Grade 10 – Textbook Answers

(Revised edition – CAPS)

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Chapter A1

Number systems

NO CALCULATOR MAY BE USED IN THIS CHAPTER!

A1.1 Number systems:

Exercise 1:

Complete:
* Natural numbers: $\mathbb{N} = \{1\,;\,2\,;\,3\,;\,\ldots\ldots\}$
* Whole numbers: $\mathbb{N}_0 = \{0\,;\,1\,;\,2\,;\,3\,;\,\ldots\ldots\}$
* Integers: $\mathbb{Z} = \{\ldots;\,-2\,;\,-1\,;0\,;1\,;\,2\,;\,3\,;\ldots\}$
* Rational numbers: $\mathbb{Q} = \left\{\frac{a}{b} \,/\, a, b \in \mathbb{Z}; b \neq 0\right\}$

A1.2 Rational numbers:

A1.2.1 Equivalent fractions:

Exercise 2:

These are possible answers!

(1) Write down three equivalent fractions for each of the following rational numbers:

(a) $\frac{-1}{4} = \frac{-2}{8} = \frac{-3}{12} = \frac{-10}{40}$

(b) $\frac{3}{7} = \frac{6}{14} = \frac{9}{21} = \frac{15}{35}$

(c) $\frac{1}{6} = \frac{2}{12} = \frac{10}{60} = \frac{100}{600}$

(d) $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{20}{30}$

(e) $\frac{12}{14} = \frac{6}{7} = \frac{24}{28} = \frac{120}{140}$

(f) $\frac{-36}{-9} = \frac{-4}{-1} = \frac{4}{1} = \frac{12}{3}$

(g) $2\frac{6}{11} = \frac{28}{11} = 2\frac{12}{22} = 2\frac{60}{110}$

(h) $5 = \frac{5}{1} = \frac{10}{2} = \frac{25}{5}$

(2) Are the following equivalent fractions or not? (Answer yes or no only.)

(a) $\frac{12}{5} = \frac{24}{10}$? : **Yes**

(b) $\frac{7}{3} = \frac{3}{7}$? : **No**

(c) $\frac{3}{-2} = \frac{6}{4}$? : **No**

(d) $\frac{3}{-5} = \frac{-9}{15}$? : **Yes**

(e) $\frac{2}{3} = \frac{4}{9}$? : **No**

(f) $\frac{3}{1} = \frac{48}{16}$? : **Yes**

(g) $\frac{4}{3} = \frac{-12}{-9}$? : **Yes**

(h) $\frac{25}{10} = \frac{5}{2}$? : **Yes**

(i) $\frac{5}{4} = \frac{4}{3}$? : **No**

A1.2.2 Order of rational numbers:

Exercise 3:

(1) Arrange the following fractions in ascending order:

(a) $\frac{3}{4}$; $\frac{2}{3}$ and $\frac{4}{5}$: $\frac{3}{4}=\frac{45}{60}$; $\frac{2}{3}=\frac{40}{60}$; $\frac{4}{5}=\frac{48}{60}$ $\therefore \frac{2}{3}<\frac{3}{4}<\frac{4}{5}$

(b) $\frac{2}{3}$; $\frac{5}{7}$ and $\frac{4}{6}$: $\frac{2}{3}=\frac{28}{42}$; $\frac{5}{7}=\frac{30}{42}$; $\frac{4}{6}=\frac{28}{42}$ $\therefore \frac{2}{3}=\frac{4}{6}<\frac{5}{7}$

(2) Arrange the following fractions in descending order:

(a) $\frac{5}{8}$; $\frac{2}{3}$ and $\frac{3}{4}$: $\frac{5}{8}=\frac{15}{24}$; $\frac{2}{3}=\frac{16}{24}$; $\frac{3}{4}=\frac{18}{24}$ $\therefore \frac{3}{4}>\frac{2}{3}>\frac{5}{8}$

(b) $-1\frac{1}{2}$; $-1\frac{2}{3}$ and $\frac{-7}{5}$: $\frac{-3}{2}=\frac{-45}{30}$; $\frac{-5}{3}=\frac{-50}{30}$; $\frac{-7}{5}=\frac{-42}{30}$ $\therefore \frac{-7}{5}>-1\frac{1}{2}>-1\frac{2}{3}$

(3) Place a rational number between each of the following numbers:

(a) $\frac{-1}{3}$ and $\frac{-3}{5}$: $\frac{-1}{3}=\frac{-5}{15}$ and $\frac{-3}{5}=\frac{-9}{15}$ $\therefore \frac{-3}{5}<\frac{-8}{15}$ or $\frac{-7}{15}$ or $\frac{-6}{15}<\frac{-1}{3}$

(b) $\frac{3}{4}$ and $\frac{7}{10}$: $\frac{3}{4}=\frac{15}{20}=\frac{30}{40}$ and $\frac{7}{10}=\frac{14}{20}=\frac{28}{40}$ $\therefore \frac{7}{10}<\frac{29}{40}<\frac{3}{4}$

A1.2.3 Conversion of common fractions to decimal fractions:

Exercise 4:

Express the following as decimal fractions, without using a calculator:

(1) $\frac{22}{7}=\frac{22{,}000\ldots}{7}=\frac{22,{}^{1}0\,{}^{3}0\,{}^{2}0}{7}=3,142\ldots$

(2) $4\frac{2}{3}=4\frac{2,{}^{2}0\,{}^{2}0\,{}^{2}0}{3}=4,66\ldots\approx 4,\dot{6}$

(3) $\frac{1}{8}=\frac{1{,}000\ldots}{8}=\frac{1,{}^{1}0\,{}^{2}0\,{}^{4}0}{8}=0,125$

(4) $\frac{7}{9}=\frac{7{,}000\ldots}{9}=\frac{7,{}^{7}0\,{}^{7}0\,{}^{7}0}{9}=0,77\ldots\approx 0,\dot{7}$

(5) $\frac{17}{25}=\frac{17{,}000\ldots}{25}=\frac{17,{}^{17}0\,{}^{20}0}{25}=0,68$

(6) $\frac{5}{100}=\frac{5{,}000\ldots}{100}=\frac{5,{}^{5}0\,{}^{50}0}{100}=0,05$

(7) $\frac{4}{11}=\frac{4,{}^{4}0\,{}^{7}0\,{}^{4}0\,{}^{7}0\,{}^{4}0}{11}=0,3636\ldots\approx 0,\dot{3}\dot{6}$

(8) $-2\frac{6}{7}=-2\frac{6,{}^{6}0\,{}^{4}0\,{}^{5}0}{7}=-2,857\ldots$

(9) $-5\frac{5}{6}=-5\frac{5,{}^{5}0\,{}^{2}0\,{}^{2}0}{6}=-5,833\ldots\approx -5,8\dot{3}$

(10) $\frac{33}{8}=\frac{33{,}000\ldots}{8}=\frac{33,{}^{1}0\,{}^{2}0\,{}^{4}0}{8}=4,125$

A1.2.4 Rounding off decimal fractions:

Exercise 5:

(1) Round off the following fractions correct to the number of decimals indicated in brackets:

(a) 3,573 (to 2 dec)

$\approx 3,57$

(b) 12,00873 (to 3 dec)

$\approx 12,009$

(c) 0,00384 (to 5 dec)

$= 0,00384$

(d) 7,3226 (to 1 dec)

$\approx 7,3$

(e) 8,39999 (to 1 dec)

$\approx 8,4$

(f) 90,9023 (to the nearest integer)

≈ 91

(g) 0,433218 (to 4 dec)

$\approx 0,4332$

(h) 1 456,6799 (to 3 dec)

$\approx 1\,456,680$

(i) 66,666 (to 2 dec)

$\approx 66,67$

(j) 13,00034 (to 3 dec)

$\approx 13,000$

(2) Consider the following and choose the correct way of rounding off in brackets:

(a) 3,47653 ≈ 3,477 correct to the nearest (tenth, hundredth or **thousandth**)

(b) 96 995,31956 ≈ 96 995,32 correct to the nearest (tenth, **hundredth** or thousandth)

A1.2.5 Conversion of decimal fractions to common fractions:

Exercise 6:

Express the following as common fractions in its simplest form:

(1) $0,125 = \frac{125}{1\,000} = \frac{125}{1\,000} \div \frac{125}{125}$

$= \frac{1}{8}$

(2) $1,25 = 1\frac{25}{100} = 1\frac{25}{100} \div \frac{25}{25}$

$= 1\frac{1}{4}$

(3) $14,6 = 14\frac{6}{10} = 14\frac{6}{10} \div \frac{2}{2}$

$= 14\frac{3}{5}$

(4) $-0,5 = -\frac{5}{10} = -\frac{5}{10} \div \frac{5}{5}$

$= -\frac{1}{2}$

(5) $-1,2 = -1\frac{2}{10} = -1\frac{2}{10} \div \frac{2}{2}$

$= -1\frac{1}{5}$

(6) $23,5 = 23\frac{5}{10} = 23\frac{5}{10} \div \frac{5}{5}$

$= 23\frac{1}{2}$

(7) $3,04 = 3\frac{4}{100} = 3\frac{4}{100} \div \frac{4}{4}$

$= 3\frac{1}{25}$

(8) $7,3 = 7\frac{3}{10}$

(9) $100,75 = 100\frac{75}{100} = 100\frac{75}{100} \div \frac{25}{25}$

$= 100\frac{3}{4}$

(10) $0,00005 = \frac{5}{100\,000} = \frac{5}{100\,000} \div \frac{5}{5}$

$= \frac{1}{20\,000}$

A1.2.6 Conversion of recurring fractions to common fractions:

Exercise 7:

Convert the following to common fractions in its simplest form: (Without a calculator.)

(1) $3,\dot{6}$

$= 3\frac{6}{9}$

$= 3\frac{6}{9} \div \frac{3}{3}$

$= 3\frac{2}{3}$

(2) $0,\dot{1}\dot{3}$

$= \frac{13}{99}$

(3) $22,3\dot{9}$

$= 22 + 0,3 + 0,0\dot{9}$

$= 22 + \frac{3}{10} + 0,\dot{9} \div 10$

$= 22 + \frac{3}{10} + \frac{9}{9} \times \frac{1}{10}$

$= 22 + \frac{3}{10} + 1 \times \frac{1}{10}$

$= 22 + \frac{3}{10} + \frac{1}{10}$

$= 22 + \frac{3+1}{10} = 22\frac{4}{10}$

$= 22\frac{2}{5}$

(4) $-1,\dot{1}\dot{3}\dot{5}$ or $-1,\overline{135}$

$= -1\frac{135}{999}$

$= -1\frac{135}{999} \div \frac{27}{27}$

$= -1\frac{5}{37}$

(5) $0,\dot{7}$

$= \frac{7}{9}$

(6) $0,00\dot{3}$

$= 0,\dot{3} \div 100$

$= \frac{3}{9} \times \frac{1}{100} = \frac{3}{900}$

$= \frac{1}{300}$

(7) $1,\overline{214}$

$= 1\frac{214}{999}$

(8) $3,2\dot{5}\dot{8}$

$= 3 + 0,2 + 0,0\dot{5}\dot{8}$

$= 3 + \frac{2}{10} + 0,\dot{5}\dot{8} \div 10$

$= 3 + \frac{2}{10} + \frac{58}{99} \times \frac{1}{10}$

$= 3 + \frac{2}{10} \times \frac{99}{99} + \frac{58}{990}$

$= 3 + \frac{198}{990} + \frac{58}{990} = 3\frac{256}{990}$

$= 3\frac{128}{495}$

☺ **Calculate the following without using a calculator:** $0,\dot{4} + \frac{2}{3}$

$$0,\dot{4} + \frac{2}{3} = \frac{4}{9} + \frac{2}{3} = \frac{4}{9} + \frac{2}{3} \times \frac{3}{3} = \frac{4}{9} + \frac{6}{9} = \frac{10}{9} = 1\frac{1}{9}$$

A1.3 Irrational and Real numbers:

Exercise 8:

(1) Which of the numbers are Rational numbers ($\mathbb{Q}$) and which are Irrational numbers ($\mathbb{Q}'$)?

(a) 14 : $\mathbb{Q}$ (b) $\frac{1}{5}$: $\mathbb{Q}$ (c) $\sqrt{81}$: $\mathbb{Q}$

(d) $0{,}12$: $\mathbb{Q}$ (e) $\sqrt{18}$: $\mathbb{Q}'$ (f) $12,\dot{2}\dot{3}$: $\mathbb{Q}$

(g) $-\sqrt{\frac{12}{3}}$: $\mathbb{Q}$ (h) $0{,}2945\ldots$: $\mathbb{Q}'$ (i) $\sqrt[3]{64}$: $\mathbb{Q}$

(j) π : $\mathbb{Q}'$ (k) $\sqrt[5]{32}$: $\mathbb{Q}$ (l) $\frac{11}{7}$: $\mathbb{Q}$

(2) Between which two integers do the following irrational numbers lie?

(a) $-\sqrt{12}$

$$-\sqrt{16} < -\sqrt{12} < -\sqrt{9}$$

$$-4 < -\sqrt{12} < -3$$

(b) $\sqrt{66}$

$$\sqrt{64} < \sqrt{66} < \sqrt{81}$$

$$8 < \sqrt{66} < 9$$

(c) $\sqrt[3]{5}$

$$\sqrt[3]{1} < \sqrt[3]{5} < \sqrt[3]{8}$$

$$1 < \sqrt[3]{5} < 2$$

(d) $\sqrt[5]{2}$

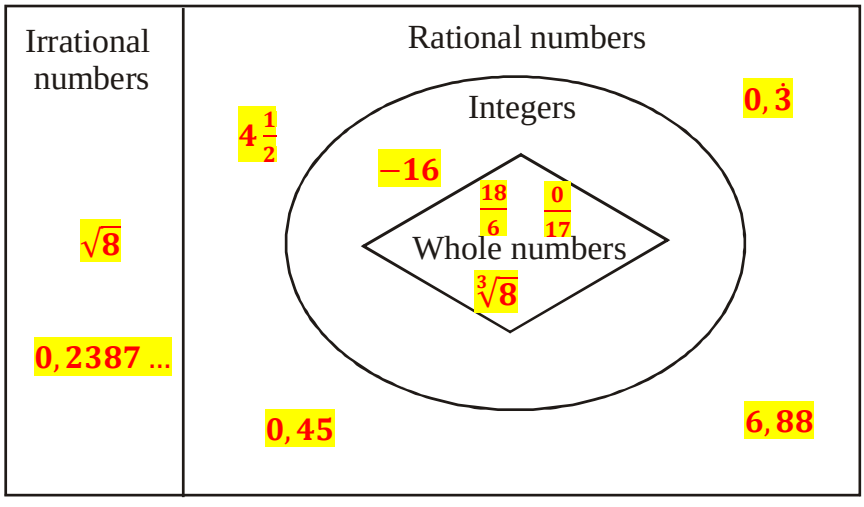
$$\sqrt[5]{1} < \sqrt[5]{2} < \sqrt[5]{32}$$

$$1 < \sqrt[5]{2} < 2$$

(3) The diagram below is a summary of all the numbers that are used on school level.
Place the following numbers in the right place on the table; simplify the number if necessary:

$4\frac{1}{2}$; $\sqrt[3]{8} = 2$; $\sqrt{8}$; -16 ; $0{,}45$; $0,\dot{3}$; $\frac{18}{6} = 3$; $0{,}2387\ldots$; $\frac{0}{17} = 0$; $6{,}88$

Real numbers:



☺ **(1) Except for the real numbers we also have the non-real numbers.**
Give an example of a non-real number. $\sqrt{-1}$; $\sqrt[3]{-2}$; ... ...

(2) What is the set called that contain all real and non-real numbers?

Complex numbers

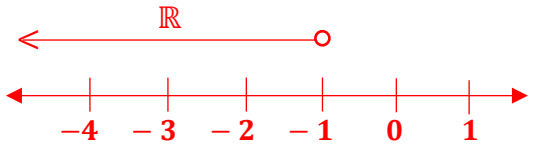
A1.4 Representation of sets of numbers:

Exercise 9:

(1) Write the following in interval notation (if applicable) and represent it on a number line:

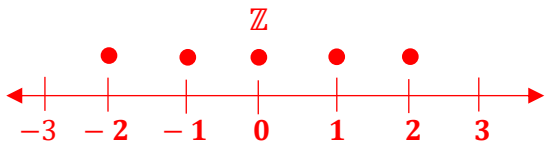
(a) $\{x : x < -1;\ x \in \mathbb{R}\}$

$x \in (-\infty\ ;\ -1)$



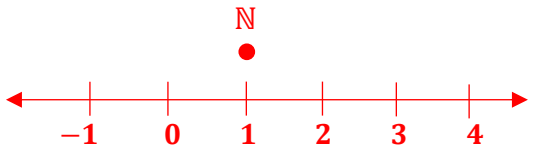
(b) $\{x : -3 < x < 3;\ x \in \mathbb{Z}\}$

No interval notation



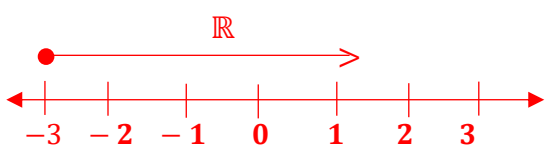
(c) $\{y : y < 2\ ;\ y \in \mathbb{N}\}$

No interval notation



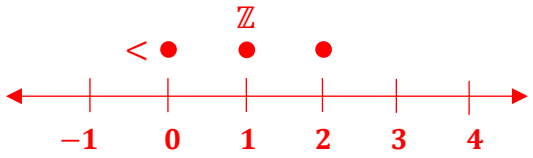
(d) $\{x : x \geq -3;\ x \in \mathbb{R}\}$

$x \in [-3\ ;\ \infty)$



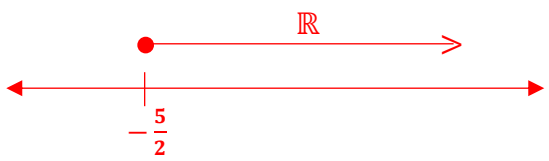
(e) $\{x\ /\ x < 3;\ x \in \mathbb{Z}\}$

No interval notation



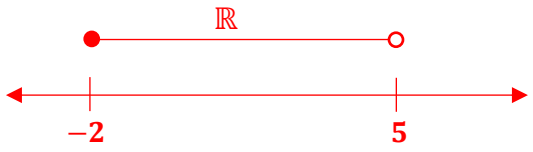
(f) $\{p\ /\ p \geq \frac{-5}{2};\ p \in \mathbb{R}\}$

$p \in [-\frac{5}{2}\ ;\ \infty)$



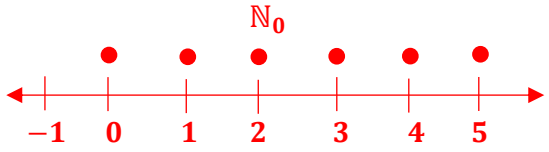
(g) $\{m : -2 \leq m < 5;\ m \in \mathbb{R}\}$

$m \in [-2\ ;\ 5)$



(h) $\{x : x \leq 5;\ x \in \mathbb{N}_0\}$

No interval notation

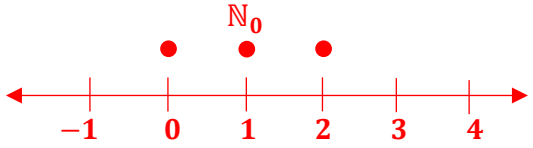


(2) Solve for x in each of the following and represent the solution on a number line:

(a) $x + 1 \leq 3;\quad x \in \mathbb{N}_0$

$x \leq 3 - 1$

$x \leq 2$



(b) $2x \geq -8;\quad x \in \mathbb{R}$

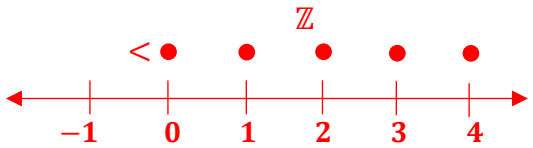
$x \geq \frac{-8}{2}$

$x \geq -4$

(c) $x - 4 \leq 0;\quad x \in \mathbb{Z}$

$x \leq 0 + 4$

$x \leq 4$

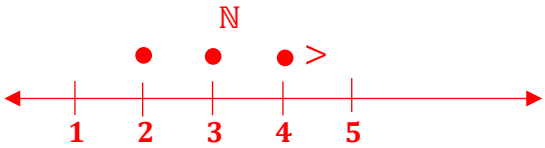


(d) $2x + 3 > 7;\quad x \in \mathbb{N}$

$2x > 7 - 3$

$x \geq \frac{4}{2}$

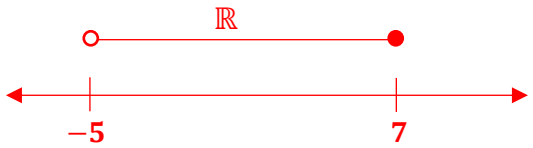
$x \geq 2$



(e) $-6 < x - 1 \leq 6;\quad x \in \mathbb{R}$

$-6 + 1 < x \leq 6 + 1$

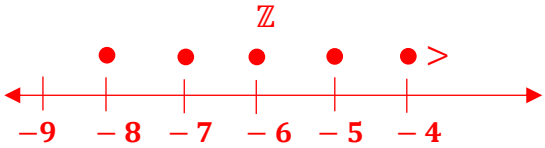
$-5 < x \leq 7$



(f) $x + 7 \geq -1;\quad x \in \mathbb{Z}$

$x \geq -1 - 7$

$x \geq -8$



A1.5 REVISION EXERCISE:

(1) Convert the following to common fractions in its simplest form: (Without a calculator.)

(a) $14,1\dot{7}$

$= 14 + 0,1 + 0,0\dot{7}$

$= 14 + \frac{1}{10} + 0,\dot{7} \div 10$

$= 14 + \frac{1}{10} + \frac{7}{9} \times \frac{1}{10}$

$= 14 + \frac{1}{10} + \frac{7}{90}$

$= 14 + \frac{1 \times 9}{10 \times 9} + \frac{7}{90}$

$= 14 + \frac{9}{90} + \frac{7}{90} = 14\frac{16}{90} = 14\frac{8}{45}$

(b) $0,\overline{1234}$

$= \frac{1\,234}{9\,999}$

(c) 4,68

$= 4\frac{68}{100}$

$= 4\frac{68}{100} \div \frac{4}{4}$

$= 4\frac{17}{25}$

(d) $5,\dot{1}$

$= 5\frac{1}{9}$

(2) Indicate, by using a ✓, all the rational numbers between 0 and 10:

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
$\sqrt{9} = 3$	-1	$\sqrt{8}$	$\frac{6}{3} = 2$	$\sqrt[3]{16}$	π	$\frac{0}{3} = 0$	$\sqrt{144} = 12$	4,124….	$\sqrt{\frac{24}{6}} = \sqrt{4} = 2$
✓			✓						✓

(3) Round off the following fractions correct to the number of decimals indicated in brackets:

(a) 7,199 (to 1 dec)

$\approx 7,2$

(b) 0,048561 (to 4 dec)

$\approx 0,0486$

(c) 234,34 (to 1 dec)

$\approx 234,3$

(d) 1 001,1989 (to the nearest integer)

$\approx 1\,001$

(e) 3,997 (to 2 dec)

$\approx 4,00$

(f) 23,7712 (to the nearest integer)

≈ 24

(4) Place any two irrational numbers between 2 and 3.

$2 = \sqrt{4}$ and $3 = \sqrt{9}$

$\therefore\ 2 < \sqrt{5} < \sqrt{6} < \sqrt{7} < \sqrt{8} < 3$

Any 2!

(5) Between which two integers do the following irrational numbers lie?

(a) $\sqrt{\frac{1}{2}}$

$\sqrt{\frac{1}{1}} > \sqrt{\frac{1}{2}} > \sqrt{\frac{1}{4}}$

$1 > \sqrt{\frac{1}{2}} > \frac{1}{2}$

or $\frac{1}{2} < \sqrt{\frac{1}{2}} < 1$

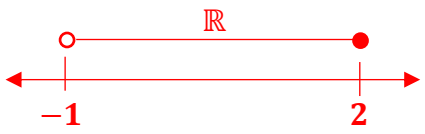
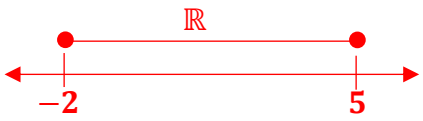
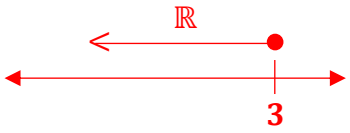
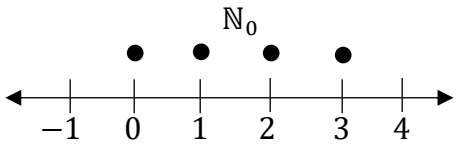
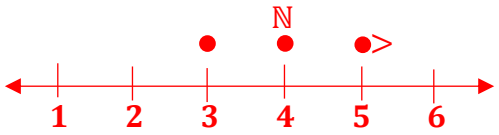
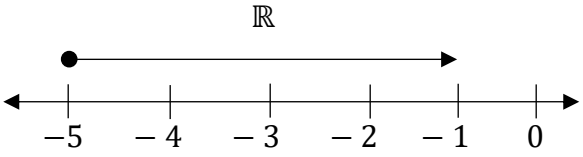
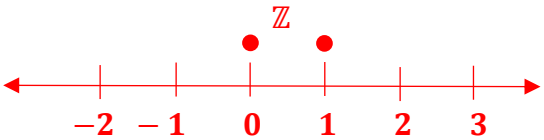
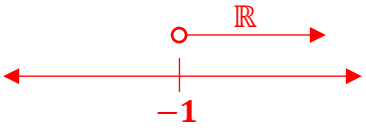
(b) $\sqrt[3]{57}$

$\sqrt[3]{27} < \sqrt[3]{57} < \sqrt[3]{64}$

$3 < \sqrt[3]{57} < 4$

$4 > \sqrt[3]{57} > 3$

(6) Complete the missing representations in the table below:

	Set builder notation:	Interval notation:	Number line:
(1)	$\{x \,/\, -1 < x \leq 2;\ x \in \mathbb{R}\}$	$x \in (-1\,;\,2]$	
(2)	$\{x : -2 \leq x \leq 5\,;x \in \mathbb{R}\}$	$x \in [-2\,;5]$	
(3)	$\{y \,/\, y \leq 3\,;y \in \mathbb{R}\}$	$y \in (-\infty\,;3]$	
(4)	$\{x \,/\, x \leq 3\,;x \in \mathbb{N}_0\}$	N/A	
(5)	$\{y \,/\ y \geq 3;\ y \in \mathbb{N}\}$	N/A	
(6)	$\{m : 0 < m \leq 4\,;m \in \mathbb{R}\}$	$m \in (0\,;4]$	
(7)	$\{t : t \geq -5\,;t \in \mathbb{R}\}$	$t \in [-5\,;\infty)$	
(8)	$\{m : m \leq 6;\ m \in \mathbb{R}\}$	$m \in (-\infty\,;6]$	
(9)	$\{x \,/\, -1 < x < 2;\ x \in \mathbb{Z}\}$	N/A	
(10)	$\{x \,/\, x > -1\,;x \in \mathbb{R}\}$	$x \in (-1;\ \infty)$	

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Chapter A2

Algebraic expressions

A2.1 Products:

A2.1.1 The law of distribution:

Exercise 1:

Determine the following products:

(1) $(y - 4)(y + 3)$

$= y^2 + 3y - 4y - 12$

$= y^2 - y - 12$

(2) $(p - 2)(p - 7)$

$= p^2 - 7p - 2p + 14$

$= p^2 - 9p + 14$

(3) $(2x + 1)(x - 5)$

$= 2x^2 - 10x + 1x - 5$

$= 2x^2 - 9x - 5$

(4) $(x - 2y)(2x - y)$

$= 2x^2 - 1xy - 4xy + 2y^2$

$= 2x^2 - 5xy + 2y^2$

(5) $(4ab + 1)(2ab - 3)$

$= 8a^2b^2 - 12ab + 2ab - 3$

$= 8a^2b^2 - 10ab - 3$

(6) $(5 - 7m)(2 - 3m)$

$= 10 - 15m - 14m + 21m^2$

$= 10 - 29m + 21m^2$

(7) $(2a - 4b)(3a + 2b)$

$= 6a^2 + 4ab - 12ab - 8b^2$

$= 6a^2 - 8ab - 8b^2$

(8) $(m + n)(2m - 1)$

$= 2m^2 - 1m + 2mn - n$

(9) $(d - 12)(12 + d)$

$= 12d + d^2 - 144 - 12d$

$= d^2 - 144$

(10) $(a^2 + 4)(a^2 + 2)$

$= a^4 + 2a^2 + 4a^2 + 8$

$= a^4 + 6a^2 + 8$

(11) $\left(\frac{1}{2}m - 6\right)(8m - 3)$

$= 4m^2 - \frac{3}{2}m - 48m + 18$

$= 4m^2 - 49\frac{1}{2}m + 18$

(12) $(-2k - 5)(5 + 3k)$

$= -10k - 6k^2 - 25 - 15k$

$= -6k^2 - 25k - 25$

(13) $\left(p + \frac{1}{p}\right)\left(8p - \frac{4}{p}\right)$

$= 8p^2 - \frac{4p}{p} + \frac{8p}{p} - \frac{4}{p^2}$

$= 8p^2 - 4 + 8 - \frac{4}{p^2}$

$= 8p^2 + 4 - \frac{4}{p^2}$

(14) $(abc - 2ac)(abc + 3bc)$

$= a^2b^2c^2 + 3ab^2c^2 - 2a^2bc^2 - 6abc^2$

(15) $(3r^3 + 2)(2r^2 - 5)$

$= 6r^5 - 15r^3 + 4r^2 - 10$

(16) $2x(x - 5y)(3x + 2y)$

$= 2x(3x^2 + 2xy - 15xy - 10y^2)$

$= 2x(3x^2 - 13xy - 10y^2)$

$= 6x^3 - 26x^2y - 20xy^2$

(17) $\left(\frac{1}{p^3q^2} - \frac{2}{p^2q}\right)\left(\frac{1}{p} + \frac{2}{q}\right)$

$= \frac{1}{p^4q^2} + \frac{2}{p^3q^3} - \frac{2}{p^3q} - \frac{4}{p^2q^2}$

(18) $\left(\frac{m^2n}{3} - \frac{6}{mn}\right)\left(\frac{mn}{2} - \frac{3}{mn^2}\right)$

$= \frac{m^3n^2}{6} - \frac{3m^2n}{3mn^2} - \frac{6mn}{2mn} + \frac{18}{m^2n^3}$

$= \frac{m^3n^2}{6} - \frac{m}{n} - 3 + \frac{18}{m^2n^3}$

Exercise 2:

Simplify:

(1) $(abc - 2)(abc + 2)$

$= a^2b^2c^2 - 4$

(2) $\left(\frac{1}{3} + 5t\right)\left(\frac{1}{3} - 5t\right)$

$= \frac{1}{9} - 25t^2$

(3) $(p - 9q)(9q + p)$

$= (p - 9q)(p + 9q)$

$= p^2 - 81q^2$

(4) $(n + 7k)(7n - k)$

$= 7n^2 - 1kn + 49kn - 7k^2$

$= 7n^2 + 48kn - 7k^2$

(5) $(-a + 4b)(-a - 4b)$

$= a^2 + 4ab - 4ab - 16b^2$

$= a^2 - 16b^2$

(6) $-x\left(\frac{1}{x} - x\right)\left(\frac{1}{x} + x\right)$

$= -x\left(\frac{1}{x^2} - x^2\right)$

$= \frac{-x}{x^2} + x^3\left(\frac{1}{x^2} - x^2\right)$

$= \frac{-1}{x} + x^3$

(7) $(x^{2m} - 8)(x^{2m} + 8)$

$= x^{2m+2m} - 64$

$= x^{4m} - 64$

(8) $(0{,}3 + 3q)(0{,}3 - 3q)$

$= 0{,}09 - 9q^2$

(9) $(b^6c^3 + 6)(b^6c^3 + 6)$

$= b^{12}c^6 + 6b^6c^3 + 6b^6c^3 + 36$

$= b^{12}c^6 + 12b^6c^3 + 36$

(10) $(4xk^5 - 7)(7 + 4xk^5)$

$= (4xk^5 - 7)(4xk^5 + 7)$

$= 16x^2k^{10} - 49$

(11) $(m - 2n)^2(m + 2n)^2$

$= [(m - 2n)(m + 2n)]^2$

$= [m^2 - 4n^2]^2$

$= m^4 - 8m^2n^2 + 16n^4$

(12) $\left(\frac{m}{n} + 2\right)\left(\frac{m^2}{n^2} + 4\right)\left(\frac{m}{n} - 2\right)$

$= \left(\frac{m}{n} + 2\right)\left(\frac{m}{n} - 2\right)\left(\frac{m^2}{n^2} + 4\right)$

$= \left(\frac{m^2}{n^2} - 4\right)\left(\frac{m^2}{n^2} + 4\right)$

$= \frac{m^4}{n^4} - 16$

A2.1.2 Squaring of a binomial:

Exercise 3:

Determine the following squares:

(1) $(y - 11)^2$

$= (y - 11)(y - 11)$

$= y^2 - 11y - 11y + 121$

$= y^2 - 22y + 121$

(2) $(3p + 2q)^2$

$= (3p + 2q)(3p + 2q)$

$= 9p^2 + 6pq + 6pq + 4q^2$

$= 9p^2 + 12pq + 4q^2$

(3) $(-4 + 5c)^2$

$= (-4 + 5c)(-4 + 5c)$

$= 16 - 20c - 20c + 25c^2$

$= 16 - 40c + 25c^2$

(4) $(mn + 3)^2$

$= (mn + 3)(mn + 3)$

$= m^2n^2 + 3mn + 3mn + 9$

$= m^2n^2 + 6mn + 9$

(5) $(k^2 + 1)^2$

$= (k^2 + 1)(k^2 + 1)$

$= k^4 + 1k^2 + 1k^2 + 1$

$= k^4 + 2k^2 + 1$

(6) $(8 - 3b)^2$

$= (8 - 3b)(8 - 3b)$

$= 64 - 24b - 24b + 9b^2$

$= 64 - 48b + 9b^2$

(7) $\left(x - \frac{1}{2}\right)^2$

$= \left(x - \frac{1}{2}\right)\left(x - \frac{1}{2}\right)$

$= x^2 - \frac{1}{2}x - \frac{1}{2}x + \frac{1}{4}$

$= x^2 - 1x + \frac{1}{4}$

(8) $\left(\frac{y}{5} - 3\right)^2$

$= \left(\frac{y}{5} - 3\right)\left(\frac{y}{5} - 3\right)$

$= \frac{y^2}{25} - \frac{3y}{5} - \frac{3y}{5} + 9$

$= \frac{y^2}{25} - \frac{6y}{5} + 9$

(9) $(5p - 2p^2)^2$

$= (5p - 2p^2)(5p - 2p^2)$

$= 25p^2 - 10p^3 - 10p^3 + 4p^4$

$= 25p^2 - 20p^3 + 4p^4$

(10) $\left(4 + \frac{3}{n}\right)^2$

$= \left(4 + \frac{3}{n}\right)\left(4 + \frac{3}{n}\right)$

$= 16 + \frac{12}{n} + \frac{12}{n} + \frac{9}{n^2}$

$= 16 + \frac{24}{n} + \frac{9}{n^2}$

(11) $(0,2 + 6y)^2$

$= (0,2 + 6y)(0,2 + 6y)$

$= 0,04 + 1,2y + 1,2y + 36y^2$

$= 0,04 + 2,4y + 36y^2$

(12) $\left(\frac{2m}{p} + \frac{p^2}{3m}\right)^2$

$= \left(\frac{2m}{p} + \frac{p^2}{3m}\right)\left(\frac{2m}{p} + \frac{p^2}{3m}\right)$

$= \frac{4m^2}{p^2} + \frac{2mp^2}{3mp} + \frac{2mp^2}{3mp} + \frac{p^4}{9m^2}$

$= \frac{4m^2}{p^2} + \frac{4p}{3} + \frac{p^4}{9m^2}$

Exercise 4:

Simplify (Use the shorter method!)

(1) $(x - 3)^2$

$= x^2 - 6x + 9$

(2) $(6m - 1)^2$

$= 36m^2 - 12m + 1$

(3) $(3y + 7)^2$

$= 9y^2 + 42y + 49$

(4) $(3 + pq)^2$

$= 9 + 6pq + p^2q^2$

(5) $(5t^2 + 8)^2$

$= 25t^4 + 80t^2 + 64$

(6) $\left(\frac{2}{3} - 6y\right)^2$

$= \frac{4}{9} - 8y + 36y^2$

(7) $(-2k - 5)^2$

$= 4k^2 + 20k + 25$

(8) $\left(\frac{3p - 2q}{5m}\right)^2$

$= \frac{9p^2 - 12pq + 4q^2}{25m^2}$

(9) $(4x^2 + 10y^2)^2$

$= 16x^4 + 80x^2y^2 + 100y^4$

(10) $(2mn + 7)(7 + 2mn)$

$= (2mn + 7)(2mn + 7)$

$= 4m^2n^2 + 28mn + 49$

(11) $(8 - 3y)(8 + 3y)$

$= 64 - 9y^2$

(12) $-2(abc - 11)^2$

$= -2(a^2b^2c^2 - 22abc + 121)$

$= -2a^2b^2c^2 + 44abc - 242$

A2.1.3 Binomials and trinomials:

Exercise 5:

Simplify the following products:

(1) $(2a - 3)(a^2 + 5a - 4)$

$= 2a^3 + 10a^2 - 8a - 3a^2 - 15a + 12$

$= 2a^3 + 7a^2 - 23a + 12$

(2) $(m + 7)(2m^2 + 3m + 3)$

$= 2m^3 + 3m^2 + 3m + 14m^2 + 21m + 21$

$= 2m^3 + 17m^2 + 24m + 21$

(3) $(1 + x)(1 - x + x^2)$

$= 1 - 1x + 1x^2 + 1x - 1x^2 + x^3$

$= 1 + x^3$

(4) $(3y - 2)(9y^2 + 6y + 4)$

$= 27y^3 + 18y^2 + 12y - 18y^2 - 12y - 8$

$= 27y^3 - 8$

(5) $\left(2m + \frac{1}{2}\right)\left(\frac{m^2}{4} + 4 - 4m\right)$

$= \frac{2m^3}{4} + 8m - 8m^2 + \frac{m^2}{8} + 2 - \frac{4m}{2}$

$= \frac{m^3}{2} + 8m - \frac{8m^2}{1} + \frac{1m^2}{8} + 2 - 2m$

$= \frac{m^3}{2} + 6m - 7\frac{7}{8}m^2 + 2$

(6) $(m^2n^2 - 5)(25 + 5m^2n^2 + m^4n^4)$

$= 25m^2n^2 + 5m^4n^4 + m^6n^6 - 125 - 25m^2n^2 - 5m^2n^2$

$= m^6n^6 - 125$

A2.1.4 The sum and difference of two cubes:

Exercise 6:

Write down the following products directly, if possible:

(1) $(a + 3)(a^2 - 3a + 9)$

$= a^3 + 27$

(2) $(2y^3 + 4)(4y^6 - 8y^3 + 16)$

$= 8y^9 + 64$

(3) $\left(\frac{x}{3} - 1\right)\left(\frac{1}{9}x^2 + \frac{1}{3}x + 1\right)$

$= \frac{x^3}{27} - 1$

(4) $\left(6a^2 - \frac{1}{2}\right)\left(36a^4 + 3a^2 + \frac{1}{4}\right)$

$= 216a^6 - \frac{1}{8}$

(5) $(5q + 7)(25q^2 - 35q + 49)$

$= 125q^3 + 343$

(6) $(8 - 3m)(9m^2 + 24m + 64)$

$= (8 - 3m)(64 + 24m + 9m^2)$

$= 512 - 27m^3$

(7) $(x - 5)(x^2 - 5x + 25)$

$= x^3 - 5x^2 + 25x - 5x^2 + 25x - 125$

$= x^3 - 10x^2 + 50x - 125$

(8) $(0{,}1 + 0{,}2y)(0{,}01 - 0{,}02y + 0{,}04y^2)$

$= 0{,}001 + 0{,}008y^3$

(9) $(9a^4 + 6a^2b + 4b^2)(3a^2 - 2b)$

$= (3a^2 - 2b)(9a^4 + 6a^2b + 4b^2)$

$= 27a^6 - 8b^3$

(10) $2(-1 + 5m)(25m^2 + 5m + 1)$

$= 2(5m - 1)(25m^2 + 5m + 1)$

$= 2(125m^3 - 1)$

$= 250m^3 - 2$

A2.1.5 Simplification of expressions:

Exercise 7:

Simplify:

(1) $2a(a + 1) - 3(a - 1)^2$

$= 2a^2 + 2a - 3(a^2 - 2a + 1)$

$= 2a^2 + 2a - 3a^2 + 6a - 3$

$= -1a^2 + 8a - 3$

(2) $(m^3 + 3m^2 - 2m - 2)m + 4(m^3 + 5m^2 - 6)$

$= m(m^3 + 3m^2 - 2m - 2) + 4(m^3 + 5m^2 - 6)$

$= m^4 + 3m^3 - 2m^2 - 2m + 4m^3 + 20m^2 - 24$

$= m^4 + 7m^3 + 18m^2 - 2m - 24$

(3) $(x - 1)(x^2 - x + 1) + 3(x - 2) - x^3$

$= x^3 - x^2 + x - x^2 + x - 1 + 3x - 6 - x^3$

$= -2x^2 + 5x - 7$

(4) $(y - 1)(y + 1)(y^2 + 1)$

$= (y^2 - 1)(y^2 + 1)$

$= y^4 - 1$

(5) $-2(2c + 1)(c - 2) - 2(2c - 1)^2$

$= -2(2c^2 - 3c - 2) - 2(4c^2 - 4c + 1)$

$= -4c^2 + 6c + 4 - 8c^2 + 8c - 2$

$= -12c^2 + 14c + 2$

(6) $\frac{1}{2}(4p - 3)^2 - \left(\frac{p}{2} + 2\right)^2$

$= \frac{1}{2}(16p^2 - 24p + 9) - \left(\frac{p^2}{4} + 2p + 4\right)$

$= 8p^2 - 12p + \frac{9}{2} - \frac{1}{4}p^2 - 2p - 4$

$= 7\frac{3}{4}p^2 - 14p + \frac{1}{2}$

(7) $a^2 + b^2 - (2a - b)^2 + (a + 2b)^2$

$= a^2 + b^2 - (4a^2 - 4ab + b^2) + a^2 + 4ab + 4b^2$

$= a^2 + b^2 - 4a^2 + 4ab - b^2 + a^2 + 4ab + 4b^2$

$= -2a^2 + 4b^2 + 8ab$

(8) $(3y + 1)(9y^2 + 1)(3y - 1)$

$= (3y + 1)(3y - 1)(9y^2 + 1)$

$= (9y^2 - 1)(9y^2 + 1)$

$= 81y^4 - 1$

(9) $7 - 3(5t - 6) + 2t - (t - 1)(t + 1)$

$= 7 - 15t + 18 + 2t - (t^2 - 1)$

$= 7 - 15t + 18 + 2t - t^2 + 1$

$= -1t^2 - 13t + 26$

(10) $(x + 3y)(x^2 - 3xy + 9y^2) + (x + 3y)(x - 3y)$

$= x^3 + 27y^3 + x^2 - 9y^2$

(11) $(2p-3)(4p^2+6p+9)(8p^3+27)$

$= (8p^3-27)(8p^3+27)$

$= 64p^6-729$

(12) $(x-y+k)(x-y-k)$

$= [(x-y)+k][(x-y)-k]$

$= [(x-y)^2-k^2]$

$= x^2-2xy+y^2-k^2$

(13) $(p-3)^3 = (p-3)(p-3)(p-3)$

$= (p-3)(p^2-6p+9)$

$= p^3-6p^2+9p-3p^2+18p-27$

$= p^3-9p^2+27p-27$

(14) $[(2m+n)(4m^2-2mn+n^2)]^2$

$= [8m^3+n^3]^2$

$= 64m^6+16m^3n^3+n^6$

(15) $(x+y)(x-y)-(x-y)^2+(x+y)^2-x(x+y)+(x-y)(x^2+2xy+y^2)$

$= x^2-y^2-(x^2-2xy+y^2)+(x^2+2xy+y^2)-x^2-xy+x^3+2x^2y+xy^2-x^2y-2xy^2-y^3$

$= x^2-y^2-x^2+2xy-y^2+x^2+2xy+y^2-x^2-xy+x^3+2x^2y+1xy^2-1x^2y-2xy^2-y^3$

$= -y^2+3xy+x^3+1x^2y-1xy^2-y^3$

☺ **Simplify:** **(a)** $(x^m+y^n)^2$

$= (x^m+y^n)(x^m+y^n)$

$= x^{2m}+x^my^n+x^my^n+y^{2n}$

$= x^{2m}+2x^my^n+y^{2n}$

(b) $(a^{x+1}-2)(a^{x+1}+2)$

$= (a^{x+1})^2-4$

$= a^{2x+2}-4$

A2.2 Factorisation:

A2.2.1 Common factor:

Exercise 8:

Factorise completely:

(1) $4am+3a^2m^4$

$= am(4+3am^3)$

(2) $12x^2-132y^2$

$= 12(x^2-11y^2)$

(3) $-p^5q^3+pq$

$= pq(-p^4q^2+1)$

(4) $4x^2-8x+1$

$= (4x^2-8x+1)$

(5) $\frac{1}{2}abc+\frac{1}{2}a^2bc^2$

$= \frac{1}{2}abc(1+ac)$

(6) $5r^7R^4+15r^5R^2$

$= 5r^5R^2(r^2R^2+3)$

(7) $7x(3y - 1) - 2(3y - 1)$

$= (3y - 1)(7x - 2)$

(8) $12x^3t - 14xt^3 + 16x^2t^2$

$= 2xt(6x^2 - 7t^2 + 8xt)$

(9) $2gh + 18g^2h + 3g^3h$

$= gh(2 + 18g + 3g^2)$

(10) $a(2m + 3) + b(3 + 2m)$

$= (2m + 3)(a + b)$

(11) $3g(3g + h) - 1(3g + h)$

$= (3g + h)(3g - 1)$

(12) $1(4b + 5) + 7a(4b + 5)$

$= (4b + 5)(1 + 7a)$

(13) $r(x - y) + (y - x) + 2t(x - y)$

$= r(x - y) - 1(x - y) + 2t(x - y)$

$= (x - y)(r - 1 + 2t)$

(14) $5ab(d - 4c) - 7a(4c - d)$

$= 5ab(d - 4c) + 7a(d - 4c)$

$= (d - 4c)(5ab + 7a)$

$= (d - 4c)(a)(5b + 7)$

(15) $4xy + 2 - k(2xy + 1)$

$= 2(2xy + 1) - k(2xy + 1)$

$= (2xy + 1)(2 - k)$

(16) $y^4(x^2 - 5) + y(x^2 - 5)$

$= (x^2 - 5)(y^4 + y)$

$= y(x^2 - 5)(y^3 + 1)$

A2.2.2 Grouping:

Exercise 9:

Factorise completely:

(1) $r - s - 5r + 5s$

$= 1(r - s) - 5(r - s)$

$= (r - s)(1 - 5)$ $= -4(r - s)$

(2) $3am^3 + 6am + 7m^2n + 14n$

$= 3am(m^2 + 2) + 7n(m^2 + 2)$

$= (m^2 + 2)(3am + 7n)$

(3) $pq - pr + q^2 - qr$

$= p(q - r) + q(q - r)$

$= (q - r)(p + q)$

(4) $4x - 8k - rtx + 2rtk$

$= 4(x - 2k) - rt(x - 2k)$

$= (x - 2k)(4 - rt)$

(5) $aw - bw + 2bw - 2aw$

$= -1aw + 1bw$

$= -1w(a - b)$ **or**

$= w(b - a)$

(6) $p^2 + p(2 + q) + 2q$

$= p^2 + 2p + pq + 2q$

$= p(p + 2) + q(p + 2)$

$= (p + 2)(p + q)$

(7) $mx + nx + rx - my - ny - ry$

$= x(m + n + r) - y(m + n + r)$

$= (m + n + r)(x - y)$

(8) $pq - 1 + p - q$

$= pq + p - 1 - q$

$= p(q + 1) - 1(1 + q)$

$= (q + 1)(p - 1)$

(9) $g(h - j) + g^2 - hj$

$= gh - gj + g^2 - hj$

$= gh + g^2 - gj - hj$

$= g(h + g) - j(g + h)$ $= (g + h)(g - j)$

(10) $3mn^3 + 2mt - 7m + 3kn^3 + 2kt - 7k$

$= m(3n^3 + 2t - 7) + k(3n^3 + 2t - 7)$

$= m(3n^3 + 2t - 7) + k(3n^3 + 2t - 7)$

$= (3n^3 + 2t - 7)(m + k)$

A2.2.3 Difference between two squares:

Exercise 10:

Factorise completely:

(1) $c^2 - 81$

$= (c - 9)(c + 9)$

(2) $1 - p^2$

$= (1 + p)(1 - p)$

(3) $x^2y^2 - 25$

$= (xy - 5)(xy + 5)$

(4) $9 + k^2$

$= (9 + k^2)$

(5) $m^{10} - 16$

$= (m^5 - 4)(m^5 + 4)$

(6) $r^6 - k^2$

$= (r^3 - k)(r^3 + k)$

(7) $(xyz)^2 - 121$

$= (xyz - 11)(xyz + 11)$

(8) $n^2 - 50$

$= (n^2 - 50)$

(9) $-9 + t^4 = t^4 - 9$

$= (t^2 + 3)(t^2 - 3)$

(10) $2x^2 - 8$

$= 2(x^2 - 4)$

$= 2(x - 2)(x + 2)$

(11) $-y^2 - 49$

$= -1(y^2 + 49)$

(12) $(x + y)^2 - 100$

$= (x + y - 10)(x + y + 10)$

(13) $p^2m + p^2 - m - 1$

$= p^2(m + 1) - 1(m + 1)$

$= (m + 1)(p^2 - 1)$

$= (m + 1)(p - 1)(p + 1)$

(14) $(m - 3)^2 - (m + 2)^2$

$= [(m - 3) - (m + 2)][(m - 3) + (m + 2)]$

$= [-5][2m - 1]$ or

$= -5(2m - 1)$

(15) $x^3 - 9x$

$= x(x^2 - 9)$

$= x(x - 3)(x + 3)$

(16) $m^4 - n^4$

$= (m^2 - n^2)(m^2 + n^2)$

$= (m - n)(m + n)(m^2 + n^2)$

(17) $ab^7x - ab^3x$

$= ab^3x(b^4 - 1)$

$= ab^3x(b^2 - 1)(b^2 + 1)$

$= ab^3x(b - 1)(b + 1)(b^2 + 1)$

(18) $y^2 - \frac{1}{4}$

$= \left(y + \frac{1}{2}\right)\left(y - \frac{1}{2}\right)$

(19) $3b(q^2 - 9) + 2(q^2 - 9)$

$= (q^2 - 9)(3b + 2)$

$= (q - 3)(q + 3)(3b + 2)$

(20) $a^3b - 8ab$

$= ab(a^2 - 8)$

(21) $5k(p^2 - 4) + 2a(p^2 - 4) - 1(p^2 - 4)$

$= (p^2 - 4)(5k + 2a - 1)$

$= (p - 2)(p + 2)(5k + 2a - 1)$

(22) $a^{12} - 81$

$= (a^6 - 9)(a^6 + 9)$

$= (a^3 - 3)(a^3 + 3)(a^6 + 9)$

(23) $(3r - 2)^2 - 4r^2$

$= [(3r - 2) - 2r][(3r - 2) + 2r]$

$= [r - 2][5r - 2]$

(24) $q^4(x^2 - 2x - 5) - k^2(x^2 - 2x - 5)$

$= (x^2 - 2x - 5)(q^4 - k^2)$

$= (x^2 - 2x - 5)(q^2 - k)(q^2 + k)$

(25) $4m^2 - n^2 + 2m + n$

$= (2m + n)(2m - n) + 1(2m + n)$

$= (2m + n)[(2m - n) + 1]$

$= (2m + n)(2m - n + 1)$

(26) $3p^2 - 2q - 3q^2 + 2p$

$= 3p^2 - 3q^2 + 2p - 2q$

$= 3(p^2 - q^2) + 2(p - q)$

$= 3(p - q)(p + q) + 2(p - q)$

$= (p - q)(3p + 3q + 2)$

(27) $16(x - y)^2 - 25(2x + 3y)^2$

$= [4(x - y) - 5(2x + 3y)][4(x - y) + 5(2x + 3y)]$

$= [4x - 4y - 10x - 15y][4x - 4y + 10x + 15y]$

$= [-6x - 19y][14x + 11y]$

(28) $(a - b + c)^2 - (a + b - c)^2$

$= [(a - b + c) - (a + b - c)][(a - b + c) + (a + b - c)]$

$= [a - b + c - a - b + c][a - b + c + a + b - c]$

$= [-2b + 2c][2a] = -2(b - c)(2a)$

$= -4a(b - c)$

A2.2.4 Trinomials:

Exercise 11:

Factorise completely:

(1) $x^2 + 5x + 6$

$= (x + 3)(x + 2)$

(2) $y^2 + 3y - 4$

$= (y + 4)(y - 1)$

(3) $m^2 - 3m - 10$

$= (m - 5)(m + 2)$

(4) $p^2 - 2p - 8$

$= (p - 4)(p + 2)$

(5) $k^2 + k - 20$

$= (k + 5)(k - 4)$

(6) $b^2 + 11b + 10$

$= (b + 10)(b + 1)$

(7) $a^2 - 5a - 14$

$= (a - 7)(a + 2)$

(8) $x^4 + 6x^2 + 9$

$= (x^2 + 3)(x^2 + 3)$

(9) $7 - 8q + q^2$

$= q^2 - 8q + 7$

$= (q - 1)(q - 7)$

(10) $s^2 + 10s - 24$

$= (s + 12)(s - 2)$

(11) $p^2 - p - 6$

$= (p - 3)(p + 2)$

(12) $d^2 + 10d + 25$

$= (d + 5)(d + 5)$

(13) $p^2 - 2p + 1$

$= (p - 1)(p - 1)$

(14) $c^6 + 5c^3 - 24$

$= (c^3 - 3)(c^3 + 8)$

(15) $12 + 8s + s^2$

$= s^2 + 8s + 12$

$= (s + 6)(s + 2)$

(16) $r^2 - 11r - 12$

$= (r - 12)(r + 1)$

(17) $k^2 - 15 - 10$

$= k^2 - 25$

$= (k - 5)(k + 5)$

(18) $g^2 + 14g - 72$

$= (g + 18)(g - 4)$

(19) $m^2 - 18m - 144$

$= (m - 24)(m + 6)$

(20) $a^2 + 2a - 63$

$= (a + 9)(a - 7)$

(21) $t^2 - 13t + 42$

$= (t - 6)(t - 7)$

(22) $2t^2 - 12t + 10$

$= 2(t^2 - 6t + 5)$

$= 2(t - 1)(t - 5)$

(23) $r^4 + 2r^2 - 24$

$= (r^2 + 6)(r^2 - 4)$

$= (r^2 + 6)(r - 2)(r + 2)$

(24) $100 - 21y - y^2$

$= -1(y^2 + 21y - 100)$

$= -1(y + 25)(y - 4)$

(25) $7k - 60 + k^2$

$= k^2 + 7k - 60$

$= (k - 5)(k + 12)$

(26) $y^3 - 3y^2 - 18y$

$= y(y^2 - 3y - 18)$

$= y(y - 6)(y + 3)$

(27) $m^2 + m + \frac{1}{4}$

$= \left(m + \frac{1}{2}\right)\left(m + \frac{1}{2}\right)$

(28) $x^2 - 4xy + 3y^2$

$= (x - 3y)(x - 1y)$

(29) $15y^2 - 8y + 1$

$= (5y - 1)(3y - 1)$

(30) $15 - 2d - d^2$

$= -1(d^2 + 2d - 15)$

$= -1(d + 5)(d - 3)$

(31) $n^3 + 8n^2 + 12n$

$= n(n^2 + 8n + 12)$

$= n(n + 2)(n + 6)$

(32) $p^2 + 3pq - 18q^2$

$= (p + 6q)(p - 3q)$

(33) $-8 + 4x + 4x^2$

$= 4(x^2 + 1x - 2)$

$= 4(x + 2)(x - 1)$

(34) $p^2q^4 - pq^2 - 12$

$= (pq^2 - 4)(pq^2 + 3)$

(35) $-k^2 + 12k - 35$

$= -1(k^2 - 12k + 35)$

$= -1(k - 5)(k - 7)$

(36) $8m^2 - 7m - 1$

$= (8m + 1)(m - 1)$

(37) $x^2(a^2 + 5a + 6) - y^2(a^2 + 5a + 6)$

$= (a^2 + 5a + 6)(x^2 - y^2)$

$= (a + 2)(a + 3)(x - y)(x + y)$

(38) $2t^4 - 26t^2 + 72$

$= 2(t^4 - 13t^2 + 36)$

$= 2(t^2 - 4)(t^2 - 9)$

$= 2(t - 2)(t + 2)(t - 3)(t + 3)$

Let $(x + 1) = k$

(39) $(x + 1)^2 + 3(x + 1) - 28$

$= k^2 + 3k - 28$

$= (k + 7)(k - 4)$

$= (x + 1 + 7)(x + 1 - 4)$

$= (x + 8)(x - 3)$

(40) $x^2(y^2 - 9) - 4x(y^2 - 9) - 12(y^2 - 9)$

$= (y^2 - 9)(x^2 - 4x - 12)$

$= (y - 3)(y + 3)(x - 6)(x + 2)$

Compare no.39 with no.42

(41) $m^4n^4 + 4m^2n^2 - 5$

$= (m^2n^2 + 5)(m^2n^2 - 1)$

$= (m^2n^2 + 5)(mn - 1)(mn + 1)$

(42) $(2c - 1)^2 + 8(2c - 1) + 16$

$= [(2c - 1) + 4][(2c - 1) + 4]$

$= [2c + 3][2c + 3]$

$= (2c + 3)^2$

A2.2.5 More trinomials:

Exercise 12:

Factorise completely:

(1) $3x^2 + 10x + 3$

$= (3x + 1)(x + 3)$

$3x \times +1 = +1x$

$x \times +3 = +9x$

$+10x$

(2) $6y^2 - 1y - 2$

$= (3y - 2)(2y + 1)$

$3y \times -2 = -4y$

$2y \times +1 = +3y$

$-1y$

(3) $5t^2 + 8t - 4$

$5t$ $-2 = -2t$

t $+2 = +10t$

$= (5t - 2)(t + 2)$

(4) $6n^2 - 29n + 9$

$3n$

$2n$

$= (3n - 1)(2n - 9)$

(5) $3p^2 - 16p + 21$

$= (3p - 7)(p - 3)$

(6) $12g^2 - 4g - 1$

$= (6g + 1)(2g - 1)$

(7) $14r^2 + 17r + 5$

$= (2r + 1)(7r + 5)$

(8) $10q^2 + q - 9$

$= (10q - 9)(q + 1)$

(9) $8c^2 - 18c + 7$

$= (2c - 1)(4c - 7)$

(10) $6m^2 - 7mn + 2n^2$

$= (2m - 1n)(3m - 2n)$

(11) $15p^2 - 1pq - 2q^2$

$= (5p - 2q)(3p + 1q)$

(12) $12 - 35x + 25x^2$

$= (4 - 5x)(3 - 5x)$

(13) $20s^2 - 44st - 15t^2$

$= (2s - 5t)(10s + 3t)$

(14) $20a^2 - 1ab - 12b^2$

$= (4a + 3b)(5a - 4b)$

(15) $9m^2 - 12m + 4$

$= (3m - 2)(3m - 2)$

(16) $x^2 - 2xy + y^2$

$= (x - y)(x - y)$

(17) $10k^2 - 18k - 10$

$= 2(5k^2 - 9k - 5)$

(18) $25a^2 - 30ab + 9b^2$

$= (5a - 3b)(5a - 3b)$

or $(5a - 3b)^2$

(19) $18c^2d^2 - 15cd - 12$

$= 3(6c^2d^2 - 5cd - 4)$

$= 3(3cd - 4)(2cd + 1))$

(20) $-12 + 23b^2 - 10b^4$

$= -1(10b^4 - 23b^2 + 12)$

$= -1(5b^2 - 4)(2b^2 - 3)$

(21) $m^2 - 4mn + 4n^2 - 9x^2$

$= (m - 2n)(m - 2n) - 9x^2$

$= (m - 2n)^2 - 9x^2$

$= (m - 2n - 3x)(m - 2n + 3x)$

(22) $4p^2 - 12pq + 9q^2 - 16y^2$

$= (2p - 3q)(2p - 3q) - 16y^2$

$= (2p - 3q)^2 - 16y^2$

$= (2p - 3q - 4y)(2p - 3q + 4y)$

(23) $x^2 - n^2 + 2mn - m^2$

$= x^2 - 1(n^2 - 2mn + m^2)$

$= x^2 - (n - m)^2$

$= [x - (n - m)][x + (n - m)]$

$= [x - n + m][x + n - m]$

(24) $9a^2 - 1 - 25y^2 + 10y$

$= 9a^2 - 1(25y^2 - 10y + 1)$

$= 9a^2 - (5y - 1)^2$

$= [3a - (5y - 1)][3a + (5y - 1)]$

$= [3a - 5y + 1][3a + 5y - 1]$

A2.2.6 Sum and difference of cubes:

Exercise 13:

Factorise completely:

(1) $a^3 - 1$

$= (a - 1)(a^2 + 1a + 1)$

(2) $m^3 - 8$

$= (m - 2)(m^2 + 2m + 4)$

(3) $1\,000 + t^3$

$= (10 + t)(100 - 10t + t^2)$

(4) $x^6 + y^9$

$= (x^2 + y^3)(x^4 - x^2y^3 + y^6)$

(5) $c^3d^3 + 64$

$= (cd + 4)(c^2d^2 - 4cd + 16)$

(6) $125 - p^{12}$

$= (5 - p^4)(25 + 5p^4 + p^8)$

(7) $8p^3 + 27q^3$

$= (2p + 3q)(4p^2 - 6pq + 9q^2)$

(8) $100 - y^6$

$= (10 - y^3)(10 + y^3)$

(9) $1 - 216t^3$

$= (1 - 6t)(1 + 6t + 36t^2)$

(10) $r^3 + \frac{1}{8}$

$= \left(r + \frac{1}{2}\right)\left(r^2 + \frac{1}{2}r + \frac{1}{4}\right)$

$T_n = -65$ $\therefore$ $2n = 65 - 11 + 2 = 56$