Grade 10 – Book B
(Revised CAPS edition)

TEACHERS GUIDELINES

CONTENTS:

This book was compiled and processed by E.J. Du Toit in 2011.
Revised edition 2023.

Website: www.abcbooks.co.za

ISBN 978-1-919957-10-4

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Chapter B1

Functions and relations

B1.1 Revision – grade 9:

Study the following table:

Number of balls bought (N):	1	2	3	4	8	12
Total cost (C):	R2	R4	R6			

In grade 9 we already saw that we could complete the table by determining the relationship between N and C and then also by compiling a formula or equation. In this case the formula is: $C = 2 \times N$

We also saw that C is called the dependent variable and N the independent variable, because C's value depends on N's value. Tables are also represented graphically.

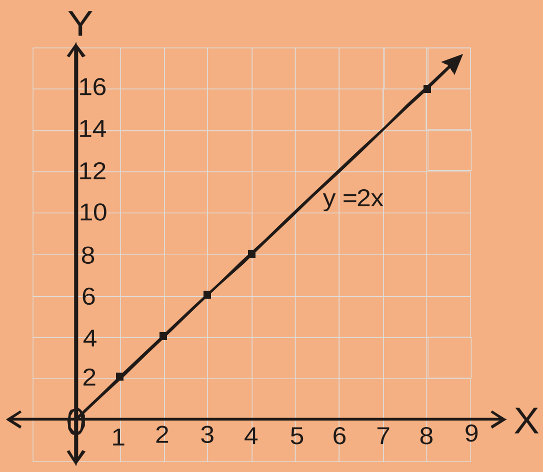
With functions and relations, x is used more times as the independent variable and y as the dependent variable. In other words, the table above will be as follows, with equation $y = 2x$.

Completed table:

Number of balls bought (x):	1	2	3	4	8
Total cost (y):	R2	R4	R6	R8	R16

From the table we can write down the following ordered pairs and then represent them graphically:

(1 ; 2)
(2 ; 4)
(3 ; 6)
(4 ; 8)
(8 ; 16)



These ordered pairs can be written as a set:

{(1 ; 2) ; (2 ; 4) ; (3 ; 6) ; (4 ; 8) ; (8 ; 16)}

From the above set, we can also write down the domain (all the possible x-coordinates in the set) and the range (all the possible y-coordinates in the set):

Domain: $x \in \{1 ; 2 ; 3 ; 4 ; 8\}$
Range: $y \in \{2 ; 4 ; 6 ; 8 ; 16\}$

B1.2 Function concept:

A set of ordered pairs, as above, is then called a **relation**.
When each element in the domain is linked with only **one** element in the range, the relation is called a **function**.

The following are examples of relations: $\{(1\,;\,1)\,;\,(1\,;\,2)\,;\,(1\,;\,3)\,;\,(2\,;\,1)\,;\,(2\,;\,2)\,;\,(2\,;3)\}$
$\{(-1\,;\,0)\,;\,(0\,;\,1)\,;\,(1\,;\,2)\,;\,(2\,;\,1)\,;\,(3\,;\,-1)\}$
$\{(1\,;\,2)\,;\,(2\,;\,4)\,;\,(3\,;\,6)\,;\,(4\,;\,8)\,;\,(8\,;\,16)\}$

However, only the following are functions: $\{(-1\,;\,0)\,;\,(0\,;\,1)\,;\,(1\,;\,2)\,;\,(2\,;\,1)\,;\,(3\,;\,-1)\}$
$\{(1\,;\,2)\,;\,(2\,;\,4)\,;\,(3\,;\,6)\,;\,(4\,;\,8)\,;\,(8\,;\,16)\}$
Because the x values are not repeated.

B1.3 Notations:

Because one frequently works with more than one function, the different functions are named. We use different notations (ways of writing) and the letters of the alphabet can be used to give the different functions "names".

B1.3.1 Set-builder notation:

$f = \{(x\,;\,y)\,/\,y = x + 1\}$ or $g = \{(x\,;\,y) : y = -2x\,;\,x > 0\}$

It reads: "f is the set of ordered pairs $(x\,;\,y)$ with y equal to $x + 1$"

or "g is the set of ordered pairs $(x\,;\,y)$ with y equal to $-2x$ where x is greater than 0"

The functions under discussion here are then: $y = x + 1$ and $y = -2x$

B1.3.2 Function notation:

$f(x) = x + 1$ or $g(x) = -2x$

Thus, it is the same as: $y = x + 1$ or $y = -2x$

This means: in stead of writing "y", $f(x)$, $g(x)$, $h(x)$ etc. is written. The f and the g therefore, indicates the "name" of each specific function, so one can differentiate between the different functions, where x indicates the elements of the domain that must be substituted into the equation to obtain the range.

B1.3.3 Map notation:

	$f: x \rightarrow x + 1$	or	$g: x \rightarrow -2x$
Thus, it is the same as:	$y = x + 1$	or	$y = -2x$
It reads: "f maps (takes) x onto $x + 1$"		or	"g maps x onto $-2x$"

E.g.1 $f: x \rightarrow -3x + 2$

(a) Write down the expression for $f(x)$.

(b) Calculate: (i) $f(-1)$ (ii) $f(m)$ (iii) x if $f(x) = 8$

****

(a) $f(x) = -3x + 2$

(b) (i) $f(-1) = -3(-1) + 2$

$f(-1) = 3 + 2$

$f(-1) = 5$

$\therefore\ y = 5$ if $x = -1$ $\therefore$ Ordered number pair is: $(-1\,;\,5)$

(ii) $f(x) = -3x + 2$

$f(m) = -3(m) + 2$

$f(-1) = -3m + 2$

$\therefore\ y = -3m + 2$ if $x = m$ $\therefore$ Ordered number pair is: $(m\,;\,-3m + 2)$

(iii) $f(x) = -3x + 2$

$8 = -3x + 2$

$3x = 2 - 8$

$3x = -6$

$x = -2$

$\therefore\ x = -2$ if $f(x) = 8 = y$ $\therefore$ Ordered number pair is: $(-2\,;\,8)$

E.g.2 $p = \{(x\,;\,y) \,/\, y = x^2 - 1\}$ and $q(x) = \frac{x+1}{2}$

Calculate: (a) $q(0)$ (b) $p\left(\frac{1}{2}\right)$

(c) $p(-2) + q(3)$ (d) x if $p(x) = q(x)$

****

(a) $q(x) = \frac{x+1}{2}$

$q(0) = \frac{0+1}{2}$

$q(0) = \frac{1}{2}$

$\therefore \left(0\,;\,\frac{1}{2}\right)$

(b) $p(x) = x^2 - 1$

$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 1$

$p\left(\frac{1}{2}\right) = \frac{1}{4} - \frac{4}{4}$

$p\left(\frac{1}{2}\right) = \frac{-3}{4}$

$\therefore \left(\frac{1}{2}\,;\,\frac{-3}{4}\right)$

(c) $p(-2) + q(3)$

$= [(-2)^2 - 1] + \left[\frac{3+1}{2}\right]$

$= [4 - 1] + \left(\frac{4}{2}\right)$

$= 3 + 2$

$= 5$

(d) If $p(x) = q(x)$

Then $x^2 - 1 = \frac{x+1}{2}$

$\therefore\ 2x^2 - 2 = x + 1$

$\therefore\ 2x^2 - x - 3 = 0$

$\therefore\ (2x - 3)(x + 1) = 0$

$\therefore\ 2x - 3 = 0$ or $x + 1 = 0$

$\therefore\ 2x = 3$ $\quad x = -1$

$\therefore\ x = \frac{3}{2}$

Exercise 1: Date: ________________

(1) If $h(x) = x + 1$, calculate: (a) $h(-7)$ (b) $h(0)$ (c) $h(p)$

(a) $h(x) = x + 1$

$h(-7) = (-7) + 1$

$h(-7) = -7 + 1$

$h(-7) = -6$

$\therefore\ (-7\,;\,-6)$

(b) $h(x) = x + 1$

$h(0) = (0) + 1$

$h(0) = 0 + 1$

$h(0) = 1$

$\therefore\ (0\,;\,1)$

(c) $h(x) = x + 1$

$h(p) = (p) + 1$

$h(p) = p + 1$

$\therefore\ (p\,;\,p + 1)$

(2) $f(x) = 5 - 2x$

Calculate: (a) $f(-1)$ (b) $f\left(\frac{1}{2}\right)$ (c) $f(0)$

(d) $f(k)$ (e) x if $f(x) = -3$

(a) $f(x) = 5 - 2x$

$f(-1) = 5 - 2(-1)$

$f(-1) = 5 + 2$

$f(-1) = 7$

$\therefore\ (-1\,;\,7)$

(b) $f(x) = 5 - 2x$

$f\left(\frac{1}{2}\right) = 5 - 2\left(\frac{1}{2}\right)$

$f\left(\frac{1}{2}\right) = 5 - 1$

$f\left(\frac{1}{2}\right) = 4$

$\therefore\ \left(\frac{1}{2}\,;\,4\right)$

(c) $f(x) = 5 - 2x$

$f(0) = 5 - 2(0)$

$f(0) = 5 - 0$

$f(0) = 5$

$\therefore\ (0\,;\,5)$

(d) $f(x) = 5 - 2x$

$f(k) = 5 - 2(k)$

$f(k) = 5 - 2k$

$\therefore\ (k\,;\,5 - 2k)$

(e) $f(x) = 5 - 2x$

$-3 = 5 - 2x$

$2x = 5 + 3$

$2x = 8$

$x = 4$

$\therefore\ (4\,;\,-3)$

(3) $g = \{(x\,;\,y) : y = 2(x - 3)\}$

Calculate: (a) $g(5)$ (b) $g(0{,}5)$ (c) $g(0)$

(d) $g(m+1)$ (e) x if $g(x) = 2$

(a) $g(x) = 2(x-3)$

$g(5) = 2(5-3)$

$g(5) = 2(2)$

$g(5) = 4$

$\therefore \quad (5\,;\,4)$

(b) $g(x) = 2(x-3)$

$g(0{,}5) = 2(0{,}5-3)$

$g(0{,}5) = 2(-2{,}5)$

$g(0{,}5) = -5$

$\therefore \quad (0{,}5\,;\,-5)$

(c) $g(x) = 2(x-3)$

$g(0) = 2(0-3)$

$g(0) = 2(-3)$

$g(0) = -6$

$\therefore \quad (0\,;\,-6)$

(d) $g(x) = 2(x-3)$

$g(m+1) = 2(m+1-3)$

$g(m+1) = 2(m-2)$

$g(m+1) = 2m-4$

$\therefore \quad (m+1\,;\,2m-4)$

(e) $g(x) = 2(x-3)$

$2 = 2(x-3)$

$2 = 2x-6$

$2+6 = 2x$

$8 = 2x$

$4 = x$ $\qquad \therefore \quad (4\,;\,2)$

(4) $t : x \to x^2 - 2x - 8$

(a) Write down an expression for $t(x)$ $\rightarrow\ t(x) = x^2 - 2x - 8$

(b) Calculate: (i) $t(-2)$ (ii) $t(-x)$ (iii) $t(1) + t(2)$

(iv) $t(0)$ (v) x if $t(x) = 0$

(i) $t(x) = x^2 - 2x - 8$

$t(-2) = (-2)^2 - 2(-2) - 8$

$t(-2) = 4 + 4 - 8$

$t(-2) = 0$ $\qquad \therefore \quad (-2\,;\,0)$

(ii) $t(x) = x^2 - 2x - 8$

$t(-x) = (-x)^2 - 2(-x) - 8$

$t(-x) = x^2 + 2x - 8$

$\therefore \quad (-x\,;\,x^2 + 2x - 8)$

(iii) $t(1) + t(2) = [(1)^2 - 2(1) - 8] + [(2)^2 - 2(2) - 8]$

$= [1 - 2 - 8] + [4 - 4 - 8]$

$= [-9] + [-8]$

$= -9 - 8 = -17$

(iv) $t(x) = x^2 - 2x - 8$

$t(0) = (0)^2 - 2(0) - 8$

$t(0) = 0 - 0 - 8$

$t(0) = -8$

$\therefore \quad (0\,;\,-8)$

(v) $t(x) = x^2 - 2x - 8$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$x = 4$ **or** $x = -2$

$\therefore \quad (4\,;\,0)$ **or** $(-2\,;\,0)$

(5) $f(x) = \frac{2}{x}$ and $g(x) = \frac{x-1}{2x}$

Calculate: (a) $g(-1)$ (b) $f(-6)$

(c) x for which $g(x)$ is undefined (d) x if $f(x) = g(x)$

(a) $g(x) = \frac{x-1}{2x}$

$g(-1) = \frac{(-1)-1}{2(-1)}$

$g(-1) = \frac{-2}{-2}$

$g(-1) = 1$

$\therefore \quad (-1\,;\,1)$

(b) $f(x) = \frac{2}{x}$

$f(-6) = \frac{2}{-6}$

$f(-6) = \frac{1}{-3}$

$\therefore \quad \left(-6\,;\,-\frac{1}{3}\right)$

(c) $g(x) = \frac{x-1}{2x}$

Undef → $2x = 0$

Undef → $x = 0$

(d) $f(x) = g(x)$

LCD = $2x$
$x \neq 0$

$\therefore \quad \frac{2}{x} = \frac{x-1}{2x}$

$2 \times 2 = x - 1$

$4 = x - 1$

$4 + 1 = x$

$\therefore \quad x = 5$

(6) $m(x) = (x+3)(3x-1)$ and $p(x) = -x$

Calculate: (a) $m(0) - p(-1)$ (b) x if $p(x) = m(1)$

(c) x if $m(x) = 0$ (d) $m(a) + 2\,p(a)$

(a) $m(0) - p(-1)$

$= [(0)+3][3(0)-1] - [-(-1)]$

$= [0+3][0-1] - [1]$

$= [3][-1] - [1]$

$= -3 - 1$

$= -4$

(b) $p(x) = m(1)$

$\therefore \quad -x = [(1)+3][3(1)-1]$

$-x = [1+3][3-1]$

$-x = [1+3][3-1]$

$-x = [4][2] = 8$

$\therefore \quad x = -8$

(c) $m(x) = (x+3)(3x-1)$

$0 = (x+3)(3x-1)$

$\therefore \quad x + 3 = 0$ or $3x - 1 = 0$

$\therefore \quad x = -3$ or $x = \frac{1}{3}$

$\therefore \quad (-3\,;\,0)$ or $\left(\frac{1}{3}\,;\,0\right)$

(d) $m(a) + 2\,p(a)$

$= [(a)+3][3(a)-1] + 2[-(a)]$

$= [a+3][3a-1] + 2[-a]$

$= 3a^2 - 1a + 9a - 3 - 2a$

$= 3a^2 + 6a - 3$

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Chapter B2
Linear function

B2.1 Standard form:

The standard form for a straight line is: $y = mx + c$
Where m represents the line's gradient and c the line's y intercept!
The gradient (m) is the incline of a line, $\therefore\ m = \frac{y_2 - y_1}{x_2 - x_1}$

B2.2 Graphic representations:

As already seen in grade 9, the linear function or straight line can then be sketched in different ways, namely table method, dual intercept method or gradient intercept method.

E.g.1 Sketch f by using the table method and write down the range of f:
$f = \{(x\,;\,y) : -x + 1 = y\,;\, x \in (-1\,;\,4]\}$

****

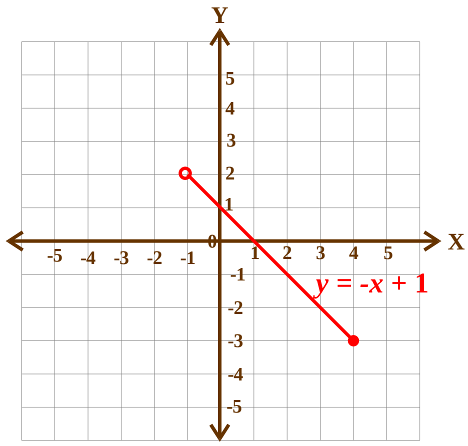
$\therefore$ Sketch $y = -x + 1$ for the interval $(-1\,;\,4]$.
$\therefore$ The domain is $x \in (-1\,;\,4]$ with -1 not included and 4 included!

$y = -(-1) + 1 = 1 + 1 = 2$ $y = -(0) + 1 = 0 + 1 = 1$ $y = -(4) + 1 = -4 + 1 = -3$

x	-1	0	4
y	2	1	-3

Ordered pairs:

$(-1\,;\,2)$
$(0\,;\,1)$
$(4\,;\,-3)$



The range of f: $R_f \in [-3\,;\,2)$

E.g. 2 Sketch the following by using the dual intercept method: $2x - 4y = 8$

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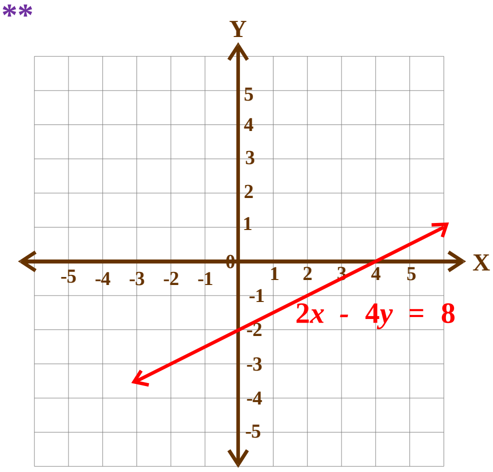
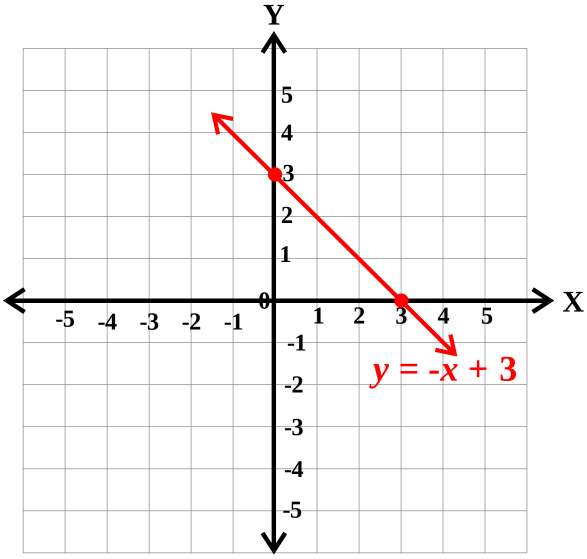
x-intercept: $(y = 0)$
$\therefore\ 2x - 4(0) = 8$
$2x - 0 = 8$
$2x = 8$
$x = 4 \quad \therefore\ (4\,;\,0)$

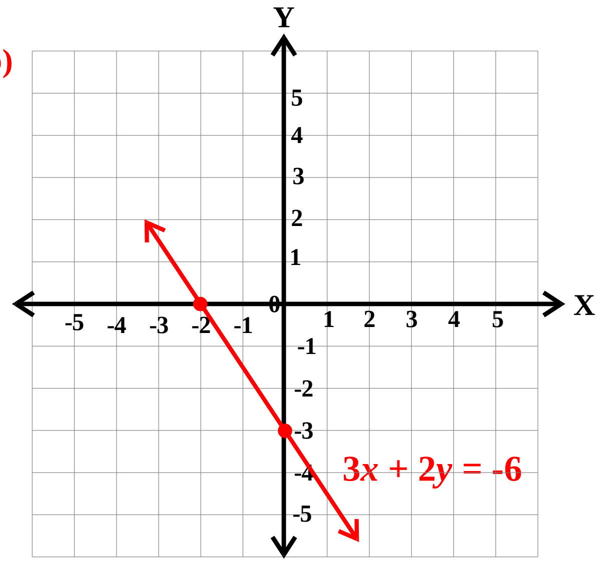
y-intercept: $(x = 0)$
$\therefore\ 2(0) - 4y = 8$
$0 - 4y = 8$
$-4y = 8$
$y = -2 \quad \therefore\ (0\,;\,-2)$



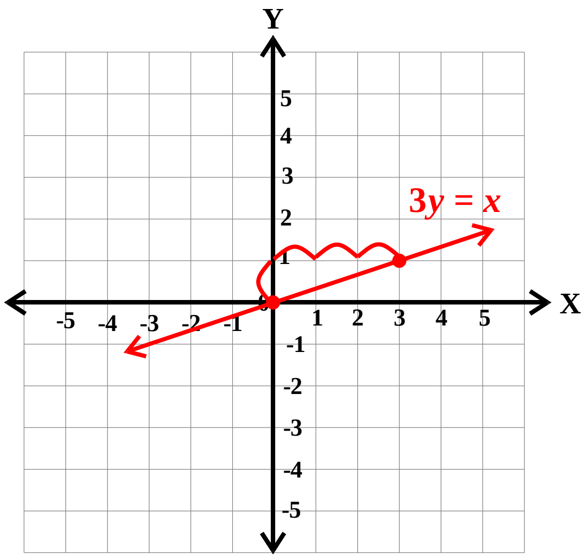
(1)(a)



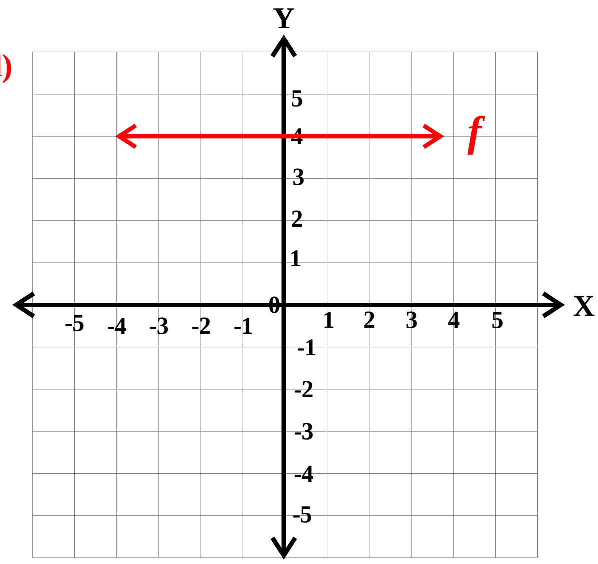
(1)(b)



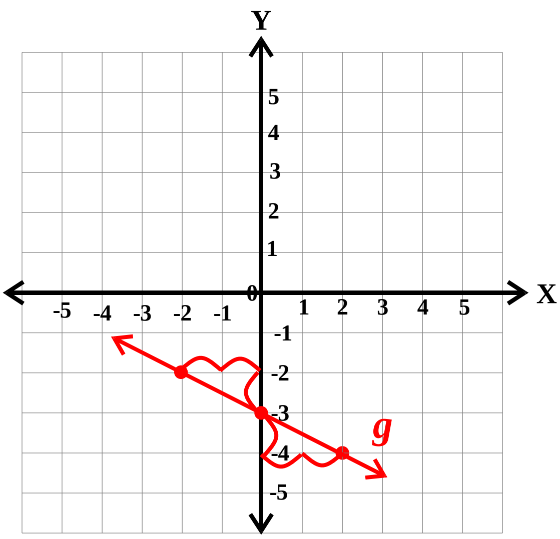
(1)(c)



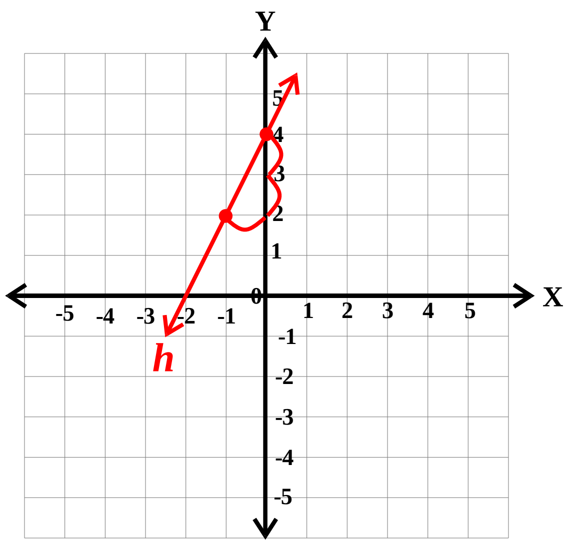
(1)(d)



(1)(e)



(1)(f)



E.g.3 (a) Sketch the following by using the gradient intercept method: $f(x) = -2x$
(b) Sketch on the same Cartesian plane: $g(x) = 3$

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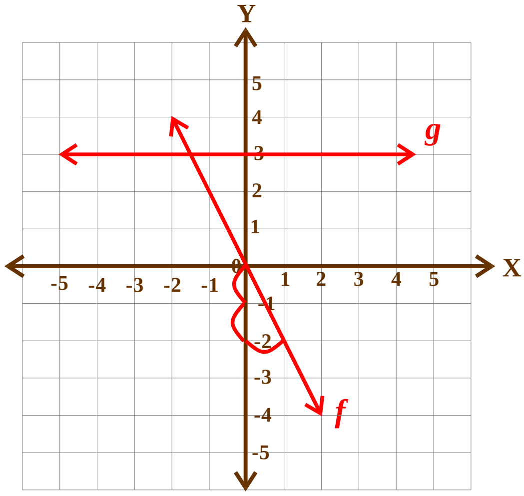
(a) $\therefore$ Sketch $y = -2x + 0$

$\therefore$ y- intercept: $c = 0$

and gradient: $m = -2$

$\therefore \; m = \frac{-2}{+1} = \frac{y\text{-difference}}{x\text{-difference}}$

(b) If $g(x) = 3$, then $y = 3$



Exercise 1: Date: ________________

(1) Sketch the following linear functions by using any method:

(a) $y = -x + 3$

x-int: $(y = 0)$
$0 = -x + 3$
$x = 3$
$(3\,;\,0)$

y-int: $(x = 0)$
$y = -(0) + 3$
$y = 3$
$(0\,;3)$

(b) $3x + 2y = -6$

x-int: $(y = 0)$
$3x + 2(0) = -6$
$3x = -6$
$x = -2$
$(-2\,;\,0)$

y-int: $(x = 0)$
$3(0) + 2y = -6$
$2y = -6$
$y = -3$
$(0\,;-3)$

(c) $3y = x$

$\therefore \; y = \frac{x}{3} = \frac{1}{3}x + 0$

$\therefore \; m = \frac{1}{3} = \frac{\Delta y}{\Delta x}$

$\therefore \; c = 0$

(d) $f : x \rightarrow 4$

$\therefore \; y = 4$

(e) $g(x) = -\frac{1}{2}x - 3$

$y = -\frac{1}{2}x - 3$

$\therefore \; m = \frac{-1}{2} = \frac{1}{-2} = \frac{\Delta y}{\Delta x}$

$\therefore \; c = -3$

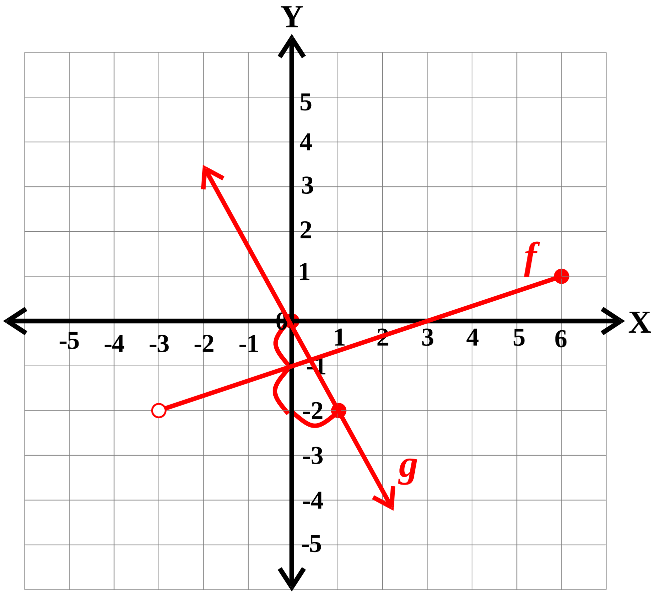
(f) $h = \{(x\,;\,y) : y - 4 = 2x\}$

$y = 2x + 4$

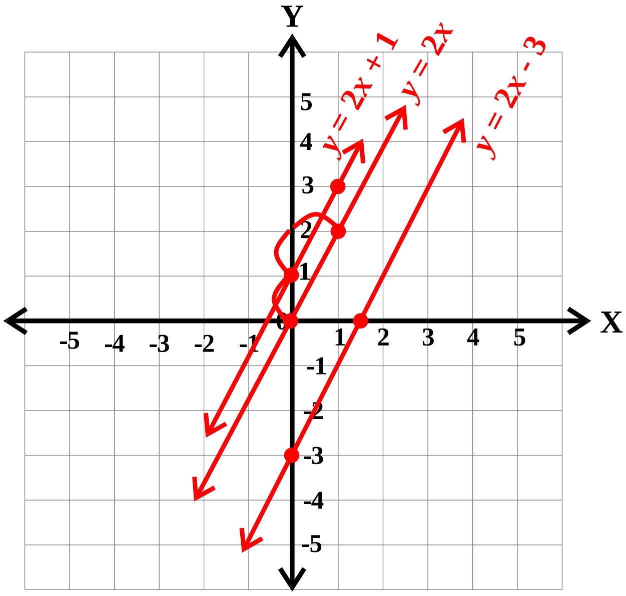
$\therefore \; m = \frac{2}{1} = \frac{-2}{-1} = \frac{\Delta y}{\Delta x}$

$\therefore \; c = 4$

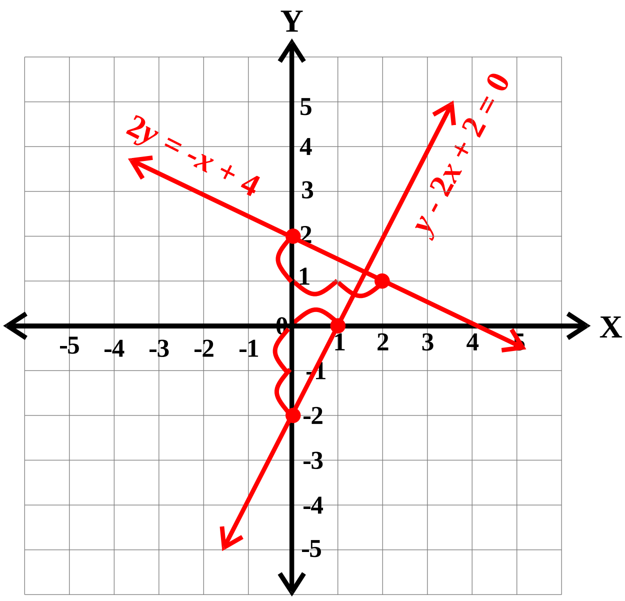
(2)(a)


Y
X
f
g
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1 1 2 3 4 5 6

(3)(a)


Y
X
y = 2x + 1
y = 2x
y = 2x - 3
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1 1 2 3 4 5

(4)(a)


Y
X
2y = -x + 4
y - 2x + 2 = 0
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1 1 2 3 4 5

(2) Sketch the following linear functions by using any method and write down each function's domain and range:

(a) $f = \{(x\,;\,y) \,/\, 3y + 3 = x\,;\, x \in (-3\,;\,6]\}$

$3y = x - 3$

$y = \frac{1}{3}x - 1$

x	-3	0	6
y	-2	-1	1

$D_f: x \in (-3\,;\,6]$

$R_f: y \in (-2\,;\,1]$

(b) $g: x \rightarrow -2x$

$y = -2x$

$\therefore \quad m = \frac{-2}{1} = \frac{2}{-1} = \frac{\Delta y}{\Delta x}$

$\therefore \quad c = 0$

$D_g: x \in \mathbb{R}$

$R_g: y \in \mathbb{R}$

(3) (a) Sketch the following linear functions on the same Cartesian plane. First write the functions in standard form.

$2y = 4x - 6$

$y = 2x - 3$

x-int: $(y = 0)$ y-int: $(x = 0)$

$0 = 2x - 3 \qquad y = 2(0) - 3$

$x = \frac{3}{2} \qquad y = -3$

$\therefore \left(\frac{3}{2}\,;\,0\right) \qquad \therefore (0\,;\,-3)$

$x = \frac{1}{2}y$

$2x = y$

$\therefore \quad m = \frac{2}{1} = \frac{-2}{-1} = \frac{\Delta y}{\Delta x}$

$\therefore \quad c = 0$

$y - 2x = 1$

$y = 2x + 1$

$\therefore \quad m = \frac{2}{1} = \frac{-2}{-1} = \frac{\Delta y}{\Delta x}$

$\therefore \quad c = 1$

(b) What do you notice from the functions in (a)'s gradients? **All gradients equal to 2.**

(c) What do you notice from the line graphs in (a)? **The lines are parallel.**

(d) What deduction can you make from (b) and (c)?

Lines with the same gradient are parallel.

(4) (a) Sketch the following linear functions on the same Cartesian plane. First write the functions in standard form.

$2y = -x + 4$ and $y - 2x + 2 = 0$

$y = -\frac{1}{2}x + 2$

$\therefore \quad m = \frac{-1}{2} = \frac{1}{-2} = \frac{\Delta y}{\Delta x}$

$\therefore \quad c = 2$

$y = 2x - 2$

$\therefore \quad m = \frac{2}{1} = \frac{-2}{-1} = \frac{\Delta y}{\Delta x}$

$\therefore \quad c = -2$

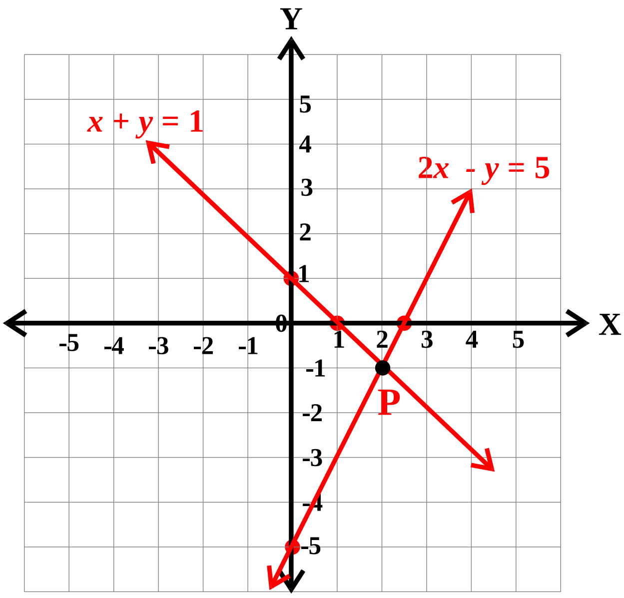
(b) What do you notice from the functions in (a)'s gradients? **Gradients are inverse.**

(c) What do you notice from the line graphs in (a)? **Lines are perpendicular.**

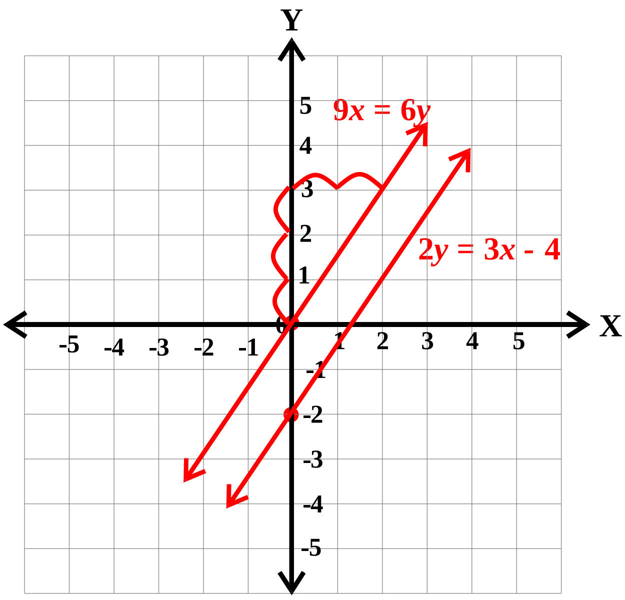
(d) What deduction can you make from (b) and (c)?

For perpendicular lines $m_1 \times m_2 = -1$.

(1)(a)


Y
X
x + y = 1
2x - y = 5
P
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1 1 2 3 4 5

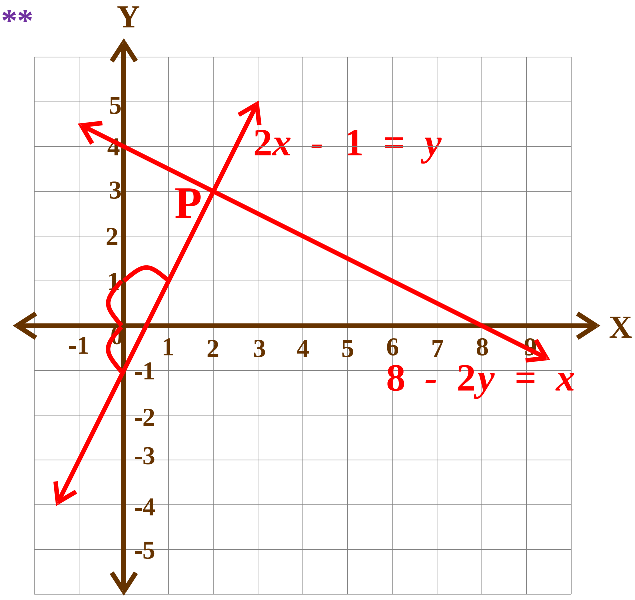
(1)(b)


Y
X
9x = 6y
2y = 3x - 4
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1 1 2 3 4 5

B2.3 Graphic solutions of simultaneous equations:

E.g.4 Determine the simultaneous solution to the following equations by first drawing the graphs: $2x - 1 = y$ **and** $8 - 2y = x$

****



Sketch $y = 2x - 1$:

$m = 2 = \frac{2}{1} = \frac{y\text{-difference} \Updownarrow}{x\text{-difference} \Leftrightarrow}$

$c = -1$

Sketch $8 - 2y = x$:

x- intercept: $(y = 0)$

$\therefore 8 - 2(0) = x \quad \therefore x = 8$

y- intercept: $(x = 0)$

$\therefore 8 - 2y = 0$

$8 = 2y \quad \therefore y = 4$

$\therefore$ Simultaneous solution is at $P(2\,;3)$

Exercise 2: Date: _______________

(1) Determine the simultaneous solution to the following equations by first drawing the graphs:

(a) $x + y = 1$ and $2x - y = 5$

x-int: $(y = 0)$

$x + 0 = 1$

$\therefore \quad x = 1$

$\therefore \quad (1\,;\ 0)$

y-int: $(x = 0)$

$0 + y = 1$

$\therefore \quad y = 1$

$\therefore \quad (0\,;\ 1)$

x-int: $(y = 0)$

$2x - 0 = 5$

$\therefore \quad x = \frac{5}{2} = 2{,}5$

$\therefore \quad (2{,}5\,;\ 0)$

y-int: $(x = 0)$

$2(0) - y = 5$

$\therefore \quad y = -5$

$\therefore \quad (0\,;\ -5)$

$\therefore \quad P(2\,;-1)$

(b) $3x - 4 = 2y$ and $9x = 6y$

$2y = 3x - 4$

$y = \frac{3}{2}x - \frac{4}{2}$

$y = \frac{3}{2}x - 2$

$\therefore \quad m = \frac{3}{2} = \frac{-3}{-2} = \frac{\Delta y}{\Delta x}$

$\therefore \quad c = -2$

$6y = 9x$

$y = \frac{9}{6}x$

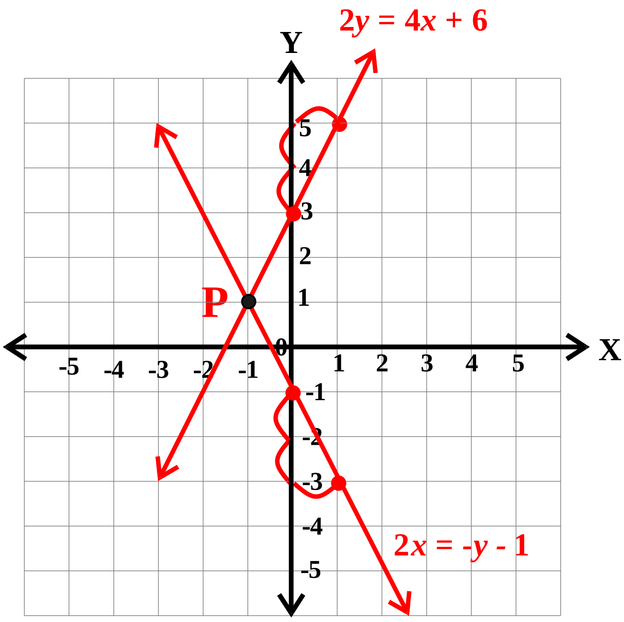
$y = \frac{3}{2}x$

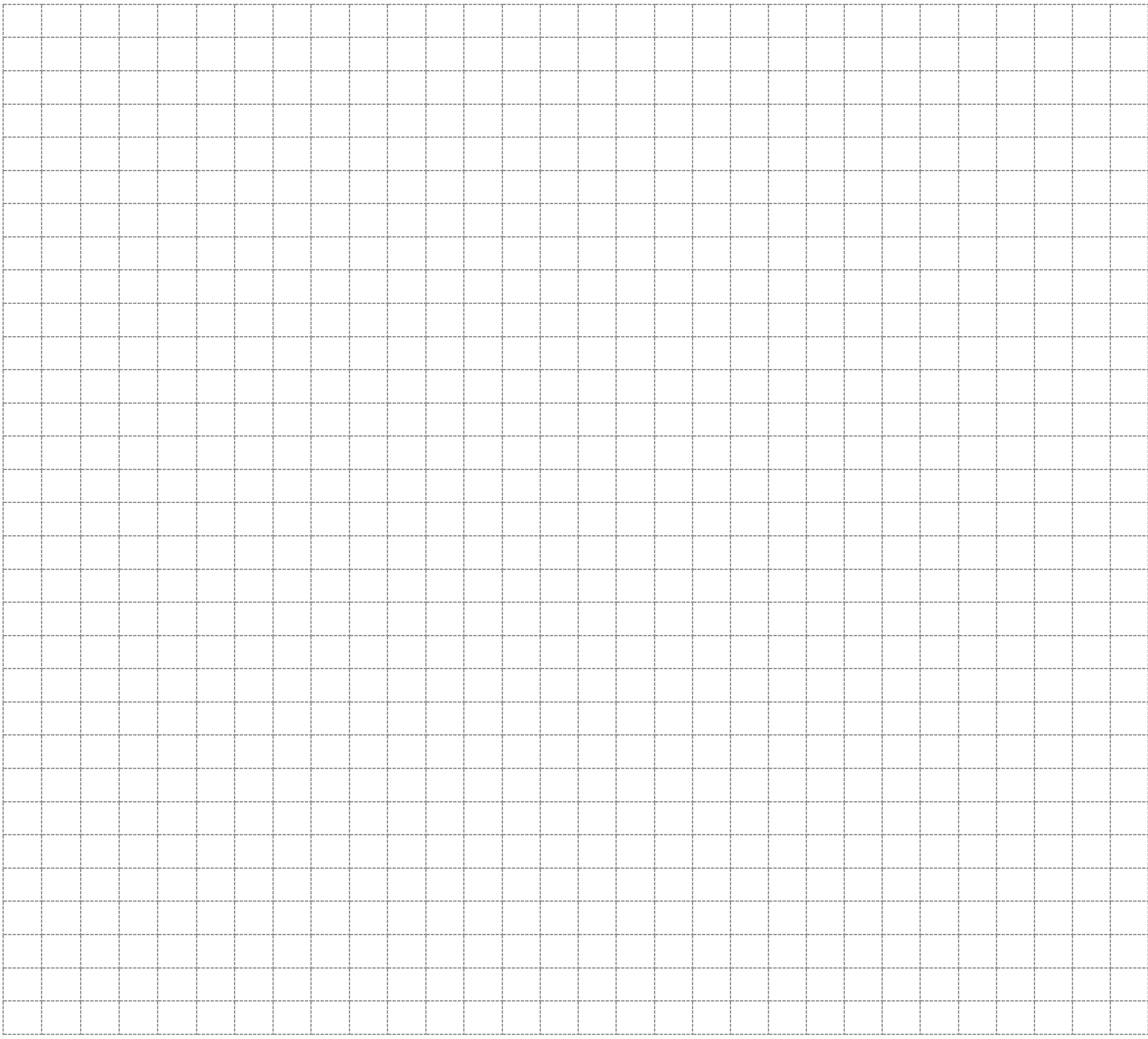
$\therefore \quad m = \frac{3}{2} = \frac{-3}{-2} = \frac{\Delta y}{\Delta x}$

$\therefore \quad c = 0$

$\therefore$ No simultaneous solution $\rightarrow$ lines are parallel

(2)





(2) Determine the solution graphically: $\{(x\,;\,y)\,/\,2x = -y - 1\} \cap \{(x\,;\,y)\,/\,2y = 4x + 6\}$

$2x = -y - 1$

$y = -2x - 1$

$\therefore \quad m = \frac{-2}{1} = \frac{2}{-1} = \frac{\Delta y}{\Delta x}$

$\therefore \quad c = -1$

$2y = 4x + 6$

$y = \frac{4}{2}x + \frac{6}{2}$

$y = 2x + 3$

$\therefore \quad m = \frac{2}{1} = \frac{-2}{-1} = \frac{\Delta y}{\Delta x}$

$\therefore \quad c = 3$

$\therefore \quad P(-1\,;\,1)$

B2.4 Parallel and perpendicular lines:

Therefore we can deduce the following from exercise 1:

From no.(3): **Parallel lines have the same gradients.** $\therefore \; m_1 = m_2$

From no.(4): **Perpendicular lines have reversed gradients.** $\therefore \; m_1 \times m_2 = -1$

E.g.5 Determine whether the following lines are parallel to each other, perpendicular to each other or neither of the two: $3x + y = 7$ **and** $-2y = 6x$

****

$y = -3x + 7$

$\therefore \; m_1 = -3$

$y = \frac{-6x}{2} = -3x$

$\therefore \; m_2 = -3$

$\therefore$ **The lines are parallel, because** $m_1 = m_2$.

E.g.6 Calculate the value(s) of p **if** $2y + 3x = 2$ **and** $y - 2px + 3 = 0$ **are perpendicular to each other.**

****

$2y + 3x = 2$ **and** $y - 2px + 3 = 0$

$\therefore \; 2y = -3x + 2$

$y = \frac{-3}{2}x + 1$

$\therefore \; m_1 = \frac{-3}{2}$

$\therefore \; y = 2px - 3$

$\therefore \; m_2 = 2p$

But the lines are perpendicular to each other, $\therefore \; m_1 \times m_2 = -1$

$\frac{-3}{2} \times \frac{2p}{1} = -1$

$\therefore \; -3p = -1$

$\therefore \quad p = \frac{-1}{-3} = \frac{1}{3}$

Exercise 3: Date: ______________

(1) Determine whether the following lines are parallel to each other, perpendicular to each other or neither of the two:

(a) $x + y + 1 = 0$ and $x - y - 3 = 0$

$y = -x - 1$

$x - 3 = y$

$y = x - 3$

$\therefore\ m_1 = -1$ $\therefore\ m_2 = 1$

∴ Lines are perpendicular because

$m_1 \times m_2 = -1 \times 1 = -1$

(b) $2x + y = 3$ and $4x + 6 = 2y$

$y = -2x + 3$

$2x + 3 = y$

$y = 2x + 3$

$\therefore\ m_1 = -2$ $\therefore\ m_2 = 2$

∴ Neither // nor ⊥

(c) $\frac{1}{3}x = y$ and $x - 3y = 4$

$y = \frac{1}{3}x$

$-3y = -x + 4$

$y = \frac{1}{3}x - \frac{4}{3}$

$\therefore\ m_1 = \frac{1}{3}$ $\therefore\ m_2 = \frac{1}{3}$

∴ Lines are parallel because

$m_1 = m_2$

(d) $6y + 4x = -3$ and $1 + 3x = 2y$

$6y = -4x - 3$

$y = -\frac{4}{6}x - \frac{3}{6}$

$y = -\frac{2}{3}x - \frac{1}{2}$

$2y = 3x + 1$

$y = \frac{3}{2}x + \frac{1}{2}$

$y = \frac{3}{2}x + \frac{1}{2}$

$\therefore\ m_1 = -\frac{2}{3}$ $\therefore\ m_2 = \frac{3}{2}$

∴ Lines are perpendicular because

$m_1 \times m_2 = -\frac{2}{3} \times \frac{3}{2} = -1$

(2) Calculate the value(s) of k if:

(a) $3x + 4y = -2$ and $6kx + 3 + y = 0$ are parallel to each other.

$4y = -3x - 2$

$y = \frac{-3}{4}x - \frac{2}{4}$ $\therefore\ m_1 = -\frac{3}{4}$

$y = -6kx - 3$

and $m_2 = -6k$

$\therefore\ -\frac{3}{4} = -6k$ → **lines are parallel**

$\therefore\ -3 = -24k$

$\therefore\ k = \frac{-3}{-24}$ → $\therefore\ k = \frac{1}{8}$

(b) $y - 2kx = 6$ and $4x - y = 1$ are perpendicular to each other.

$y = 2kx + 6$

$4x - 1 = y$

$\therefore\ m_1 = 2k$ and $m_2 = 4$

For perpendicular lines → $m_1 \times m_2 = -1$

$\therefore\ 2k \times 4 = -1$

$\therefore\ 8k = -1$

$\therefore\ k = -\frac{1}{8}$