quad 6y^2 - 1y - 2 \quad \begin{array}{l} 3y \nearrow -2 = -4y \\ 2y \searrow +1 = +3y \\ \hline -1y \end{array}$$

$$= (3y-2)(2y+1)$$

$$(3) \quad 5t^2 + 8t - 4$$

$$= (5t - 2)(t + 2)$$

$$(4) \quad 6n^2 - 29n + 9$$

$$= (3n - 1)(2n - 9)$$

$$(5) \quad 3p^2 - 16p + 21$$

$$= (3p - 7)(p - 3)$$

$$(6) \quad 12g^2 - 4g - 1$$

$$= (6g + 1)(2g - 1)$$

$$(7) \quad 14r^2 + 17r + 5$$

$$= (2r + 1)(7r + 5)$$

$$(8) \quad 10q^2 + q - 9$$

$$= (10q - 9)(q + 1)$$

$$(9) \quad 8c^2 - 18c + 7$$

$$= (2c - 1)(4c - 7)$$

$$(10) \quad 6m^2 - 7mn + 2n^2$$

$$= (2m - 1n)(3m - 2n)$$

$$(11) \quad 15p^2 - 1pq - 2q^2$$

$$= (5p - 2q)(3p + 1q)$$

$$(12) \quad 12 - 35x + 25x^2$$

$$= (4 - 5x)(3 - 5x)$$

$$(13) \quad 20s^2 - 44st - 15t^2$$

$$= (2s - 5t)(10s + 3t)$$

$$(14) \quad 20a^2 - 1ab - 12b^2$$

$$= (4a + 3b)(5a - 4b)$$

$$(15) \quad 9m^2 - 12m + 4$$

$$= (3m - 2)(3m - 2)$$

$$(16) \quad x^2 - 2xy + y^2$$

$$= (x - y)(x - y)$$

$$(17) \quad 10k^2 - 18k - 10$$

$$= 2(5k^2 - 9k - 5)$$

$$(18) \quad 25a^2 - 30ab + 9b^2$$

$$= (5a - 3b)(5a - 3b)$$

of $(5a - 3b)^2$

$$(19) \quad 18c^2d^2 - 15cd - 12$$

$$= 3(6c^2d^2 - 5cd - 4)$$

$$= 3(3cd - 4)(2cd + 1)$$

$$(20) \quad -12 + 23b^2 - 10b^4$$

$$= -1(10b^4 - 23b^2 + 12)$$

$$= -1(5b^2 - 4)(2b^2 - 3)$$

$$\begin{aligned}
 (21) \quad & \underline{m^2 - 4mn + 4n^2} - 9x^2 \\
 & = \underline{(m - 2n)(m - 2n)} - 9x^2 \\
 & = (m - 2n)^2 - 9x^2 \\
 & = (m - 2n - 3x)(m - 2n + 3x)
 \end{aligned}$$

$$\begin{aligned}
 (23) \quad & x^2 - \underline{n^2 + 2mn - m^2} \\
 & = x^2 - 1(n^2 - 2mn + m^2) \\
 & = x^2 - (n - m)^2 \\
 & = [x - (n - m)][x + (n - m)] \\
 & = [x - n + m][x + n - m]
 \end{aligned}$$

$$\begin{aligned}
 (22) \quad & \underline{4p^2 - 12pq + 9q^2} - 16y^2 \\
 & = \underline{(2p - 3q)(2p - 3q)} - 16y^2 \\
 & = (2p - 3q)^2 - 16y^2 \\
 & = (2p - 3q - 4y)(2p - 3q + 4y)
 \end{aligned}$$

$$\begin{aligned}
 (24) \quad & 9a^2 - 1 - \underline{25y^2 + 10y} \\
 & = 9a^2 - 1(25y^2 + 10y + 1) \\
 & = 9a^2 - (5y + 1)^2 \\
 & = [3a - (5y + 1)][3a + (5y + 1)] \\
 & = [3a - 5y + 1][3a + 5y + 1]
 \end{aligned}$$

A2.2.6 Som en verschil van derdemagte:

Oefening 13:

Faktoriseer volledig:

$$\begin{aligned}
 (1) \quad & a^3 - 1 \\
 & = (a - 1)(a^2 + 1a + 1)
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad & 1\,000 + t^3 \\
 & = (10 + t)(100 - 10t + t^2)
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad & c^3d^3 + 64 \\
 & = (cd + 4)(c^2d^2 - 4cd + 16)
 \end{aligned}$$

$$\begin{aligned}
 (7) \quad & 8p^3 + 27q^3 \\
 & = (2p + 3q)(4p^2 - 6pq + 9q^2)
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad & 1 - 216t^3 \\
 & = (1 - 6t)(1 + 6t + 36t^2)
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & m^3 - 8 \\
 & = (m - 2)(m^2 + 2m + 4)
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad & x^6 + y^9 \\
 & = (x^2 + y^3)(x^4 - x^2y^3 + y^6)
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad & 125 - p^{12} \\
 & = (5 - p^4)(25 + 5p^4 + p^8)
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad & 100 - y^6 \\
 & = (10 - y^3)(10 + y^3)
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad & r^3 + \frac{1}{8} \\
 & = \left(r + \frac{1}{2}\right)\left(r^2 + \frac{1}{2}r + \frac{1}{4}\right)
 \end{aligned}$$