Grade 10 – Textbook

(Revised edition – CAPS)

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Chapter A1

Number systems

NO CALCULATOR MAY BE USED IN THIS CHAPTER!

A1.1 Number systems:

Exercise 1:

Complete: * Natural numbers: $\mathbb{N}$ = {______________}

* Whole numbers: $\mathbb{N}_0$ = {______________}

* Integers: $\mathbb{Z}$ = {______________}

* Rational numbers: $\mathbb{Q}$ = {______________}

A1.2 Rational numbers:

A1.2.1 Equivalent fractions:

E.g.1 Write down two equivalent fractions for $\frac{2}{3}$:

$\frac{2\times 3}{3\times 3} = \frac{6}{9}$ **or** $\frac{2\times 5}{3\times 5} = \frac{10}{15}$

Exercise 2:

(1) Write down three equivalent fractions for each of the following rational numbers:

(a) $\frac{-1}{4}$ (b) $\frac{3}{7}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$

(e) $\frac{12}{14}$ (f) $\frac{-36}{-9}$ (g) $2\frac{6}{11}$ (h) 5

(2) Are the following equivalent fractions or not? (Answer yes or no only.)

(a) $\frac{12}{5} = \frac{24}{10}$? (b) $\frac{7}{3} = \frac{3}{7}$? (c) $\frac{3}{-2} = \frac{6}{4}$?

(d) $\frac{3}{-5} = \frac{-9}{15}$? (e) $\frac{2}{3} = \frac{4}{9}$? (f) $\frac{3}{1} = \frac{48}{16}$?

(g) $\frac{4}{3} = \frac{-12}{-9}$? (h) $\frac{25}{10} = \frac{5}{2}$? (i) $\frac{5}{4} = \frac{4}{3}$?

A1.2.2 Order of rational numbers:

E.g.2 (a) Arrange the following fractions in ascending order: $\frac{1}{2}$; $\frac{3}{4}$ and $\frac{2}{3}$:

$$\frac{1}{2} = \frac{6}{12} \quad ; \quad \frac{3}{4} = \frac{9}{12} \quad \text{and} \quad \frac{2}{3} = \frac{8}{12}$$

$$\therefore \frac{1}{2} < \frac{2}{3} < \frac{3}{4}$$

(b) Write down a rational number between $\frac{3}{4}$ and $\frac{1}{3}$:

$$\frac{3}{4} = \frac{9}{12} \quad \text{and} \quad \frac{1}{3} = \frac{4}{12}$$

$$\therefore \frac{1}{3} < \frac{5}{12} \text{ or } \frac{6}{12} \text{ or } \frac{7}{12} \text{ or } \frac{8}{12} < \frac{3}{4}$$

Exercise 3:

(1) Arrange the following fractions in ascending order:

(a) $\frac{3}{4}$; $\frac{2}{3}$ and $\frac{4}{5}$

(b) $\frac{2}{3}$; $\frac{5}{7}$ and $\frac{4}{6}$

(2) Arrange the following fractions in descending order:

(a) $\frac{5}{8}$; $\frac{2}{3}$ and $\frac{3}{4}$

(b) $-1\frac{1}{2}$; $-1\frac{2}{3}$ and $\frac{-7}{5}$

(3) Place a rational number between each of the following numbers:

(a) $\frac{-1}{3}$ and $\frac{-3}{5}$

(b) $\frac{3}{4}$ and $\frac{7}{10}$

A1.2.3 Conversion of common fractions to decimal fractions:

E.g.3 Express the following as decimal fractions, without using a calculator:

(a) $\frac{3}{8} = \frac{3{,}000\ldots}{8} = \frac{3{,}{}^{3}0{}^{6}0{}^{4}0}{8} = 0{,}375$

(b) $1\frac{2}{3} = 1\frac{2{,}000\ldots}{3} = 1\frac{2{,}{}^{2}0{}^{2}0{}^{2}0\ldots}{3} = 1{,}66\ldots = 1{,}\dot{6}$

Exercise 4:

Express the following as decimal fractions, without using a calculator:

(1) $\frac{22}{7}$

(2) $4\frac{2}{3}$

(3) $\frac{1}{8}$

(4) $\frac{7}{9}$

(5) $\frac{17}{25}$

(6) $\frac{5}{100}$

(7) $\frac{4}{11}$

(8) $-2\frac{6}{7}$

(9) $-5\frac{5}{6}$

(10) $\frac{33}{8}$

A1.2.4 Rounding off decimal fractions:

E.g.4 Round off the following fractions correct to the number of decimals indicated in brackets:

(a) $\mathbf{4,34712}$ **(3 dec)**

$= \mathbf{4,347\underline{1}2}$ **Consider the underlined number**

$\approx \mathbf{4,347}$

(b) $\mathbf{290,09832}$ **(2 dec)**

$= \mathbf{290,09\underline{8}32}$

$\approx \mathbf{290,10}$

Exercise 5:

(1) Round off the following fractions correct to the number of decimals indicated in brackets:

(a) 3,573 (to 2 dec)

(b) 12,00873 (to 3 dec)

(c) 0,00384 (to 5 dec)

(d) 7,3226 (to 1 dec)

(e) 8,39999 (to 1 dec)

(f) 90,9023 (to the nearest integer)

(g) 0,433218 (to 4 dec)

(h) 1 456,6799 (to 3 dec)

(i) 66,666 (to 2 dec)

(j) 13,00034 (to 3 dec)

(2) Consider the following and choose the correct way of rounding off in brackets:

(a) 3,47653 $\approx$ 3,477 correct to the nearest (tenth, hundredth or thousandth)

(b) 96 995,31956 $\approx$ 96 995,32 correct to the nearest (tenth, hundredth or thousandth)

A1.2.5 Conversion of decimal fractions to common fractions:

E.g.5 Express the following as common fractions in its simplest form:

(a) $\mathbf{4,5 = 4\frac{5}{10}\left(\div\frac{5}{5}\right) = 4\frac{1}{2}}$

(b) $\mathbf{-0,12 = -\frac{12}{100}\left(\div\frac{4}{4}\right) = -\frac{3}{25}}$

Exercise 6:

Express the following as common fractions in its simplest form:

(1) 0,125

(2) 1,25

(3) 14,6

(4) −0,5

(5) −1,2

(6) 23,5

(7) 3,04

(8) 7,3

(9) 100,75

(10) 0,00005

A1.2.6 Conversion of recurring fractions to common fractions:

E.g.6 Convert the following to common fraction in its simplest form:

(a) $0,\dot{1} = \frac{1}{9}$; $0,\dot{3} = \frac{3}{9} = \frac{1}{3}$; $0,\dot{5} = \frac{5}{9}$; $0,\dot{8} = \frac{8}{9}$

(b) $3,\dot{2}\dot{4} = 3\frac{24}{99} = 3\frac{8}{33}$; $0,\dot{4}\dot{2}\dot{1} = \frac{421}{999}$; $15,\dot{1}\dot{6}\dot{5}\dot{3} = 15\frac{1653}{9999} = 15\frac{551}{3333}$

(c) $0,0\dot{3} = 0,\dot{3} \div 10 = \frac{3}{9} \div \frac{10}{1} = \frac{3}{9} \times \frac{1}{10} = \frac{3}{90} = \frac{1}{30}$

(d) $0,00\dot{4}\dot{6} = 0,\dot{4}\dot{6} \div 100 = \frac{46}{99} \div \frac{100}{1} = \frac{46}{99} \times \frac{1}{100} = \frac{46}{9900} = \frac{23}{4950}$

(e) $0,5\dot{7} = 0,5 + 0,0\dot{7} = 0,5 + 0,\dot{7} \div 10 = \frac{5}{10} + \frac{7}{9} \times \frac{1}{10} = \frac{5\times 9}{10\times 9} + \frac{7}{90} = \frac{45+7}{90} = \frac{52}{90} = \frac{26}{45}$

Exercise 7:

Convert the following to common fractions in its simplest form:

(1) $3,\dot{6}$ (2) $0,\dot{1}\dot{3}$ (3) $22,3\dot{9}$ (4) $-1,\dot{1}\dot{3}\dot{5}$ or $-1,\overline{135}$

(5) $0,\dot{7}$ (6) $0{,}00\dot{3}$ (7) $1,\overline{214}$ (8) $3{,}2\dot{5}\dot{8}$

☺ **Calculate the following without using a calculator:** $0,\dot{4} + \frac{2}{3}$

A1.3 Irrational and Real numbers:

Irrational numbers cannot be expressed as a ratio between two integers. These numbers are non-terminating and non-recurring decimals.

E.g. 7 Irrational numbers:

- $\sqrt{2}$ or $\sqrt{7}$ or $\sqrt{\frac{3}{4}}$ etc. because 2 ; 7 and 3 are not perfect squares!
- $\sqrt[3]{12}$ or $\sqrt[3]{100}$ etc. because 12 and 100 are not perfect cubes!

Whereas the following numbers are rational numbers:

- $\sqrt{4}$ or $\sqrt{0,01}$ or $\sqrt{\frac{25}{9}}$ etc. because 4 ; 0,01 ; 25 and 9 are perfect squares!
- $\sqrt[3]{27}$ or $\sqrt[3]{125}$ etc. because 27 and 125 are perfect cubes!

The real numbers, $\mathbb{R}$ consist of the rational numbers, $\mathbb{Q}$ and the irrational numbers, $\mathbb{Q}'$. Remember that all terminating and recurring decimals are rational numbers.

E.g.8 Determine the two integers between which the irrational number $\sqrt{7}$ lies.

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Choose the two perfect squares on either side of 7:

$$\sqrt{4} < \sqrt{7} < \sqrt{9}$$

$$\therefore\ 2 < \sqrt{7} < 3$$

<u>Exercise 8</u>:

(1) Which of the numbers are Rational numbers ($\mathbb{Q}$) and which are Irrational numbers ($\mathbb{Q}'$)?

(a) 14 (b) $\frac{1}{5}$ (c) $\sqrt{81}$

(d) $0{,}12$ (e) $\sqrt{18}$ (f) $12{,}\dot{2}\dot{3}$

(g) $-\sqrt{\frac{12}{3}}$ (h) $0{,}2945\ldots$ (i) $\sqrt[3]{64}$

(j) π (k) $\sqrt[5]{32}$ (l) $\frac{11}{7}$

(2) Between which two integers do the following irrational numbers lie?

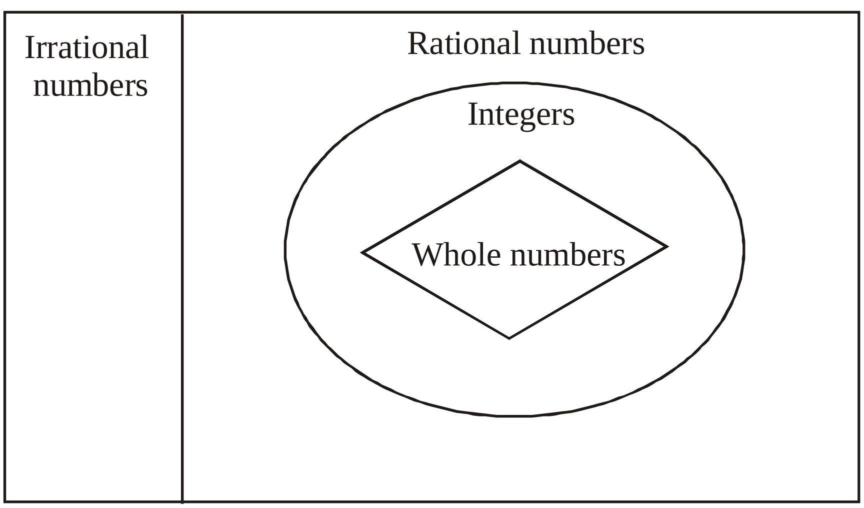
(a) $-\sqrt{12}$ (b) $\sqrt{66}$

(c) $\sqrt[3]{5}$ (d) $\sqrt[5]{2}$

(3) The diagram below is a summary of all the numbers that are used on school level.
Place the following numbers in the right place on the table; simplify the number if necessary:

$4\frac{1}{2}$; $\sqrt[3]{8}$; $\sqrt{8}$; -16 ; $0{,}45$; $0{,}\dot{3}$; $\frac{18}{6}$; $0{,}2387\ldots$; $\frac{0}{17}$; $6{,}88$

Real numbers:



☺ **(1) Except for the real numbers we also have the non-real numbers.**
Give an example of a non-real number.

(2) What is the set called that contain all real and non-real numbers?

A1.4 Representation of sets of numbers:

Sets of numbers can be represented or written in the following ways:

A1.4.1 Set builder notation:

E.g. 9 Write the following sets of numbers in set builder notation:

(a) All natural numbers greater than 6: $\{x \,/\, x > 6\,; x \in \mathbb{N}\}$

(b) All real numbers between -2 and 5: $\{m : -2 < m < 5\ ;\ m \in \mathbb{R}\}$

A1.4.2 Interval notation:

Only sets that form part of real numbers can be represented using interval notation!

E.g. 10 Write the following in interval notation:

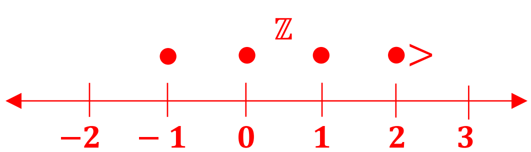
(a) The real numbers between -2 and 4, including 4: $x \in (-2\,; 4]$ **Open, closed interval!**

(b) $\{m \,/\, m > 7;\ m \in \mathbb{R}\}$: $m \in (7\ ;\ \infty)$ **Open interval!**

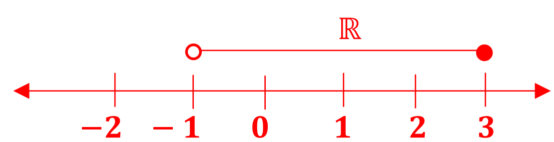
A1.4.3 Number lines:

E.g. 11 Represent the following on a number line:

(a) $\{-1\,;\,0\,;\,1\,;\,2\,;\,\ldots\ldots\ldots\}$



(b) $\{x :\ -1 < x \leq 3\ ;\ x \in \mathbb{R}\}$



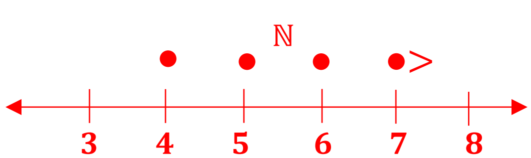
A1.4.4 Solving of linear inequalities:

E.g. 12 Solve for x in each of the following and represent the solution on a number line:

(a) $x - 2 \geq 2$ if $x \in \mathbb{N}$

$x \geq 2 + 2$

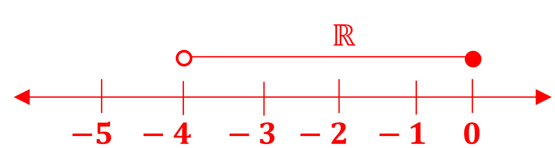
$x \geq 4$



(b) $-3 < x + 1 \leq 1$ if $x \in \mathbb{R}$

$-3 - 1 < x \leq 1 - 1$

$-4 < x \leq 0$



Exercise 9:

(1) Write the following in interval notation (if applicable) and represent it on a number line:

(a) $\{x : 2x < -2;\ x \in \mathbb{R}\}$

(b) $\{x : -2 \leq x + 1 \leq 4;\ x \in \mathbb{Z}\}$

(c) $\{y : y - 3 < -1;\ y \in \mathbb{N}\}$

(d) $\{x : x \leq -1;\ x \in \mathbb{R}\}$

(e) $\{x \,/\, x < 3;\ x \in \mathbb{Z}\}$

(f) $\{p \,/\, 2p \geq -5;\ p \in \mathbb{R}\}$

(g) $\{m : -1 \leq 2m - 1 < 7;\ m \in \mathbb{R}\}$

(h) $\{x : 2x - 3 < 7;\ x \in \mathbb{N}_0\}$

(2) Solve for x in each of the following and represent the solution on a number line:

(a) $x + 1 \leq 3;\quad x \in \mathbb{N}_0$

(b) $2x \geq -8;\quad x \in \mathbb{R}$

(c) $x - 4 \leq 0;\quad x \in \mathbb{Z}$

(d) $2x + 3 > 7;\quad x \in \mathbb{N}$

(e) $-6 < x - 1 \leq 6;\quad x \in \mathbb{R}$

(f) $x + 7 \geq -1;\quad x \in \mathbb{Z}$

A1.5 REVISION EXERCISE:

(1) Convert the following to common fractions in its simplest form: (Without a calculator.)

(a) $14{,}1\dot{7}$

(b) $0,\overline{1234}$

(c) $4{,}68$

(d) $5,\dot{1}$

(2) Indicate, by using a ✓, all the rational numbers between 0 and 10:

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
$\sqrt{9}$	-1	$\sqrt{8}$	$\frac{6}{3}$	$\sqrt[3]{16}$	π	$\frac{0}{3}$	$\sqrt{144}$	$0{,}124\ldots.$	$\sqrt{\frac{24}{6}}$

(3) Round off the following fractions correct to the number of decimals indicated in brackets:

(a) $7{,}199$ (to 1 dec)

(b) $0{,}048561$ (to 4 dec)

(c) $234{,}34$ (to 1 dec)

(d) $1\,001{,}1989$ (to the nearest integer)

(e) $3{,}997$ (to 2 dec)

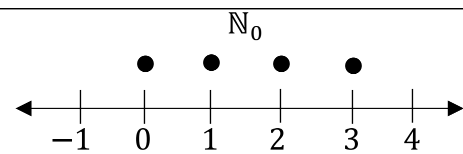
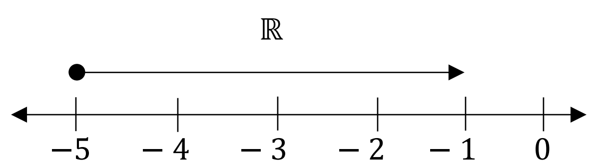
(f) $23{,}712$ (to the nearest integer)

(4) Place any two irrational numbers between 2 and 3.

(5) Between which two integers do the following irrational numbers lie?

(a) $\sqrt{\frac{1}{2}}$ (b) $\sqrt[3]{57}$

(6) Complete the missing representations in the table below:

	Set builder notation:	Interval notation:	Number line:
(a)	$\{x \,/\, -1 < x \leq 2;\ x \in \mathbb{R}\}$		
(b)		$x \in [-2\,;5]$	
(c)		$y \in (-\infty\,;3]$	
(d)			
(e)	$\{y \,/\, y \geq 3;\ y \in \mathbb{N}\}$		
(f)		$m \in (0\,;4]$	
(g)			
(h)	$\{m : m \leq 6;\ m \in \mathbb{R}\}$		
(i)	$\{x \,/\, -1 < x < 2;\ x \in \mathbb{Z}\}$		
(j)		$x \in (-1;\ \infty)$	

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Chapter A2

Algebraic expressions

A2.1 Products:
A2.1.1 The law of distribution:

E.g. 1 Determine the following products by using the law of distribution:

(a) $(x - 2)(x + 2)$ **(b)** $(3m + n)(2m + 5n)$

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(a) $x(x + 2) - 2(x + 2)$
$= x^2 + 2x - 2x - 4$
$= x^2 - 4$

(b) $(3m + 1n)(2m + 5n)$
$= 6m^2 + 15mn + 2mn + 5n^2$
$= 6m^2 + 17mn + 5n^2$

Exercise 1:

Determine the following products:

(1) $(y - 4)(y + 3)$

(2) $(p - 2)(p - 7)$

(3) $(2x + 1)(x - 5)$

(4) $(x - 2y)(2x - y)$

(5) $(4ab + 1)(2ab - 3)$

(6) $(5 - 7m)(2 - 3m)$

(7) $(2a - 4b)(3a + 2b)$

(8) $(m + n)(2m - 1)$

(9) $(d - 12)(12 + d)$

(10) $(a^2 + 4)(a^2 + 2)$

(11) $\left(\frac{1}{2}m - 6\right)(8m - 3)$

(12) $(-2k - 5)(5 + 3k)$

(13) $\left(p + \frac{1}{p}\right)\left(8p - \frac{4}{p}\right)$

(14) $(abc - 2ac)(abc + 3bc)$

(15) $(3r^3 + 2)(2r^2 - 5)$

(16) $2x(x - 5y)(3x + 2y)$

(17) $\left(\frac{1}{p^3q^2} - \frac{2}{p^2q}\right)\left(\frac{1}{p} + \frac{2}{q}\right)$

(18) $\left(\frac{m^2n}{3} - \frac{6}{mn}\right)\left(\frac{mn}{2} - \frac{3}{mn^2}\right)$

E.g. 2 Simplify:

(a) $(2a + 1)(2a - 1) = 4a^2 - 2a + 2a - 1 = 4a^2 - 1$

(b) $(m^2 - 5n)(m^2 + 5n) = m^4 + 5m^2n - 5m^2n - 25n^2 = m^4 - 25n^2$

Or shorter

(c) $(xy + 3)(xy - 3) = x^2y^2 - 9$

(d) $\left(\frac{ab}{4} - \frac{1}{7}\right)\left(\frac{ab}{4} + \frac{1}{7}\right) = \frac{a^2b^2}{16} - \frac{1}{49}$

Exercise 2:

Simplify:

(1) $(abc - 2)(abc + 2)$

(2) $\left(\frac{1}{3} + 5t\right)\left(\frac{1}{3} - 5t\right)$

(3) $(p - 9q)(9q + p)$

(4) $(n + 7k)(7n - k)$

(5) $(-a + 4b)(-a - 4b)$

(6) $-x\left(\frac{1}{x} - x\right)\left(\frac{1}{x} + x\right)$

(7) $(x^{2m} - 8)(x^{2m} + 8)$

(8) $(0{,}3 + 3q)(0{,}3 - 3q)$

(9) $(b^6c^3 + 6)(b^6c^3 + 6)$

(10) $(4xk^5 - 7)(7 + 4xk^5)$

(11) $(m - 2n)^2(m + 2n)^2$

(12) $\left(\frac{m}{n} + 2\right)\left(\frac{m^2}{n^2} + 4\right)\left(\frac{m}{n} - 2\right)$

A2.1.2 Squaring of a binomial:

E.g. 3 Determine the following products:

(a) $(2x + 1)^2$

$= (2x + 1)(2x + 1)$

$= 4x^2 + 2x + 2x + 1$

$= 4x^2 + 4x + 1$

****

(b) $\left(m - \frac{1}{m}\right)^2$

$= \left(m - \frac{1}{m}\right)\left(m - \frac{1}{m}\right)$

$= m^2 - \frac{m}{m} - \frac{m}{m} + \frac{1}{m^2}$

$= m^2 - 2 + \frac{1}{m^2}$

Exercise 3:

Determine the following squares:

(1) $(y - 11)^2$

(2) $(3p + 2q)^2$

(3) $(-4 + 5c)^2$

(4) $(mn + 3)^2$

(5) $(k^2 + 1)^2$

(6) $(8 - 3b)^2$

(7) $\left(x - \frac{1}{2}\right)^2$

(8) $\left(\frac{y}{5} - 3\right)^2$

(9) $(5p - 2p^2)^2$

(10) $\left(4 + \frac{3}{n}\right)^2$

(11) $(0{,}2 + 6y)^2$

(12) $\left(\frac{2m}{p} + \frac{p^2}{3m}\right)^2$

E.g. 4 Simplify the following: (Shorter method!)

(a) $(m + 5n)^2 = (m)^2 + 2(m)(5n) + (5n)^2 = m^2 + 10mn + 25n^2$

(b) $(pq - 2)^2 = p^2q^2 - 4pq + 4$

(c) $\left(\frac{1}{3} + 3x\right)^2 = \frac{1}{9} + 2x + 9x^2$

Exercise 4:

Simplify (Use the shorter method!)

(1) $(x - 3)^2$

(2) $(6m - 1)^2$

(3) $(3y + 7)^2$

(4) $(3 + pq)^2$

(5) $(5t^2 + 8)^2$

(6) $\left(\frac{2}{3} - 6y\right)^2$

(7) $(-2k - 5)^2$

(8) $\left(\frac{3p - 2q}{5m}\right)^2$

(9) $(4x^2 + 10y^2)^2$

(10) $(2mn + 7)(7 + 2mn)$

(11) $(8 - 3y)(8 + 3y)$

(12) $-2(abc - 11)^2$

A2.1.3 Binomials and trinomials:

E.g. 5 Simplify the following products: $(4y + 1)(y^2 - y + 5)$

$= 4y^3 - 4y^2 + 20y + 1y^2 - 1y + 5$

$= 4y^3 - 3y^2 + 19y + 5$

Exercise 5:

Simplify the following products:

(1) $(2a - 3)(a^2 + 5a - 4)$

(2) $(m + 7)(2m^2 + 3m + 3)$

(3) $(1 + x)(1 - x + x^2)$

(4) $(3y - 2)(9y^2 + 6y + 4)$

(5) $\left(2m + \frac{1}{2}\right)\left(\frac{m^2}{4} + 4 - 4m\right)$

(6) $(m^2n^2 - 5)(25 + 5m^2n^2 + m^4n^4)$

A2.1.4 The sum and difference of two cubes:

E.g. 6 Consider the following:

Product:

(a) $(x - 3)(x^2 + 3x + 9) = x^3 + 3x^2 + 9x - 3x^2 - 9x - 27 = x^3 - 27$

(b) $(y + 5)(y^2 - 5y + 25) = y^3 - 5y^2 + 25y + 5y^2 - 25y + 125 = y^3 + 125$

Pattern:

(c) $(4m - 1)(16m^2 + 4m + 1) = (4m - 1)[(4m)^2 + (4m)(1) + (1)^2] = 64m^3 - 1$

Pattern:

(d) $(n^2 + 2)(n^4 - 2n^2 + 4) = (n^2 + 2)[(n^2)^2 - (2)(n^2) + (2)^2] = n^6 + 8$

Exercise 6:

Write down the following products directly, if possible:

(1) $(a + 3)(a^2 - 3a + 9)$

(2) $(2y^3 + 4)(4y^6 - 8y^3 + 16)$

(3) $\left(\frac{x}{3} - 1\right)\left(\frac{1}{9}x^2 + \frac{1}{3}x + 1\right)$

(4) $\left(6a^2 - \frac{1}{2}\right)\left(36a^4 + 3a^2 + \frac{1}{4}\right)$

(5) $(5q + 7)(25q^2 - 35q + 49)$

(6) $(8 - 3m)(9m^2 + 24m + 64)$

(7) $(x - 5)(x^2 - 5x + 25)$

(8) $(0{,}1 + 0{,}2y)(0{,}01 - 0{,}02y + 0{,}04y^2)$

(9) $(9a^4 + 6a^2b + 4b^2)(3a^2 - 2b)$

(10) $2(-1 + 5m)(25m^2 + 5m + 1)$

A2.1.5 Simplification of expressions:

Remember: Order of operations!

E.g. 7 Simplify the following:

(a) $x(x-2) + 3(x-2)(x+2)$

$= x^2 - 2x + 3(x^2 - 4)$

$= x^2 - 2x + 3x^2 - 12$

$= 4x^2 - 2x - 12$

(b) $(m - 4n)^2 - (n + m)^2$

$= m^2 - 8mn + 16n^2 - 1(n^2 + 2mn + m^2)$

$= 1m^2 - 8mn + 16n^2 - 1n^2 - 2mn - 1m^2$

$= 15n^2 - 10mn$

Exercise 7:

Simplify:

(1) $2a(a + 1) - 3(a - 1)^2$

(2) $(m^3 + 3m^2 - 2m - 2)m + 4(m^3 + 5m^2 - 6)$

(3) $(x-1)(x^2 - x + 1) + 3(x-2) - x^3$

(4) $(y-1)(y+1)(y^2+1)$

(5) $-2(2c+1)(c-2) - 2(2c-1)^2$

(6) $\frac{1}{2}(4p-3)^2 - \left(\frac{p}{2} + 2\right)^2$

(7) $a^2 + b^2 - (2a - b)^2 + (a + 2b)^2$

(8) $(3y+1)(9y^2+1)(3y-1)$

(9) $7 - 3(5t-6) + 2t - (t-1)(t+1)$

(10) $(x+3y)(x^2 - 3xy + 9y^2) + (x+3y)(x-3y)$

(11) $(2p-3)(4p^2+6p+9)(8p^3+27)$

(12) $(x - y + k)(x - y - k)$

(13) $(p - 3)^3$

(14) $[(2m+n)(4m^2 - 2mn + n^2)]^2$

(15) $(x+y)(x-y) - (x-y)^2 + (x+y)^2 - x(x+y) + (x-y)(x^2 + 2xy + y^2)$

☺ **Simplify:** **(a)** $(x^m + y^n)^2$ **(b)** $(a^{x+1} - 2)(a^{x+1} + 2)$

A2.2 <u>Factorisation</u>:
A2.2.1 <u>Common factor</u>:

E.g.8 Factorise completely: **Check your answer:**

(a) $12mx^2 - 2mx = 2mx(6x - 1)$ $\leftrightarrow$ $2mx \times 6x + 2mx \times -1 = 12mx^2 - 2mx$

(b) $p^3q^2r + p^2qr^3 = p^2qr(pq + r^2)$

(c) $2(m + n) + y(m + n) = (m + n)(2 + y)$

(d) $2a(x - y) - 5(y - x) = 2a(x - y) - 5[-(-y + x)]$

$= 2a(x - y) + 5(x - y) = (x - y)(2a + 5)$

<u>Exercise 8</u>:

Factorise completely:

(1) $4am + 3a^2m^4$ (2) $12x^2 - 132y^2$ (3) $-p^5q^3 + pq$

(4) $4x^2 - 8x + 1$ (5) $\frac{1}{2}abc + \frac{1}{2}a^2bc^2$ (6) $5r^7R^4 + 15r^5R^2$

(7) $7x(3y - 1) - 2(3y - 1)$ (8) $12x^3t - 14xt^3 + 16x^2t^2$

(9) $2gh + 18g^2h + 3g^3h$ (10) $a(2m + 3) + b(3 + 2m)$

(11) $3g(3g + h) - (3g + h)$ (12) $4b + 5 + 7a(4b + 5)$

(13) $r(x - y) + (y - x) + 2t(x - y)$ (14) $5ab(d - 4c) - 7a(4c - d)$

(15) $4xy + 2 - k(2xy + 1)$ (16) $y^4(x^2 - 5) + y(x^2 - 5)$

A2.2.2 <u>Grouping</u>:

E.g. 9 Factorise completely:

(a) $ax + ay + bx + by = a(x + y) + b(x + y)$

$= (x + y)(a + b)$

(b) $4m^2 + n - pn - 4m^2p = 1(4m^2 + n) - p(n + 4m^2)$

$= (4m^2 + n)(1 - p)$

Exercise 9:

Factorise completely:

(1) $r - s - 5r + 5s$

(2) $3am^3 + 6am + 7m^2n + 14n$

(3) $pq - pr + q^2 - qr$

(4) $4x - 8k - rtx + 2rtk$

(5) $aw - bw + 2bw - 2aw$

(6) $p^2 + p(2 + q) + 2q$

(7) $mx + nx + rx - my - ny - ry$

(8) $pq - 1 + p - q$

(9) $g(h - j) + g^2 - hj$

(10) $3mn^3 + 2mt - 7m + 3kn^3 + 2kt - 7k$

A2.2.3 Difference between two squares:

E.g. 10 Factorise completely:

(a) $x^2 - y^2 = (x - y)(x + y)$

(b) $144r^2 - p^{16} = (12r + p^8)(12r - p^8)$

(c) $m^2n^2 - (2mn + 1)^2 = [mn - (2mn + 1)][mn + (2mn + 1)]$

$= [mn - 2mn - 1][mn + 2mn + 1]$

$= [-mn - 1][3mn + 1]$

Exercise 10:

Factorise completely:

(1) $c^2 - 81$

(2) $1 - p^2$

(3) $x^2y^2 - 25$

(4) $9 + k^2$

(5) $m^{10} - 16$

(6) $r^6 - k^2$

(7) $(xyz)^2 - 121$

(8) $n^2 - 50$

(9) $-9 + t^4$

(10) $2x^2 - 8$

(11) $-y^2 - 49$

(12) $(x + y)^2 - 100$

(13) $p^2m + p^2 - m - 1$

(14) $(m - 3)^2 - (m + 2)^2$

(15) $x^3 - 9x$

(16) $m^4 - n^4$

(17) $ab^7x - ab^3x$

(18) $y^2 - \frac{1}{4}$

(19) $3b(q^2 - 9) + 2(q^2 - 9)$

(20) $a^3b - 8ab$

(21) $5k(p^2 - 4) + 2a(p^2 - 4) - (p^2 - 4)$

(22) $a^{12} - 81$

(23) $(3r - 2)^2 - 4r^2$

(24) $q^4(x^2 - 2x - 5) - k^2(x^2 - 2x - 5)$

(25) $4m^2 - n^2 + 2m + n$

(26) $3p^2 - 2q - 3q^2 + 2p$

(27) $16(x - y)^2 - 25(2x + 3y)^2$

(28) $(a - b + c)^2 - (a + b - c)^2$

A2.2.4 Trinomials:

E.g. 11 Factorise completely:

Pattern:

(a) $x^2 + 4x + 3 = (x + 3)(x + 1)$ → $+3 \times +1 = +3$ and $+3 + 1 = +4$

(b) $x^2 - 7x + 12 = (x - 3)(x - 4)$ → $-3 \times -4 = +12$ and $-3 - 4 = -7$

(c) $y^2 + 6y - 7 = (y + 7)(y - 1)$ → $+7 \times -1 = -7$ and $+7 - 1 = +6$

(d) $m^2 - 1m - 12 = (m - 4)(m + 3)$ → $-4 \times +3 = -12$ and $-4 + 3 = -1$

Exercise 11:

Factorise completely:

(1) $x^2 + 5x + 6$ (2) $y^2 + 3y - 4$ (3) $m^2 - 3m - 10$

(4) $p^2 - 2p - 8$ (5) $k^2 + k - 20$ (6) $b^2 + 11b + 10$

(7) $a^2 - 5a - 14$ (8) $x^4 + 6x^2 + 9$ (9) $7 - 8q + q^2$

(10) $s^2 + 10s - 24$ (11) $p^2 - p - 6$ (12) $d^2 + 10d + 25$

(13) $p^2 - 2p + 1$ (14) $c^6 + 5c^3 - 24$ (15) $12 + 8s + s^2$

(16) $r^2 - 11r - 12$ (17) $k^2 - 15 - 10$ (18) $g^2 + 14g - 72$

(19) $m^2 - 18m - 144$ (20) $a^2 + 2a - 63$ (21) $t^2 - 13t + 42$

(22) $2t^2 - 12t + 10$ (23) $r^4 + 2r^2 - 24$ (24) $100 - 21y - y^2$

(25) $7k - 60 + k^2$ (26) $y^3 - 3y^2 - 18y$ (27) $m^2 + m + \frac{1}{4}$

(28) $x^2 - 4xy + 3y^2$ (29) $15y^2 - 8y + 1$ (30) $15 - 2d - d^2$

(31) $n^3 + 8n^2 + 12n$ (32) $p^2 + 3pq - 18q^2$ (33) $-8 + 4x + 4x^2$

(34) $p^2q^4 - pq^2 - 12$ (35) $-k^2 + 12k - 35$ (36) $8m^2 - 7m - 1$

(37) $x^2(a^2 + 5a + 6) - y^2(a^2 + 5a + 6)$ (38) $2t^4 - 26t^2 + 72$

(39) $(x + 1)^2 + 3(x + 1) - 28$ (40) $x^2(y^2 - 9) - 4x(y^2 - 9) - 12(y^2 - 9)$

(41) $m^4n^4 + 4m^2n^2 - 5$ (42) $(2c - 1)^2 + 8(2c - 1) + 16$

A2.2.5 More trinomials:

E.g. 12 Factorise completely: (a) $8x^2 + 2x - 15$

$4x \quad -5 \qquad -10x$

$2x \quad +3 \qquad +12x$

$\qquad\qquad\qquad + 2x$

$= (4x - 5)(2x + 3)$

(b) $5a^2 + 17ab + 6b^2$

$5a \quad +2b \qquad + 2ab$

$1a \quad +3b \qquad + 15ab$

$\qquad\qquad\qquad 17ab$

$= (5a + 2b)(1a + 3b)$

Exercise 12:

Factorise completely:

(1) $3x^2 + 10x + 3$

(2) $6y^2 - y - 2$

(3) $5t^2 + 8t - 4$

(4) $6n^2 - 29n + 9$

(5) $3p^2 - 16p + 21$

(6) $12g^2 - 4g - 1$

(7) $14r^2 + 17r + 5$

(8) $10q^2 + q - 9$

(9) $8c^2 - 18c + 7$

(10) $6m^2 - 7mn + 2n^2$

(11) $15p^2 - pq - 2q^2$

(12) $12 - 35x + 25x^2$

(13) $20s^2 - 44st - 15t^2$

(14) $20a^2 - ab - 12b^2$

(15) $9m^2 - 12m + 4$

(16) $x^2 - 2xy + y^2$

(17) $10k^2 - 18k - 10$

(18) $25a^2 - 30ab + 9b^2$

(19) $18c^2d^2 - 15cd - 12$

(20) $-12 + 23b^2 - 10b^4$

(21) $m^2 - 4mn + 4n^2 - 9x^2$

(22) $4p^2 - 12pq + 9q^2 - 16y^2$

(23) $x^2 - n^2 + 2mn - m^2$

(24) $9a^2 - 1 - 25y^2 + 10y$

A2.2.6 Sum and difference of cubes:

E.g. 13 Factorise completely: (a) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

(b) $x^3 - y^3 = (x + y)(x^2 - xy + y^2)$

See A2.1.4 on page 13 (The sum and difference between two cubes) for the pattern.

Exercise 13:

Factorise completely:

(1) $a^3 - 1$

(2) $m^3 - 8$

(3) $1\,000 + t^3$

(4) $x^6 + y^9$

(5) $c^3d^3 + 64$

(6) $125 - p^{12}$

(7) $8p^3 + 27q^3$

(8) $100 - y^6$

(9) $1 - 216t^3$

(10) $r^3 + \frac{1}{8}$

(11) $3b^4 - 3b$

(12) $-t^3 - s^3$

(13) $\frac{2x^3}{y^9} + 54$

(14) $0{,}001a - ab^6$

(15) $(x - y)^3 + y^3$

(16) $m^6(p^6 + 8) - 8(p^6 + 8)$

☺ **Factorise completely:** $x^2 - 4x + 4 - 3xy + 6y$

[Hint: use grouping!]

A2.3 Algebraic fractions:

A2.3.1 Multiplication and division:

E.g.14 Simplify (assume that no denominator is zero!):

(a) $\frac{12y - 3y^2}{6y} = \frac{{}^{1}\cancel{3y}(4 - y)}{{}^{2}\cancel{6y}} = \frac{1(4 - y)}{2} = \frac{4 - y}{2}$

(b) $\frac{x^2(1-y) + x(1-y) - 6(1-y)}{(x+3)(y-1)} = \frac{(1-y)(x^2 + x - 6)}{(x+3)(y-1)} = \frac{-\cancel{(y-1)}(x^2 + x - 6)}{(x+3)\cancel{(y-1)}}$

$= \frac{-(x^2 + x - 6)}{(x + 3)} = \frac{-1\cancel{(x+3)}(x - 2)}{\cancel{(x+3)}} = -(x - 2) = -x + 2$

(c) $\frac{6ab^2}{5ac} \times 3b^3c^2 \div \frac{18a^2bc}{10a} = \frac{6a^1b^2}{5a^1c^1} \times \frac{3b^3c^2}{1} \times \frac{10a^1}{18a^2b^1c^1}$

$$= \frac{180a^2b^5\cancel{c^2}}{90a^3b^1\cancel{c^2}} = \frac{(180 \div 90)\, b^{5-1}}{a^{3-2}} = \frac{2b^4}{a}$$

(d) $\frac{y^2 + y}{y^2 + 2y + 1} \div \frac{y^3 + y^2 - 2y}{y^2 - 1} = \frac{y(\cancel{y + 1})}{(y+1)(\cancel{y+1})} \times \frac{y^2 - 1}{y^3 + y^2 - 2y}$

$$= \frac{y}{(y+1)} \times \frac{(y + 1)(y - 1)}{y(y^2 + y - 2)}$$

$$= \frac{\cancel{y}}{(\cancel{y+1})} \times \frac{(\cancel{y+1})(\cancel{y-1})}{\cancel{y}(y + 2)(\cancel{y-1})} = \frac{1}{(y + 2)}$$

Exercise 14:

Simplify: (No denominator is zero.)

(1) $\frac{-28x^3y}{8x^2y^2}$ (2) $\frac{6m^3}{30m^3n}$ (3) $\frac{ab + b}{b}$

(4) $\frac{3d^2 + d}{3d^2}$ (5) $\frac{x^2 + 2x}{x^2 - 4}$ (6) $\frac{(c + 1)(c - 5)}{2c + 2}$

(7) $\frac{3t(p-2) + (p-2)}{(3t+1)}$ (8) $\frac{2q^2 + q - 6}{2q^2 + 4q}$ (9) $\frac{2x^2 - 18}{12x - 4x^2}$

(10) $\frac{m^2n^2 - 25}{5 + mn}$ (11) $\frac{a^2 - 3a - 4}{a^2 - 16}$

(12) $\frac{3(t + 3) + 7r(3 + t)}{2t + 6}$ (13) $\frac{4y^2 - 16}{4y^2 + 16}$

(14) $\frac{a^2 - 4}{a^2 + a} \times \frac{a + 1}{a + 2}$ (15) $\frac{mn - m^2}{m^2 - n^2} \div \frac{m^2}{n^2 + mn}$

(16) $\frac{p^2(q - 5) + 6p(q - 5) + 8(q - 5)}{(q - 5)^2(p + 2)}$ (17) $\frac{(b - 1)(b + 4) - 3(b - 1)}{b^2 - 1}$

(18) $\frac{p^2 + p - 12}{p^2 + 4p} \times \frac{p^3 - 3p^2}{p^2 - 9}$ (19) $\frac{q^2 - 4}{q^2 + q - 6} \div \frac{q^2 + q - 2}{q^2 + 4q + 3}$

(20) $\frac{b^2 + 6b + 9}{b^2 + 2b - 3} \div \frac{b^2 - 1}{b^2 + b - 6}$ (21) $\frac{3y^2 + 27}{2y + 6} \times \frac{6y - 18}{y^4 - 81}$

(22) $\frac{5 - 15x}{3x^2 - 10x + 3} \div \frac{15 - 2x - x^2}{x^2 + 4x - 5}$ (23) $\frac{p^2 + p - 2}{2p - 4} \times \frac{4 - p^2}{p + 1}$

(24) $\dfrac{4-8t}{4t^2-4t+1} \div \dfrac{2t+2}{1-t-2t^2}$

(25) $\dfrac{x^3-8y^3}{2-2y} \div \dfrac{x^2+2xy+4y^2}{x+y}$

(26) $\dfrac{m^2+2m-8}{m^3-m} \times \dfrac{1+m}{16-m^2} \div \dfrac{m-4}{m^3+6m^2-7m}$

(27) $\dfrac{xy+2x}{xy-2x} \div \left(\dfrac{y^2+2y}{y+3} \times \dfrac{y^2}{xy^2-3yx+2x}\right)$

A2.3.2 Addition and subtraction:

E.g.15 Simplify. Indicate all restrictions!

(a) $\dfrac{2m}{m-n} + \dfrac{3m}{m-n} = \dfrac{2m+3m}{(m-n)} = \dfrac{5m}{m-n}$

LCM $= (m-n)$

Restrictions: $m-n \neq 0 \quad \therefore \quad m \neq n$

(b) $\dfrac{12}{y^2-4} - \dfrac{5}{y^2+2y}$

$= \dfrac{12}{(y-2)(y+2)} - \dfrac{5}{y(y+2)}$

LCM $= y(y-2)(y+2)$

$= \dfrac{12}{(y-2)(y+2)} \times \dfrac{y}{y} - \dfrac{5}{y(y+2)} \times \dfrac{(y-2)}{(y-2)}$

$= \dfrac{12y}{y(y-2)(y+2)} - \dfrac{5(y-2)}{y(y+2)(y-2)}$

$= \dfrac{12y-5(y-2)}{y(y-2)(y+2)}$

$= \dfrac{12y-5y+10}{y(y-2)(y+2)}$

$= \dfrac{7y+10}{y(y-2)(y+2)}$

Restrictions: * $y \neq 0$

* $(y-2) \neq 0 \quad \therefore \quad y \neq 2$

* $(y+2) \neq 0 \quad \therefore \quad y \neq -2$

Exercise 15:

Simplify. Indicate all restrictions where necessary!

(1) $\dfrac{x-3}{2} + \dfrac{5x}{2}$

(2) $\dfrac{3-2m}{mn} + \dfrac{m+1}{mn}$

(3) $\dfrac{4}{x-1} - \dfrac{2}{x-2}$

(4) $\dfrac{-1}{q+p} - \dfrac{3}{q-p}$

(5) $\dfrac{5}{(y+2)^2} + \dfrac{3}{y^2-4}$

(6) $\dfrac{m}{n^2-n} + \dfrac{3m}{n^2-2n}$

(7) $\dfrac{4}{p^2+2p-3} - \dfrac{2}{p^2-1}$

(8) $\dfrac{8}{9-y^2} - \dfrac{3}{y-3}$

(9) $\dfrac{3}{x} + \dfrac{2}{x^2+x} - \dfrac{1}{x^2-1}$

(10) $\dfrac{2}{q^2+3q-10} - \dfrac{1}{(q-2)^2}$

(11) $\dfrac{2t-3}{t+2} - \dfrac{t+7}{t^2+5t+6}$

(12) $\dfrac{2}{3d^2-6d} + \dfrac{3d}{2d^3-2d^2}$

(13) $\dfrac{3}{m+3} - \dfrac{2}{3-m} + \dfrac{18}{m^2-9}$

(14) $\dfrac{3}{(x-1)^2} - \dfrac{2}{x^2-1} + \dfrac{1}{(1-x)^2} - \dfrac{4}{x+1}$

☺ **If $y + \frac{1}{y} = 3$, calculate, without the use of a calculator, the value of:**

[Hint: Square the given and use factorisation!]

(a) $y^2 + \dfrac{1}{y^2}$ **(b)** $y^3 + \dfrac{1}{y^3}$

A2.4 <u>REVISION EXERCISE</u>:

(1) Simplify the following products:

(a) $-2(b-9)(b+9)$

(b) $(q+3p)^2$

(c) $c^2(c^2-b^2)(c^2+b^2)$

(d) $(2x-5)(4x^2+10x+25)$

(e) $(b^2+9b+9)(b-3)$

(f) $-(3xy+3m)^2$

(g) $\left(\frac{x}{y}-4\right)^2$

(h) $(m-6n)(m^2+36n^2)(m+6n)$

(i) $(pq+rs)(pq-rs)-(pq+rs)^2$

(j) $(3c+5)(2c)(1-2c)$

(k) $\left(\frac{1}{3}m-\frac{1}{2}n\right)^2$

(l) $[(mn+3)(mn-3)]^2$

(m) $\{2y[(x-2y)-4(3y-2x)]\}$

(n) $a^2+3a^2(a+1)-(4a-1)^2+(a^2-7)a-(a-1)(a+1)$

(2) Factorise completely:

(a) $7x^2 - 28y^2$

(b) $x^2 - 5x - xy + 5y$

(c) $q^3 - 2q^2 + q$

(d) $8p^3 + 125$

(e) $4 - m(5 - m)$

(f) $10t^2 + 22t + 4$

(g) $b^8 - 1$

(h) $7(p^4 - q^2) + 3y(p^4 - q^2)$

(i) $2t^2 - \frac{r^2}{2}$

(j) $16b^3 - 432$

(k) $4d^2 - 2(5d + 3)$

(l) $9 - 9(m + n)^2$

(m) $(x + 2y)^2 - 4(x + 2y) - 12$

(n) $(2x + 3y)^2 - (x - y)^2$

(3) Simplify. Indicate all restrictions where necessary!

(a) $\frac{c^2 - 2c - 15}{c^2 - 3c - 10}$

(b) $\frac{p^2 - 4q^2}{5p + 10q} \times \frac{5q + 5p}{7p - 14q}$

(c) $\frac{m}{m + n} - \frac{n}{m - n} - 1$

(d) $\frac{3y^2 + 9y - 30}{3y^2 + 12y} \div \frac{y^2 - 25}{y^2 - y - 20}$

(e) $\frac{2}{a^2 - b^2} - \frac{2}{(a - b)^2}$

(f) $\frac{16y^2 - 49}{2y^2 - y - 1} \div \frac{4y^2 + y - 14}{2y^2 + 5y + 2}$

(g) $\frac{x^2 + xm + xn + mn}{x^2 - xm + m^2} \times \frac{x^4 - x^3m + x^2m^2}{2x + 2n}$

(h) $\left(\frac{x}{y - x} - \frac{x}{y + x}\right) \div \left(\frac{x^2}{x^2 + y^2} - \frac{x^2}{x^2 - y^2}\right)$

(i) $\frac{5m - 11}{m^2 - 5m + 6} + \frac{m - 3}{m - 2} - \frac{2m + 1}{3 - m}$

(j) $\dfrac{2 - \frac{1}{x + 3}}{\frac{4}{x - 1} + 3}$

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