

Graad 11 – Boek A OH

(CAPS Uitgawe)

INHOUD:

| | <u>Bladsy:</u> |
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| 1. Getalle stelsels, eksponente | 3 |
| 2. Algebraïese uitdrukkings en vergelykings en ongelykhede | 27 |
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Kontak nommer: 086 618 3709 (Faks!)

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Hoofstuk A1

Getalstelsels en eksponente

A1.1 Getalstelsels:

Oefening 1:

Datum: _____

(1) Voltooi:

* Natuurlike getalle: $\mathbb{N} = \underline{\{1; 2; 3; \dots\}}$

* Telgetalle: $\mathbb{N}_0 = \underline{\{0; 1; 2; 3; \dots\}}$

* Heelgetalle: $\mathbb{Z} = \underline{\{\dots; -2; -1; 0; 1; 2; \dots\}}$

* Rasionale getalle: $\mathbb{Q} = \underline{\left\{ \frac{a}{b} \mid a, b \in \mathbb{Z}; b \neq 0 \right\}}$

* Reële getalle: $\mathbb{R} = \underline{\{ \text{Rasionale getalle} \} \cup \{ \text{Irrasionale getalle} \}}$

(2) Gee drie voorbeelde van Irrasionale getalle: $\underline{\sqrt{3}; \sqrt[3]{4}; \sqrt{7}}$

(3) Beskou: $x(x - 6)(x^2 - 5)(2x^2 + x - 3) = 0$.

Los op vir x en skryf die waarde(s) van x waarvoor die oplossing van die uitdrukking:

(a) irrasionale wortels het.

(c) heelgetallige wortels het.

(b) natuurlike wortels het.

$$\underline{x(x-6)(x^2-5)(2x+3)(x-1) = 0}$$

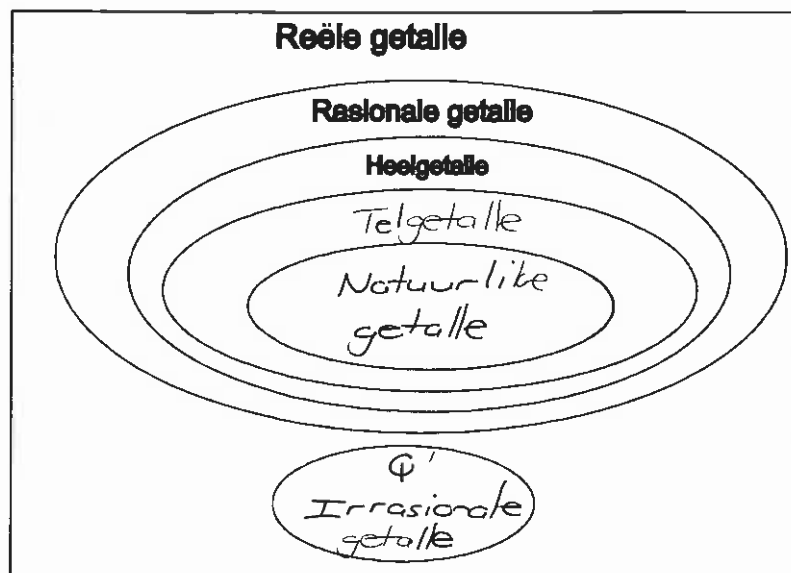
$$\underline{x=0 \text{ of } x=6 \text{ of } x^2=5 \text{ of } x=-\frac{3}{2} \text{ of } x=1}$$

$$\underline{x = \pm\sqrt{5}}$$

(a) $x = \pm\sqrt{5}$ (c) $x = 0$ of $x = 6$

(b) $x = 6$ of $x = 1$ of $x = 1$

(4) Voltooi die volgende figuur wat die samestelling van die stelsel reële getalle verteenwoordig:



A1.2 Nie-Reële getalle:

Voorbeelde van nie-reële getalle is: $\sqrt{-2}$: $\sqrt{-9}$ of $\sqrt[3]{-5}$

Maar nie $\sqrt[3]{-8}$ nie, want $-2 \times -2 \times -2 = -8 \therefore \sqrt[3]{-8} = -2$

Oefening 2:

Datum: _____

(1) Bepaal of die volgende getalle reël of nie-reël is. Indien dit reël is, dui aan of die getal rasionaal of irrasionaal sal wees.

- | | | | |
|----------------------|--|---------------------------------|--|
| (a) 7 | <u>Reël \rightarrow rasionaal</u> | (b) $-\sqrt{3}$ | <u>Reël \rightarrow irrasionaal</u> |
| (c) π | <u>Reël \rightarrow irrasionaal</u> | (d) $\sqrt{-16}$ | <u>Nie-reël</u> |
| (e) $0,3$ | <u>Reël \rightarrow rasionaal</u> | (f) $\frac{12}{36}$ | <u>Reël \rightarrow rasionaal</u> |
| (g) $\sqrt[3]{-125}$ | <u>$= -5$ Reël \rightarrow rasionaal</u> | (h) $\frac{1+\sqrt{9}}{=1+3=4}$ | <u>Reël \rightarrow rasionaal</u> |
| (i) $\sqrt{(-2)^3}$ | <u>$= \sqrt{-8}$ Nie-reël</u> | (j) 0 | <u>Reël \rightarrow rasionaal</u> |

(2) Sê of die volgende bewerings waar of vals is:

- (a) Die produk van twee heelgetalle is altyd weer 'n heelgetal. Waar
- (b) Die produk van twee irrasionale getalle is altyd weer 'n irrasionale getal. Vals
Bv. $\sqrt{2} \times \sqrt{2} = 2$
- (c) As m 'n natuurlike getal is, dan sal $\sqrt{4m}$ ook 'n natuurlike getal wees. Waar
 $= 2m$
- (d) Die verskil tussen twee rasionale getalle is altyd weer 'n rasionale getal. Waar
- (e) Die kwosiënt van 'n rasionale getal en 'n irrasionale getal is altyd rasionaal. Vals
Bv. $\frac{2}{\sqrt{2}} \rightarrow$ irrasionaal

(3) Vir watter waardes van x is die volgende bewerings: (i) ongedefinieerd (ii) nie-reël

- (a) $\frac{x+3}{x}$: (i) $x=0$ (ii) Geen
- (b) $\sqrt{x-1}$: (i) Geen (ii) $x-1 < 0 \therefore x < 1$
- (c) $\frac{\sqrt{x}}{x+2}$: (i) $x=-2$ (ii) $x < 0$

(4) Gegee: $P = \sqrt[3]{3y} - 1$. Tot watter van die volgende getallestelsel(s) sal P behoort indien:

[Getallestelsels: \mathbb{N} ; \mathbb{N}_0 ; \mathbb{Z} ; \mathbb{Q} ; \mathbb{Q}' ; \mathbb{R} of \mathbb{R}']

- | | | |
|---|--|--|
| (a) $y = \frac{1}{3}$ $P = \sqrt[3]{3 \times \frac{1}{3}} - 1$ $= \sqrt[3]{1} - 1$ <u>$= 1 - 1 = 0$</u> <u>$\mathbb{N}_0; \mathbb{Z}; \mathbb{Q}; \mathbb{R}$</u> | (b) $y = -1$ $P = \sqrt[3]{3 \times -1} - 1$ $= \sqrt[3]{-3} - 1$ <u>\mathbb{R}'</u> | (c) $y = 5$ $P = \sqrt[3]{3 \times 5} - 1$ $= \sqrt[3]{15} - 1$ <u>$\mathbb{Q}'; \mathbb{R}$</u> |
|---|--|--|

A1.3 Voorstelling van die reële getalle:

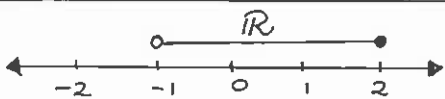
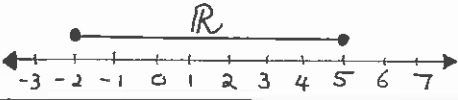
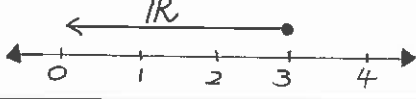
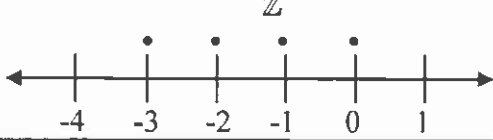
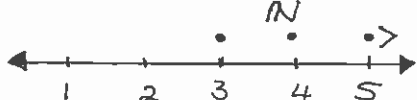
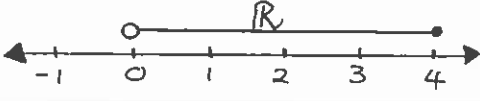
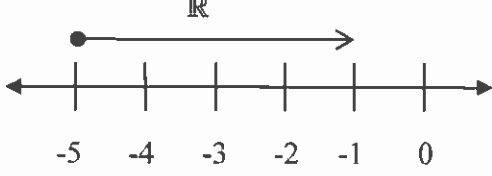
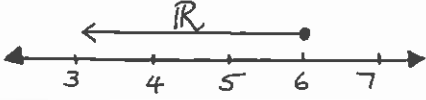

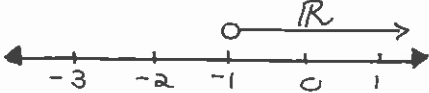
Soos reeds gesien in die vorige grade, kan die reële getalle op een van die volgende maniere voorgestel word:

- (a) Intervalnotasie
 (b) Op 'n getallelyn.
 (c) As ongelykhede in versameling keurdernotasie. Onthou die volgende simbole:
 \cup \rightarrow die vereniging van twee of meer intervale of versamelings.
 \cap \rightarrow die snyding tussen twee of meer intervale of versamelings.

Oefening 3:

Datum: _____

Voltooi die volgende tabel:

| | Versamelingskeurdernotasie: | Intervalnotasie: | Getallelyn: |
|------|--|----------------------|--|
| (1) | $\{x / -1 < x \leq 2; x \in \mathbb{R}\}$ | $x \in (-1; 2]$ |  |
| (2) | $\{x / -2 \leq x \leq 5; x \in \mathbb{R}\}$ | $x \in [-2; 5]$ |  |
| (3) | $\{y / y \leq 3; y \in \mathbb{R}\}$ | $y \in (-\infty; 3]$ |  |
| (4) | $\{x / -3 \leq x \leq 0; x \in \mathbb{Z}\}$ of $\{x / -4 < x < 1; x \in \mathbb{Z}\}$ | Geen |  |
| (5) | $\{y / y \geq 3; y \in \mathbb{N}\}$ | Geen |  |
| (6) | $\{m / 0 < m \leq 4; m \in \mathbb{R}\}$ | $m \in (0; 4]$ |  |
| (7) | $\{x / x \geq -5; x \in \mathbb{R}\}$ | $x \in [-5; \infty)$ |  |
| (8) | $\{m : m \leq 6; m \in \mathbb{R}\}$ | $m \in (-\infty; 6]$ |  |
| (9) | $\{x / -1 < x < 2; x \in \mathbb{Z}\}$ | Geen |  |
| (10) | $\{x / x > -1; x \in \mathbb{R}\}$ | $x \in (-1; \infty)$ |  |

☉ (1) Gee 'n sinoniem vir "nie-reële getalle": Denkbeeldig

(2) Doen navorsing oor komplekse getalle en gee twee voorbeelde van komplekse getalle:

Word voorgestel deur \mathbb{C} . Getalle soos $\sqrt{-1}$ of $\sqrt{-7}$
 Kan geskryf word as die som of verskil van reële en
 nie-reële getalle bv. $1 + \sqrt{-1}$ of $3 - 2i$

A1.4 Eksponente en wortelvorme:

A1.4.1 Eksponente:

Basiese eksponentwette en eienskappe:

$$(1) x^m \times x^n = x^{m+n}$$

$$(2) x^m \div x^n = x^{m-n}$$

$$(3) (x^m)^n = x^{mn}$$

$$(4) (xy)^m = x^m y^m \quad \text{of} \quad \left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$$

$$(5) x^0 = 1$$

$$(6) x^{-m} = \frac{1}{x^m}$$

$$(7) x^{\frac{m}{n}} = \sqrt[n]{x^m} \quad (m, n \in \mathbb{Z} \text{ en } n > 0 \text{ met } n \neq 1)$$

Vb.1 Vereenvoudig en skryf jou antwoord as 'n positiewe eksponent:

$$(a) \frac{(x^3 \cdot y^{-2})^2}{x^3(xy)^3} = \frac{x^6 \cdot y^{-4}}{x^3 \cdot x^3 \cdot y^3} = \frac{x^6 \cdot y^{-4}}{x^6 \cdot y^3} = x^{6-6} \cdot y^{-4-3} = x^0 \cdot y^{-7} = \frac{x^0}{y^7} = \frac{1}{y^7}$$

$$(b) \sqrt{x} \times x^{\frac{1}{2}} \div x^{\frac{1}{2}} = x^{\frac{1}{2}} \times x^{\frac{1}{2}} \div x^{\frac{1}{2}} = x^{\frac{1}{2} + \frac{1}{2} - \frac{1}{2}} = x^{\frac{1+1-1}{2}} = x^{\frac{1}{2}}$$

$$(c) \frac{25^{n+1} \cdot 10^n}{8^{n-1} \cdot 5^{3n} \cdot (2^{-1})^{2n}} = \frac{(5^2)^{n+1} \cdot (2 \times 5)^n}{(2^3)^{n-1} \cdot 5^{3n} \cdot 2^{-2n}}$$

$$= \frac{5^{2n+2} \cdot 2^n \cdot 5^n}{2^{3n-3} \cdot 5^{3n} \cdot 2^{-2n}} = \frac{5^{3n+2} \cdot 2^n}{2^{n-3} \cdot 5^{3n}}$$

$$= 5^{3n+2-3n} \cdot 2^{n-(n-3)} = 5^2 \cdot 2^{n-n+3} = \underline{5^2 \cdot 2^3} = \underline{200}$$

$$(d) \frac{2^x - 2^{x+1}}{2^{x-1} + 2^x}$$

$$= \frac{2^x - 2^x \cdot 2^1}{2^x \cdot 2^{-1} + 2^x}$$

$$= \frac{2^x(1 - 2^1)}{2^x(2^{-1} + 1)} = \frac{1 - 2}{\frac{1}{2} + 1} = -1 \div \left(\frac{1+2}{2}\right) = -1 \div \frac{3}{2} = -1 \times \frac{2}{3} = \underline{\underline{-\frac{2}{3}}}$$

$$(e) \frac{x - x^{\frac{1}{2}} - 6}{x - 4} = \frac{\left(x^{\frac{1}{2}} - 3\right)\left(x^{\frac{1}{2}} + 2\right)}{\left(x^{\frac{1}{2}} - 2\right)\left(x^{\frac{1}{2}} + 2\right)} = \frac{\left(x^{\frac{1}{2}} - 3\right)}{\left(x^{\frac{1}{2}} - 2\right)}$$

Oefening 4:

Datum: _____

Vereenvoudig, sonder 'n sakrekenaar: (Skryf antwoorde as positiewe eksponente!)

$$\begin{aligned}
 (1) \quad & (125x^6)^{\frac{1}{3}} \\
 &= (5^3 x^6)^{\frac{1}{3}} \\
 &= (5^3)^{\frac{1}{3}} (x^6)^{\frac{1}{3}} \\
 &= 5x^2
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & (x^{\frac{1}{2}} - 2)^2 \\
 &= (x^{\frac{1}{2}} - 2)(x^{\frac{1}{2}} - 2) \\
 &= (x^{\frac{1}{2}})^2 - 2x^{\frac{1}{2}} - 2x^{\frac{1}{2}} + 4 \\
 &= x - 4x^{\frac{1}{2}} + 4
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad & \sqrt[3]{-8x^9y^{-3}} \\
 &= \sqrt[3]{(-2)^3 x^9 y^{-3}} \\
 &= (-2)^{\frac{3}{3}} x^{\frac{9}{3}} y^{-\frac{3}{3}} \\
 &= -2x^3 y^{-1} \\
 &= \frac{-2x^3}{y}
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad & 3y^{\frac{1}{2}} \div (3y)^{-\frac{1}{2}} \\
 &= 3y^{\frac{1}{2}} \div 3^{-\frac{1}{2}} y^{-\frac{1}{2}} \\
 &= 3^{1 - (-\frac{1}{2})} y^{\frac{1}{2} - (-\frac{1}{2})} \\
 &= 3^{1\frac{1}{2}} y^{\frac{1}{2} + \frac{1}{2}} \\
 &= 3^{1\frac{1}{2}} y^1 \\
 &= 3^{1\frac{1}{2}} y
 \end{aligned}$$

$x^{\frac{1}{4}} \times x^4 = x^{\frac{2}{4}} = x^{\frac{1}{2}}$

$$\begin{aligned}
 (5) \quad & (0,25m^{\frac{1}{4}})^2 \\
 &= \left(\frac{25}{100} m^{\frac{1}{4}}\right)^2 \\
 &= \left(\frac{1}{4} m^{\frac{1}{4}}\right)^2 \\
 &= \left(\frac{1}{4}\right)^2 (m^{\frac{1}{4}})^2 = \frac{1}{16} m^{\frac{1}{2}} = \frac{m^{\frac{1}{2}}}{16}
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad & (x^{\frac{1}{2}} + 4)(x^{\frac{1}{2}} - 2)(x^{\frac{1}{2}} + 2) \\
 &= (x^{\frac{1}{2}} + 4)(x^{\frac{1}{2}} - 4) \\
 &= (x^{\frac{1}{2}})^2 - 16 \\
 &= x - 16
 \end{aligned}$$

$$\begin{aligned}
 (7) \quad & \frac{x^{\frac{1}{2}} \cdot \sqrt{x^3}}{x^{\frac{1}{3}}} \\
 &= \frac{x^{\frac{1}{2}} \cdot x^{\frac{3}{2}}}{x^{\frac{1}{3}}} \\
 &= x^{\frac{1}{2} + \frac{3}{2} - \frac{1}{3}} \\
 &= x^{2 - \frac{1}{3}} \\
 &= x^{1\frac{2}{3}}
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad & \left(\frac{-12x^4y^4z^{\frac{1}{2}}}{-3x^2z^3}\right)^{\frac{1}{2}} \\
 &= (4x^2y^4z^{-2})^{\frac{1}{2}} \\
 &= (2^2)^{\frac{1}{2}} (x^2)^{\frac{1}{2}} (y^4)^{\frac{1}{2}} (z^{-2})^{\frac{1}{2}} \\
 &= 2^1 x^1 y^2 z^{-1} \\
 &= \frac{2xy^2}{z}
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad & \frac{m^{-2} - 3}{m^{-3} - 3m^{-1}} \\
 &= \frac{(m^{-2} - 3)}{m^{-1}(m^{-2} - 3)} \\
 &= \frac{1}{m^{-1}} \\
 &= m^1
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad & \frac{(9x^{\frac{2}{3}}y^4)^{\frac{3}{2}}}{3xy} \\
 &= \frac{(3^2)^{\frac{3}{2}} (x^{\frac{2}{3}})^{\frac{3}{2}} (y^4)^{\frac{3}{2}}}{3xy} \\
 &= \frac{3^3 x^{-1} y^6}{3^1 x^1 y^1} \\
 &= 3^{-4} x^{-2} y^5 \\
 &= \frac{y^5}{3^4 x^2}
 \end{aligned}$$

$$\begin{aligned}
 (11) \quad & \frac{(x+y)^1}{x^1 - y^1} \\
 &= \frac{1}{(x+y)} \div \left(\frac{1}{x} - \frac{1}{y} \right) \\
 &= \frac{1}{(x+y)} \div \left(\frac{y-x}{xy} \right) \\
 &= \frac{1}{(x+y)} \times \frac{xy}{(y-x)} = \frac{xy}{y^2 - x^2}
 \end{aligned}$$

$$\begin{aligned}
 (12) \quad & \frac{2^{2n} - 3 \cdot 2^n + 2}{2^n - 2} \\
 &= \frac{(2^n)^2 - 3 \cdot (2^n) + 2}{2^n - 2} \\
 &= \frac{(2^n - 2)(2^n - 1)}{(2^n - 2)} \\
 &= \underline{2^n - 1}
 \end{aligned}$$

$$\begin{aligned}
 (13) \quad & (m^{\frac{1}{3}} + n^{\frac{1}{3}})^2 \\
 &= (m^{\frac{1}{3}} + n^{\frac{1}{3}})(m^{\frac{1}{3}} + n^{\frac{1}{3}}) \\
 &= (m^{\frac{1}{3}})^2 + m^{\frac{1}{3}} n^{\frac{1}{3}} + m^{\frac{1}{3}} n^{\frac{1}{3}} + (n^{\frac{1}{3}})^2 \\
 &= \underline{m^{\frac{2}{3}} + 2m^{\frac{1}{3}} n^{\frac{1}{3}} + n^{\frac{2}{3}}}
 \end{aligned}$$

$$\begin{aligned}
 (14) \quad & (a^{\frac{1}{3}} - 5)(5 + a^{\frac{1}{3}}) \\
 &= (a^{\frac{1}{3}} - 5)(a^{\frac{1}{3}} + 5) \\
 &= (a^{\frac{1}{3}})^2 - 25 \\
 &= \underline{a^{\frac{2}{3}} - 25}
 \end{aligned}$$

$$\begin{aligned}
 (15) \quad & \sqrt[3]{(0.125)^{-2}} + (125^2)^{\frac{1}{3}} \\
 &= \sqrt[3]{\left(\frac{1}{8}\right)^{-2}} + \left((5^3)^2\right)^{\frac{1}{3}} \\
 &= \sqrt[3]{\left(\frac{1}{2^3}\right)^{-2}} + (5^6)^{\frac{1}{3}} \\
 &= \sqrt[3]{\frac{1}{2^{-6}}} + 5^{\frac{6 \times \frac{1}{3}}{1}} \\
 &= \sqrt[3]{2^6} + 5^2 \\
 &= 2^{\frac{6}{3}} + 5^2 \\
 &= 2^2 + 25 \\
 &= 4 + 25 = \underline{29}
 \end{aligned}$$

$$\begin{aligned}
 (16) \quad & \frac{12^{n+1} \cdot 9^{n-2}}{18^{2n-1} \cdot 3^n} \\
 &= \frac{(2^2 \times 3^1)^{n+1} \cdot (3^2)^{n-2}}{(2^1 \times 3^2)^{2n-1} \cdot 3^n} \\
 &= \frac{2^{2n+2} \times 3^{n+1} \cdot 3^{2n-4}}{2^{2n-1} \times 3^{4n-2} \cdot 3^n} \\
 &= \frac{2^{2n+2} \times 3^{3n-3}}{2^{2n-1} \times 3^{3n-2}} \\
 &= 2^{2n+2 - (2n-1)} \times 3^{3n-3 - (3n-2)} \\
 &= 2^{2n+2-2n+1} \times 3^{3n-3-3n+2} \\
 &= 2^3 \times 3^{-1} \\
 &= \underline{\frac{8}{3}}
 \end{aligned}$$

$$\begin{aligned}
 (17) \quad & \frac{5^{n+1} \cdot 25^{n-1}}{125^{n-2}} \\
 &= \frac{5^{n+1} \cdot (5^2)^{n-1}}{(5^3)^{n-2}} \\
 &= \frac{5^{n+1} \cdot 5^{2n-2}}{5^{3n-6}} \\
 &= 5^{n+1+2n-2 - (3n-6)} \\
 &= 5^{3n-1-3n+6} = 5^5 \\
 &= \underline{3125}
 \end{aligned}$$

$$\begin{aligned}
 (18) \quad & \frac{3^{2n} - 9^{n+1}}{3^{2n}} \\
 &= \frac{3^{2n} - (3^2)^{n+1}}{3^{2n}} \\
 &= \frac{3^{2n} - 3^{2n} \cdot 3^2}{3^{2n}} \\
 &= \frac{3^{2n}(1 - 3^2)}{3^{2n}} \\
 &= 1 - 9 = \underline{-8}
 \end{aligned}$$

$$\begin{aligned}
 (19) \quad & \frac{3 \times 2^x + 2^{x+1}}{5 \times 2^x} \\
 &= \frac{3 \times 2^x + 2^x \cdot 2^1}{5 \times 2^x} \\
 &= \frac{2^x (3 + 2^1)}{5 \times 2^x} \\
 &= \frac{2^x (5)}{5 \times 2^x} \\
 &= \underline{\underline{1}}
 \end{aligned}$$

$$\begin{aligned}
 (20) \quad & \frac{3^2 \cdot 5^0 \cdot 4^{n-1}}{2^{2n+1} - 2^{2n}} \\
 &= \frac{3^2 \cdot 1 \cdot (2^2)^{n-1}}{2^{2n} \cdot 2^1 - 2^{2n}} \\
 &= \frac{9 \cdot 2^{2n} \cdot 2^{-2}}{2^{2n} (2^1 - 1)} \\
 &= \frac{9 \cdot 2^{-2}}{2-1} \\
 &= \frac{9 \cdot 2^{-2}}{1} \\
 &= 9 \times \frac{1}{2^2} \\
 &= \underline{\underline{\frac{9}{4}}}
 \end{aligned}$$

$$\begin{aligned}
 (21) \quad & \frac{3^{-2x} \cdot 36^{x+1} \cdot 3}{4^{x-1} \cdot (0.5)^2} \\
 &= \frac{3^{-2x} \cdot (2^2 \times 3^2)^{x+1} \cdot 3^1}{(2^2)^{x-1} \cdot \left(\frac{1}{2}\right)^2} \\
 &= \frac{3^{-2x} \cdot 2^{2x+2} \cdot 3^{2x+2} \cdot 3^1}{2^{2x-2} \cdot (2^{-1})^2} \\
 &= \frac{3^{-2x+2x+2+1} \cdot 2^{2x+2}}{2^{2x-2} \cdot 2^{-2}} \\
 &= \frac{3^3 \cdot 2^{2x+2}}{2^{2x-2-2}} \\
 &= 3^3 \cdot 2^{2x+2-(2x-4)} \\
 &= 3^3 \cdot 2^{2x+2-2x+4} \\
 &= \underline{\underline{3^3 \cdot 2^6}}
 \end{aligned}$$

$$\begin{aligned}
 (22) \quad & \frac{5 \cdot 5^{y-1} + 5^{-2y} \cdot 5^y}{3 \cdot 5^{-y} - 5^{1-y}} \\
 &= \frac{5 \cdot 5^{-y} \cdot 5^{-1} + 5^{-2y} \cdot 5^y}{3 \cdot 5^{-y} - 5^{1-y}} \\
 &= \frac{5^{1-1} \cdot 5^{-y} + 5^{-2y+y}}{3 \cdot 5^{-y} - 5^1 \cdot 5^{-y}} \\
 &= \frac{5^0 \cdot 5^{-y} + 5^{-y}}{3 \cdot 5^{-y} - 5 \cdot 5^{-y}} \\
 &= \frac{5^{-y} (5^0 + 1)}{5^{-y} (3 - 5)} \\
 &= \frac{1+1}{-2} \\
 &= \underline{\underline{-1}}
 \end{aligned}$$

A1.4.2 Wortelvormen:

Onthou: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ ($m, n \in \mathbb{Z}$ en $n \geq 2$)

Vb.2 Vereenvoudig: (a) $3\sqrt{2} + 7\sqrt{2} = \underline{10\sqrt{2}}$

$$(b) \quad \sqrt{8} \times \sqrt{2} = \sqrt{8 \times 2} = \sqrt{16} = \underline{4}$$

$$\begin{aligned}
 (c) \quad (\sqrt{3} + 1)^2 &= (\sqrt{3} + 1)(\sqrt{3} + 1) = 3 + 1\sqrt{3} + 1\sqrt{3} + 1 \\
 &= \underline{\underline{4 + 2\sqrt{3}}}
 \end{aligned}$$

$$\begin{aligned}
 (d) \quad & \sqrt{18} + \sqrt{50} - 2\sqrt{8} \\
 &= \sqrt{9 \times 2} + \sqrt{25 \times 2} - 2\sqrt{4 \times 2} \\
 &= \sqrt{9} \times \sqrt{2} + \sqrt{25} \times \sqrt{2} - 2\sqrt{4} \times \sqrt{2} \\
 &= 3\sqrt{2} + 5\sqrt{2} - 2 \times 2\sqrt{2} \\
 &= 8\sqrt{2} - 4\sqrt{2} \\
 &= \underline{4\sqrt{2}}
 \end{aligned}$$

Vb.3 Rasionaliseer die noemer: $\frac{2 + \sqrt{8}}{\sqrt{2}}$

$$\begin{aligned}
 &= \frac{2 + \sqrt{8}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
 &= \frac{2 \times \sqrt{2} + \sqrt{8} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\
 &= \frac{2\sqrt{2} + \sqrt{16}}{\sqrt{4}} = \frac{2\sqrt{2} + 4}{2} = \frac{2\sqrt{2}}{2} + \frac{4}{2} = \underline{\underline{\sqrt{2} + 2}}
 \end{aligned}$$

Oefening 5:

Datum: _____

(1) Vereenvoudig, sonder die gebruik van 'n sakrekenaar:

| | | |
|------------------------------------|--|--------------------------------------|
| (a) $(\sqrt{3} - 2)(\sqrt{3} + 2)$ | (b) $\sqrt{8} + \sqrt{50} - \sqrt{18}$ | (c) $(\sqrt{8} - 2\sqrt{2})^2$ |
| $= (\sqrt{3})^2 - 4$ | $= \sqrt{4 \times 2} + \sqrt{25 \times 2} - \sqrt{9 \times 2}$ | $= (\sqrt{4 \times 2} - \sqrt{2})^2$ |
| $= 3 - 4$ | $= \sqrt{4} \sqrt{2} + \sqrt{25} \sqrt{2} - \sqrt{9} \sqrt{2}$ | $= (2\sqrt{2} - \sqrt{2})^2$ |
| $= -1$ | $= 2\sqrt{2} + 5\sqrt{2} - 3\sqrt{2}$ | $= (1\sqrt{2})^2$ |
| $\underline{\underline{-1}}$ | $= 4\sqrt{2}$ | $= 2$ |

| | |
|--|---|
| (d) $\sqrt[3]{27x^6} + \sqrt[5]{32x^{10}}$ | (e) $(4\sqrt{2} - 3)^2$ |
| $= \sqrt[3]{3^3 x^6} + \sqrt[5]{2^5 x^{10}}$ | $= (4\sqrt{2} - 3)(4\sqrt{2} - 3)$ |
| $= 3^{\frac{3}{3}} x^{\frac{6}{3}} + 2^{\frac{5}{5}} x^{\frac{10}{5}}$ | $= (4\sqrt{2})^2 - 12\sqrt{2} - 12\sqrt{2} + 9$ |
| $= 3x^2 + 2x^2$ | $= 16 \times 2 - 24\sqrt{2} + 9$ |
| $= 3x^2 + 2x^2$ | $= 32 - 24\sqrt{2} + 9$ |
| $= 5x^2$ | $= 41 - 24\sqrt{2}$ |

$$\begin{aligned}
 \text{(f)} \quad & m \times \sqrt{27m^6} - \sqrt{12m^8} \\
 &= m \times \sqrt{9 \times 3 m^6} - \sqrt{4 \times 3 m^8} \\
 &= m \times 3\sqrt{3} m^{\frac{6}{2}} - 2\sqrt{3} m^{\frac{8}{2}} \\
 &= m^1 \times 3\sqrt{3} m^3 - 2\sqrt{3} m^4 \\
 &= 3\sqrt{3} m^4 - 2\sqrt{3} m^4 \\
 &= \underline{1\sqrt{3} m^4}
 \end{aligned}$$

$$\begin{aligned}
 \text{(g)} \quad & \sqrt{3} (\sqrt{48} - 3\sqrt{75} + 2\sqrt{108}) \\
 &= \sqrt{3} (\sqrt{16 \times 3} - 3\sqrt{25 \times 3} + 2\sqrt{36 \times 3}) \\
 &= \sqrt{3} (4\sqrt{3} - 3 \times 5\sqrt{3} + 2 \times 6\sqrt{3}) \\
 &= \sqrt{3} (4\sqrt{3} - 15\sqrt{3} + 12\sqrt{3}) \\
 &= \sqrt{3} (1\sqrt{3}) \\
 &= \underline{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(h)} \quad & \frac{\sqrt{18} - \sqrt{98}}{\sqrt{200}} \\
 &= \frac{\sqrt{9 \times 2} - \sqrt{49 \times 2}}{\sqrt{100 \times 2}} \\
 &= \frac{3\sqrt{2} - 7\sqrt{2}}{10\sqrt{2}} \\
 &= \frac{-4\sqrt{2}}{10\sqrt{2}} \\
 &= \underline{\frac{-2}{5}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(i)} \quad & \frac{\sqrt{\sqrt{64}} - \sqrt{12}}{\sqrt{18} - \sqrt{27}} \\
 &= \frac{\sqrt{8} - \sqrt{12}}{\sqrt{18} - \sqrt{27}} \\
 &= \frac{\sqrt{4 \times 2} - \sqrt{4 \times 3}}{\sqrt{9 \times 2} - \sqrt{9 \times 3}} \\
 &= \frac{2\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} - 3\sqrt{3}} \\
 &= \frac{2(\sqrt{2} - \sqrt{3})}{3(\sqrt{2} - \sqrt{3})} = \underline{\frac{2}{3}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(j)} \quad & \frac{(2 + \sqrt{3})(4 - \sqrt{3})}{\sqrt{100} + \sqrt{48}} \\
 &= \frac{8 - 2\sqrt{3} + 4\sqrt{3} - (\sqrt{3})^2}{10 + \sqrt{16 \times 3}} \\
 &= \frac{8 + 2\sqrt{3} - 3}{10 + 4\sqrt{3}} \\
 &= \frac{5 + 2\sqrt{3}}{10 + 4\sqrt{3}} \\
 &= \frac{(5 + 2\sqrt{3})}{2(5 + 2\sqrt{3})} \\
 &= \underline{\frac{1}{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(k)} \quad & \frac{\sqrt[3]{27x^6} + \sqrt[4]{16x^8}}{\sqrt[3]{125x^{27}}} \\
 &= \frac{\sqrt[3]{3^3} \sqrt[3]{x^6} + \sqrt[4]{2^4} \sqrt[4]{x^8}}{\sqrt[3]{5^3} \sqrt[3]{x^{27}}} \\
 &= \frac{3^{\frac{3}{3}} x^{\frac{6}{3}} + 2^{\frac{4}{4}} x^{\frac{8}{4}}}{5^{\frac{3}{3}} x^{\frac{27}{3}}} \\
 &= \frac{3x^2 + 2x^2}{5x^9} \\
 &= \frac{5x^2}{5x^9} \\
 &= \underline{\frac{1}{x^7}}
 \end{aligned}$$

